MASSACHUSETTS INSTITUTE of TECHNOLOGY
Department of Electrical Engineering and Computer Science

Problem Set No. 9 | *6.637 Optical Signals, Devices and Systems* | Issued Thurs. 4/24/2003
Spring Term, 2003 | Due Thurs. 5/1/2003

Reading recommendation: 6.637 Class Notes

Problem 9.1

In the classical two-lens coherent optical processor with lenses of focal length F, two signal transparencies of transmittance $g_a(x_1, y_1)$ and $g_b(x_1, y_1)$ are inserted in the $\bar{\rho}_1$ (input) plane with centers at $(a, 0, 0)$ and $(-a, 0, 0)$ respectively. The Fourier-plane filter $\underline{t}_h(\bar{\rho})$ is a sinusoidal grating with amplitude transmittance

$$\underline{t}_h(\bar{\rho}) = \underline{t}_h(F\lambda\,\bar{f}) = \frac{1}{2} + \frac{1}{2}\sin(2\pi\, f_g x + \phi)$$

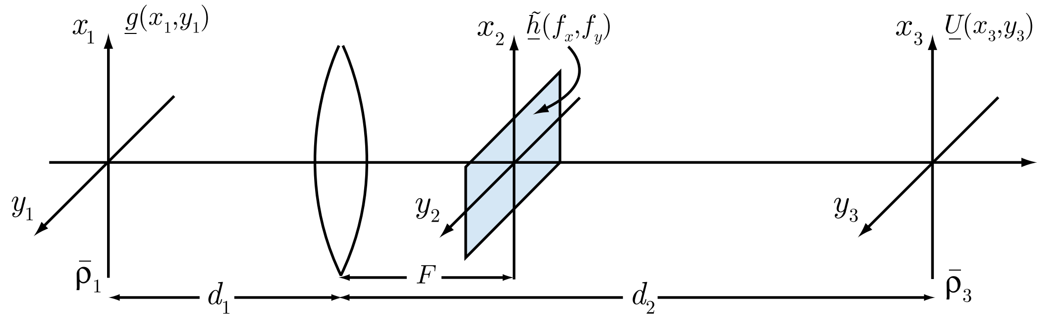
where f_g and ϕ represent the grating frequency and position respectively.

(a) Calculate the amplitude distribution at the output plane of the processor.

(b) For the special case where f_g= a/λF and $\phi = 0$, comment on the significance of the output. Repeat the process for $\phi = 90^\circ$.

Problem 9.2

The system shown below has been proposed as a scheme for performing single-lens convolution of two signals $\underline{g}(x, y)$ and $\underline{h}(x, y)$ when these signals are positioned as shown.



(a) Compute the output signal $\underline{U}(x_3, y_3)$ for the case where $\overline{\rho}_3$ is the image plane of $\overline{\rho}_1$ with $d_1 > F$, the focal length of the lens. [Hint: use $1/d_1 + 1/d_2 = 1/F$ (the imaging condition of the system) to simplify your expressions].

(b) From part (a), show that the signal in the $\overline{\rho}_3$ plane is scaled by d_2/d_1 relative to the signal in the $\overline{\rho}_1$ plane.

(c) For (1) $d_1 \neq d_2$ and (2) $d_1 = d_2 = 2F$, comment of the differences between $\underline{U}(x_3, y_3)$ and the desired ideal convolution operation.

Problem 9.3

The Mellin Transform has been proposed as an alternative to the Fourier transform for applications involving correlation detection and matched filtering with coherent light. The Mellin transform is defined by

$$\hat{g}(f_x, f_y) = \int_0^\infty \int_0^\infty g(x,y) x^{-(j2\pi f_x + 1)} y^{-(j2\pi f_y + 1)} dx dy$$

where the ˆ is a label denoting Mellin transformation.

(a) Show that the Mellin transform and the Fourier transform are related by

$$\hat{g}(f_x, f_y) = M\{g(x,y)\} = \mathcal{F}\{g(e^x, e^y)\}$$

(b) The basic advantage of the Mellin transform over the Fourier transform is the scale-invariant property of its magnitude. Show that indeed the Mellin transform has this property.

(c) It is known that the magnitude of the Fourier transform is shift-invariant. Derive the shift property of the Mellin transform, comment on its invariance, and compare it with that of the Fourier transform.

(d) A computer-generated optical Mellin transform transparency with transmittance $\underline{t}_M(x,y) = \hat{g}(f_x, f_y)$ is imaged with unity magnification onto a recording medium and interferometrically combined with an off-axis plane wave reference beam to synthesize a complex spatial filter, $\underline{h}(f_x, f_y)$, as shown below. Write down an expression for the transmittance of the filter.

(e) This filter is now placed in the Fourier plane of a coherent two-lens optical processor. If $\underline{U}_1(\overline{\rho}_1)$ is the input signal to this processor, calculate the resultant amplitudes of the terms in the output plane of the processor and comment of the significance of each term.