

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

Problem Set No. 7 6.632 Electromagnetic Wave Theory
Spring Term 2003

Reading assignment: Section 3.5, J. A. Kong, “*Electromagnetic Wave Theory*”

Problem P7.1

- (a) Consider a 50Ω polyethylene ($\epsilon = 2.25\epsilon_0$) filled coaxial cable with an inner diameter of 0.81 mm and an outer diameter of 2.946 mm (Figure 7.1). The cutoff frequency of the next higher-order waveguide mode above TEM is:

$$f_c \simeq \frac{v_{TEM}}{\pi(a + b)}$$

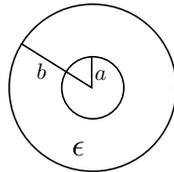


Figure 7.1

where v_{TEM} is the velocity of the TEM waves in the medium that fills the space between the conductors. Calculate the cutoff frequency of the lowest high-order mode of the given coaxial cable.

- (b) In part (a), if the center conductor is removed and replaced by polyethylene, what is the lowest propagating mode? What is cutoff frequency of the lowest propagating mode? What is the next higher order propagating mode and its cutoff frequency? What is the single-mode operating frequency range of the dielectric-filled circular metallic waveguide?

Problem P7.2

Consider an air-filled metallic rectangular waveguide operating at frequency $f = 7.5$ GHz ($k = 50\pi \text{ m}^{-1}$) with dimensions $a = 3$ cm and $b < a$.

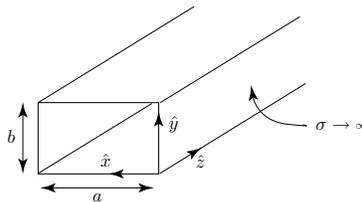


Figure 7.2.1

- (a) For the fundamental mode (TE_{10} mode), determine
- (i) the cutoff frequency
 - (ii) the propagation constant k_z
 - (iii) the phase velocity
- (b) Consider a propagating mode inside the waveguide operating at the same frequency $f = 7.5$ GHz ($k = 50\pi \text{ m}^{-1}$). The plots of E versus time t at $z = 0$ and $z = 1$ cm are shown below.

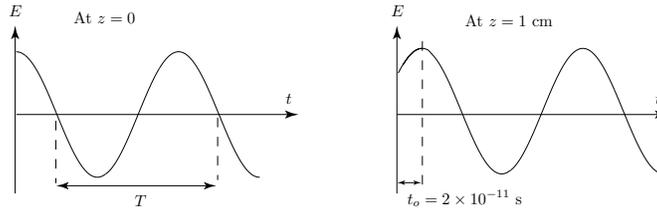
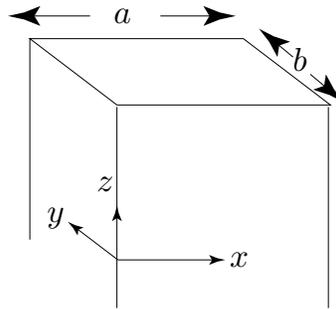


Figure 7.2.2

- (i) Determine the period T .
- (ii) Calculate the phase velocity.
- (iii) Show that $k_z = 30\pi \text{ m}^{-1}$.
- (iv) Find the mode designation (i.e., TE_{mn} , TM_{mn}) for the propagating mode in the waveguide by assigning numerical values to m and n .
- (v) Calculate the length b in cm.

Problem P7.3



Consider a metallic rectangular waveguide with the longer dimension $a = 1.5\text{cm}$.

- (a) What is the fundamental mode? What is the frequency range for single mode operation? What is the limitation on b in order to maximize the bandwidth?
- (b) Let the electric field in the waveguide,

$$\vec{E} = \hat{y}E_0 \sin k_x x e^{\pm ik_z z}$$

be the fundamental mode operating at 15GHz.

- (i) What is the group velocity v_g , and phase velocity v_p ?
- (ii) Find the current distribution $\vec{J}_s(x)$ at $z = 0$.
- (iii) The breakdown voltage of air is 30kV/cm, find the maximum time averaged power deliverable by the waveguide at 15GHz in the positive z -direction, while maintaining maximum single mode bandwidth. What is b ?