MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

Problem Set No. 6 6.632 Electromagnetic Wave Theory
Spring Term 2003

Reading assignment: Section 3.4, 3.5, J. A. Kong, "*Electromagnetic Wave Theory*"

Problem P6.1

An AM radio in an automobile can not receive any signal when the car is inside a tunnel. Consider, for example, the Lincoln Tunnel under the Hudson River, which was built in 1939. A cross-section of the tunnel is shown in the figure below. In your analysis, ignore the air ducts, assume they are closed. Model the tunnel as a rectangular waveguide of dimension $6.55m \times 4.19m$.

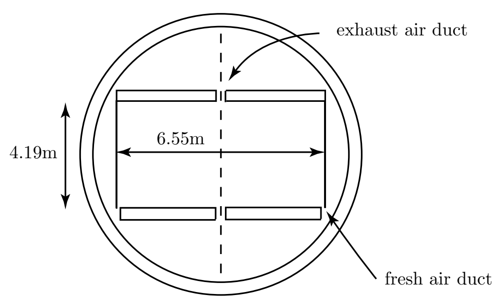

Figure 6.1

(a) Give the range of frequencies for which only the dominant mode, TE_{10}, may propagate.

(b) Explain why AM signals can not be received.

(c) Can FM signals be received? Above what frequencies?

Problem P6.2

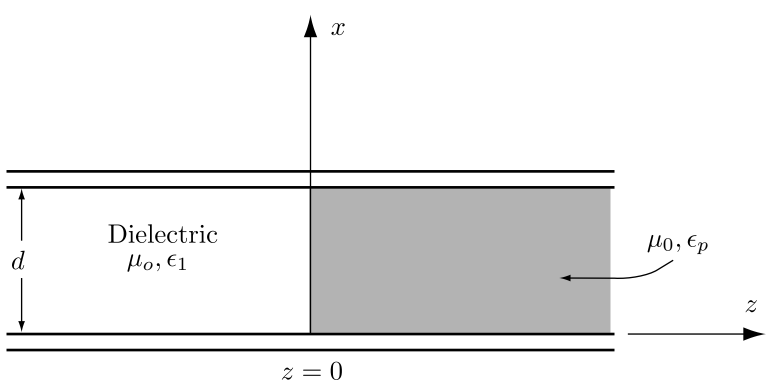

Fig. 6.2

As shown in Fig. 6.2, a parallel-plate waveguide is filled with plasma medium for $z > 0$ and with dielectric medium for $z < 0$. The plasma medium has permittivity $\epsilon_p = \epsilon_0 \left[1 - \omega_p^2/\omega^2\right]$, and the dielectric medium has permittivity $\epsilon_1 = 3\epsilon_0$. Let $d =$

$\sqrt{15}\,\pi/(\omega\sqrt{\mu_0\epsilon_0})$.

(a) Consider the case when $\epsilon_p = \epsilon_1$ (namely the whole waveguide is filled with dielectric). How many propagating TM modes can be guided?

(b) Let $\omega_p = (1/2)\omega$. For waves propagating in the $+z$ direction, which of the above TM modes will be totally reflected at the dielectric-plasma boundary? Why?

(c) One of the above TM modes will be totally transmitted (no reflection). Which one and why?

(d) For a given excitation frequency ω, at what plasma frequency ω_p will all of the above TM modes be totally reflected?

Problem P6.3

Although waves are usually guided by at least two plane interfaces, it is also possible to guide a wave with a single plane interface between two media. Such waves are called surface waves. Field components of a surface wave decay away from the interface exponentially. Consider a plane boundary surface at $z = 0$ separating two media with μ_0, ϵ_0 for $z > 0$ and μ_0, ϵ_p for $z < 0$.

(a) For TE waves with

$$\overline{E} = \hat{y}E_0 e^{ik_x x} \begin{cases} e^{-\alpha_0 z} & z > 0 \\ e^{\alpha_p z} & z < 0 \end{cases}$$

is it possible to have a TE surface wave? If so, find the dispersion relation.

(b) For TM waves with

$$\overline{H} = \hat{y}H_0 e^{ik_x x} \begin{cases} e^{-\alpha_0 z} & z > 0 \\ e^{\alpha_p z} & z < 0 \end{cases}$$

show that it is possible to have TM surface waves only if the permittivities of the upper and lower regions are real and opposite in sign.

Problem P6.4

Explain the following question:

(a) why there is no TE_0 mode in a parellel-plate waveguide?

(b) why there is no TM_{m0} or TM_{0n} mode in a rectangular waveguide?

(c) Is TE_{00} mode in a rectangular waveguide is a propogating mode?