

MASSACHUSETTS INSTITUTE of TECHNOLOGY  
Department of Electrical Engineering and Computer Science

Problem Set No. 3  
Spring Term, 2003

6.637 Optical Signals, Devices and Systems

Issued Tues. 2/25/2003  
Due Tues. 3/4/2003

**Reading recommendation:** 6.637 Class Notes, Chapter 4;

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**Problem 3.1**

In the coordinate system below, the following definitions hold:

$$\bar{\rho} = x\hat{x} + y\hat{y}$$

$$\bar{r} = x\hat{x} + y\hat{y} + z\hat{z}$$

$$\bar{k} = k_x\hat{x} + k_y\hat{y} + k_z\hat{z}$$

Show the direction of propagation, and algebraically derive, sketch and describe the shape of the wavefront associated with the following elementary unit amplitude waves:

(a)  $\underline{U}(\bar{r}) = e^{j\bar{k}\cdot\bar{r}}$

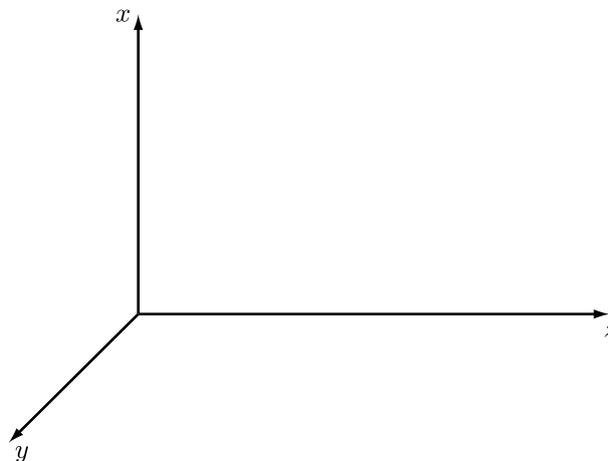
(b)  $\underline{U}(\bar{r}) = e^{jkz}e^{-j\frac{k\rho^2}{2F}}$

(c)  $\underline{U}(\bar{r}) = e^{j\frac{k}{2z}(x^2+y^2)}$

(d)  $\underline{U}(\bar{r}) = e^{j\frac{k}{2z}[(x-x_0)^2+(y-y_0)^2]}$

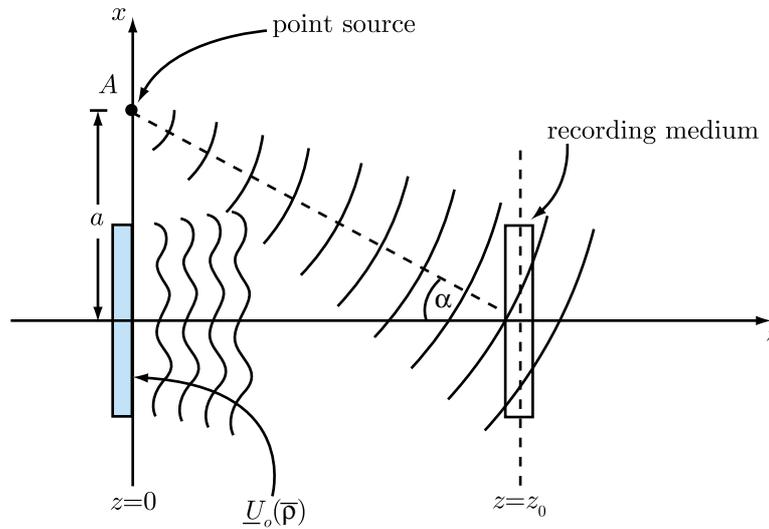
(e)  $\underline{U}(\bar{r}) = e^{jk(z^2+x^2+y^2)^{1/2}}$

(f)  $\underline{U}(\bar{r}) = e^{jk[z^2+(x-x_0)^2+(y-y_0)^2]^{1/2}}$



**Problem 3.2**

A hologram is made with waves derived from an on-axis object  $\underline{U}_o(\vec{\rho})$  located in the  $z = 0$  plane and a point source of amplitude  $A$  located on the  $x$ -axis at a distance  $a$  above the principal axis as shown.



- Write an expression for the object wave,  $\underline{U}_o(\vec{\rho}, z_0)$ , at the recording medium.
- Write an expression for the spherical reference wave,  $\underline{U}_r(\vec{\rho}, z_0)$ , at the recording medium.
- Assuming the recording medium has a transmitted amplitude response that is proportional to exposure, write an expression for the amplitude transmittance of the resulting hologram.
- The hologram is to be read out with the conjugate of the reference wave. Write an expression for the conjugate reference wave at the  $z = z_0$  plane and draw a diagram to show the wavefronts of the conjugate reference wave reading out the hologram.
- Compute the output terms corresponding to the waves generated by the hologram when read out with the conjugate reference beam, and illustrate these on the diagram you made in part (e).

**Problem 3.3**

Describe with the aid of diagrams (and equations if necessary) how you would make a phase-compensating hologram, which when placed a distance  $d$  from a pane of frosted shower glass would allow you to see through the shower glass pane.

