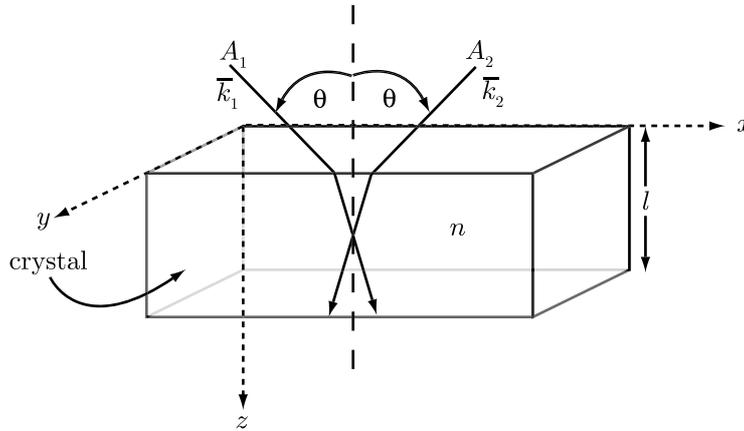


Reading recommendation: 6.637 Class Notes, Chapter 5;

Problem 4.1

Two mutually coherent TE-polarized plane-wave light beams of unequal amplitudes (A_1 and A_2) traveling in the x - z plane intersect in a photorefractive crystal at an angle 2θ as shown. The wavelength of the two writing beams is λ_w , the unperturbed refractive index of the crystal is n_0 , and the thickness of the crystal is l in the z -direction. Assume the writing beams overfill the crystal.



- (a) Derive an expression for the intensity fringe spacing, Λ , within the photorefractive crystal.
- (b) Let us assume that before saturation occurs, the local number of mobile charges generated per unit volume, $N(x)$, is proportional to the local exposure, $\mathcal{E}(x)$, write an expression for $N(x)$.
- (c) Assuming that the more mobile negative charges move by diffusion (motion owing to concentration gradients) towards the least bright regions within the crystal, write an expression for the resulting space-charge density, $\rho_{sc}(x)$.
- (d) Using the result from part (c) write an expression for the associated space-charge electric field, $E_{sc}(x)$.
- (e) Further assume that the electric-field induced refractive index change in the crystal is given by

$$\Delta n(x) = -\frac{1}{2}n_0^3 r E_{sc}(x)$$

where r is the electro-optic coefficient. Make a plot of $I(x)$, $\rho_{sc}(x)$, $E_{sc}(x)$ and $\Delta n(x)$ all on the same graph, one below the other.

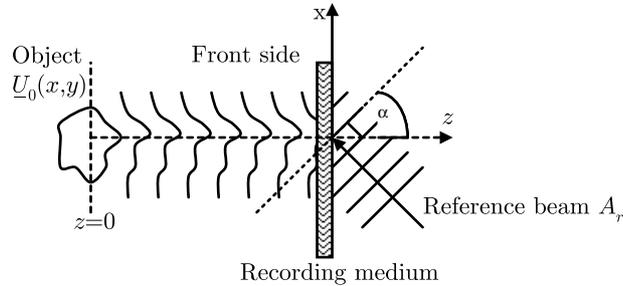
- (f) Consider the case where the crystal can be considered a “thin” grating, and is read out with a plane wave of wavelength λ_r and amplitude A_r propagating along the $+z$ axis:
 - (1) Write an expression for the transmitted wave at the output face of the crystal.
 - (2) What is the photorefractively-induced phase change, $\Delta\phi(x)$, that the crystal imparts to the readout beam?
 - (3) What is the intensity of the light that is diffracted into the m th order?

- (g) Now consider the case where the crystal is thick, $\lambda_r > \lambda_w$, and the crystal is readout at the Bragg angle. What is this angle? What is the expected first-order diffraction efficiency?

Problem 4.2

Thick reflection holograms are used for decorative purposes on credit cards, and are also worn (usually around the neck) as decorative jewelry. Large reflection holograms can also be found as art media in museums (e.g., MIT Museum).

A “thick” reflection hologram of an object $\underline{U}_0(x, y)$ is made in a recording medium of refractive index n using the setup shown below. The writing light has wavelength λ_w . The plane-wave reference beam has amplitude A_r and is incident from below in the $x - z$ plane at an angle α .



- (a) For the case where the recording medium is thick, draw and describe the location of the images when the hologram is read out with the conjugate of the reference beam.
- (b) Next, the thick hologram is read out in an optimal way (Bragg matched) with a plane wave of wavelength $3\lambda_w/2$ also incident from above on the front side.
 - (1) What is the optimal angle, ψ , between the z -axis and this readout beam?
 - (2) Describe the characteristics of the images produced, and draw a diagram of this optical readout configuration showing the location of the images.
- (c) Using the results from part (b), describe the output images that are obtained when this hologram is read out with collimated white light incident from above on the front side.

Problem 4.3

A carbon-dioxide laser beam of wavelength $10.6\mu\text{m}$ is incident on a plasma containing 10^{15} electrons/ m^3 .

- (a) What is the plasma frequency?
- (b) What is the refractive index of this plasma?
- (c) Does the laser penetrate the plasma? Why?
- (d) What minimum density must the plasma have in order for the laser to be totally reflected by the plasma? Is such a density possible?