MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

Problem Set No. 2 6.632 Electromagnetic Wave Theory
Spring Term 2003

Reading assignment: Section 3.2, 3.3 J. A. Kong, "*Electromagnetic Wave Theory*"

Problem P2.1

An electromagnetic wave with the following electric field

$$\overline{E} = \hat{x}\sin\left[\frac{k}{\sqrt{2}}(y+z)-\omega t\right] + \frac{1}{\sqrt{2}}\left[A\hat{y}+\hat{z}\right]\cos\left[\frac{k}{\sqrt{2}}(y+z)-\omega t\right]$$

is propagating in a plasma medium characterized by the dispersion relation

$$k = \frac{1}{c}\sqrt{\omega^2 - 4\pi^2\times 10^{12}}$$

where ω is the frequency in rad/sec, and c is the speed of light in free space.

(a) What is the value of A?
(b) In which direction is the wave propagating and what is wave vector $\overline{k}$?
(c) What is the polarization of the wave?
(d) The permeability of the plasma medium is μ_o of free space, what is the permittivity ϵ of the medium in terms of ω and permittivity of free space ϵ_o?
(e) What is the magnetic field vector of the wave ?
(f) What is the Poynting power density vector of the wave ?
(g) Show that the plasma frequency is $f_p = 10^6$ Hz.
(h) If $\omega = \sqrt{5}\pi\times 10^6$ rad/sec, what is k and what are the phase velocity v_p and group velocity v_g?
(i) If $\omega = \sqrt{3}\pi\times 10^6$ rad/sec, what is k and what is the expression for $\overline{E}$?
(j) If $\omega = \sqrt{3}\pi\times 10^6$ rad/sec, what is the time-averaged Poynting power density ?

Problem P2.2

Consider an electromagnetic wave propagating in the $\hat{z}$-direction with

$$\overline{E} = \hat{x}e_x\cos(kz-\omega t+\psi_x) + \hat{y}e_y\cos(kz-\omega t+\psi_y)$$

where e_x, e_y, ψ_x, and ψ_y are all real numbers.

(a) Let $e_x = 2$, $e_y = 1$, $\psi_x = \pi/2$, $\psi_y = \pi/4$. What is the polarization?
(b) Let $e_x = 1$, $e_y = \psi_x = 0$. This is a linearly polarized wave. Prove that it can be expressed as the superposition of a right-hand circularly polarized wave and a left-hand circularly polarized wave.
(c) Let $e_x = 1$, $\psi_x = \pi/4$, $\psi_y = -\pi/4$, $e_y = 1$. This is a circularly polarized wave. Prove that it can be decomposed into two linearly polarized waves.

Problem P2.3

In a uniaxial medium with

$$\overline{H} = \nu\overline{B}, \quad \overline{E} = \overline{\overline{\kappa}}\cdot\overline{D}, \quad \overline{\overline{\kappa}} = \begin{bmatrix}\kappa & 0 & 0\\ 0 & \kappa & 0\\ 0 & 0 & \kappa_z\end{bmatrix}$$

a plane wave propagates in the $\hat{y}$-direction.

(a) Write down the electric field vector of an ordinary wave. What is its spatial frequency k and its speed of propagation?

(b) Write down the electric field vector of an extraordinary wave. What is its spatial frequency k and its speed of propagation?

(c) If κ is real and $\kappa_z = -iK$ is purely imaginary, $K \gg \kappa$, what are the penetration depths of the ordinary and extraordinary waves?