

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

Problem Set No. 11 6.632 Electromagnetic Wave Theory
Spring Term 2003

Reading assignment: Section 5.1 J. A. Kong, “*Electromagnetic Wave Theory*”

Problem P11.1

By the image theorem, a vertical monopole antenna on a conducting plane is equivalent to a dipole with the conductor removed. In radio broadcasting stations, the Earth is used as the conducting plane. Calculate the power and the gain for a monopole on a conducting plane.

Problem P11.2

Let the tangential \overline{E} field and tangential \overline{H} field on an enclosed surface S be related by an impedance matrix $\overline{Z}(\overline{r})$ such that

$$\hat{n} \times \overline{E} = \overline{Z}(\overline{r}) \cdot (\hat{n} \times \overline{H})$$

where \hat{n} is inward normal to the surface S bounding the region of interest V

$$Z_{ij}(\overline{r}) = \hat{s}_i Z_{ij}(\overline{r}) \hat{s}_j$$

and \hat{s}_i and \hat{s}_j with $i, j = 1, 2$ are unit vectors tangential to the surface S such that \hat{s}_1, \hat{s}_2 , and \hat{n} form an orthogonal coordinate system. Show that

$$\begin{aligned} E_1 &= -Z_{21}H_2 + Z_{22}H_1 \\ E_2 &= Z_{11}H_2 - Z_{12}H_1 \end{aligned}$$

Let $\delta\overline{E} = E_1\hat{s}_1 + E_2\hat{s}_2 + E_n\hat{n}$ and $\delta\overline{H} = H_1\hat{s}_1 + H_2\hat{s}_2 + H_n\hat{n}$, then

$$\begin{aligned} \hat{n} \cdot \delta\overline{E} \times \delta\overline{H}^* &= (-Z_{21}H_2H_2^* + Z_{22}H_1H_2^* - Z_{11}H_2H_1^* + Z_{12}H_1H_1^*) \\ \hat{n} \cdot \delta\overline{E}^* \times \delta\overline{H} &= (-Z_{21}^*H_2^*H_2 + Z_{22}^*H_1^*H_2 - Z_{11}^*H_2^*H_1 + Z_{12}^*H_1^*H_1) \end{aligned}$$

Determine the conditions for Z_{ij} such that the uniqueness theorem holds.

Problem P11.3

Consider a plane wave incident on a conducting plate of length a in the \hat{x} direction and width b in the \hat{y} direction. Assume that the plate size is very large and that the field is negligible behind the plate. Solve the scattered field by two approximate source distributions.

- (a) The tangential electric field at the conducting plate is zero. Under the physical optics approximation, the tangential magnetic field at the conducting plate is twice that of the incident field, so that

$$\overline{J}_s = 2\hat{n} \times \overline{H}^i = \hat{x} \frac{2E_0}{\eta}$$

Show that the scattered electric field is

$$\bar{E}_s = (\hat{\phi} \sin \phi - \hat{\theta} \cos \theta \cos \phi) \frac{2ikE_0 e^{ikr}}{\pi r} \frac{\sin(k_x a/2)}{k_x} \frac{\sin(k_y b/2)}{k_y}$$

- (b) The induction theorem makes use of the source distribution for the scattered field, given by $\bar{J}_s = -\hat{n} \times (\bar{H} - \bar{H}_i) = -\hat{n} \times \bar{H}_i$ and $\bar{M}_s = -\hat{n} \times (\bar{E} - \bar{E}_i) = \hat{n} \times \bar{E}_i = \hat{y}E_0$, where \bar{E} and \bar{H} represent the total field at the scatterer. These are impressed currents that radiate in the presence of the conductor. By the image theorem, the source responsible for the scattered field in the absence of the scatterer is $2\bar{M}_s$, since the total field \bar{E} at the conducting plate is zero and \bar{J}_s does not radiate in front of perfect conductors. Show that the radiated electric field vector is

$$\bar{E}_s = (\hat{\phi} \cos \theta \sin \phi - \hat{\theta} \cos \phi) \frac{2ikE_0 e^{ikr}}{\pi r} \frac{\sin(k_x a/2)}{k_x} \frac{\sin(k_y b/2)}{k_y}$$

- (c) Compare the result obtained with the two approaches. Show that the results are identical in the backscattered direction. For a plane wave normally incident on a large plate, the backscattering cross-section, or radar cross-section, or echo area A_e is

$$A_e \equiv \lim_{r \rightarrow \infty} \left(4\pi r^2 \frac{S^s}{S^i} \right) = \frac{k^2 (\text{Area})^2}{\pi}$$

where $S^i = |E_o|^2/2\eta$ and S^s is the scattered power density.

Problem P11.4

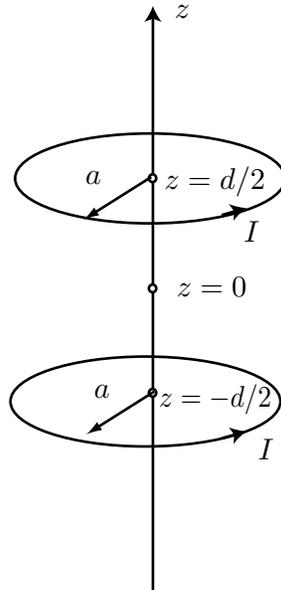


Fig. 10.4

In MRI, it is required to generate a uniform B field in the vicinity of the a certain point. Helmholtz coil, as shown in Figure 2, can realize this. The two identical circular loops of radius a , separated by distance d , carry the same current I in the same direction.

- (a) Show the magnetic field along the z -axis is

$$B_z = \frac{\mu_0 I a^2}{2 [(d/2 - z)^2 + a^2]^{\frac{3}{2}}} + \frac{\mu_0 I a^2}{2 [(d/2 + z)^2 + a^2]^{\frac{3}{2}}}$$

- (b) Show when $d = a$, the B_z is most uniform in the vicinity of $z = 0$, i.e.: $\frac{dB_z}{dz} = 0$ and $\frac{d^2 B_z}{dz^2} = 0$ when $z = 0$.
- (c) plot the $B_z(z)$ when $d = a = 5$ cm and $I = 2$ A for $-\frac{d}{2} \leq z \leq \frac{d}{2}$.