6.642 Continuum Electromechanics
Fall 2008

Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
6.642 Continuum Electromechanics

Problem Set #4 — Issued: 9/24/08
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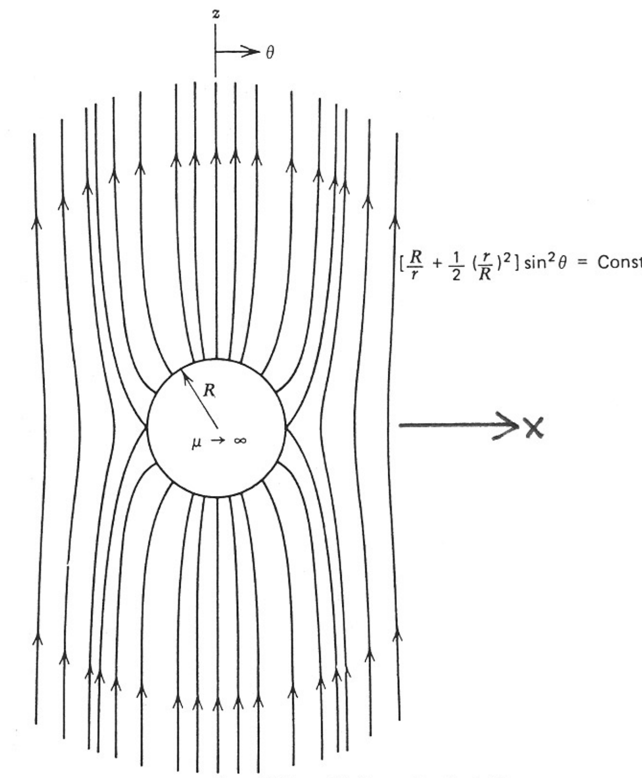
Problem 1

A sphere of radius R and infinite magnetic permeability is placed within a uniform magnetic field at infinity,

$$\bar{H}(r \to \infty, \theta) = H_0 \bar{i}_z = H_0(\bar{i}_r \cos\theta - \bar{i}_\theta \sin\theta).$$

The medium outside the sphere is free space.

a) Find the magnetic flux density $\bar{B}(r,\theta)$ for $r > R$.
b) Find the equation of the magnetic field lines
$$\frac{dr}{rd\theta} = \frac{B_r}{B_\theta}$$
c) Find the vector potential $\bar{A}(r,\theta)$.
d) For $r > R$, the governing equation for the vector potential is $\nabla^2 \bar{A} = 0$. Show that the solution of part (c) satisfies $\nabla^2 \bar{A} = 0$.
e) For the separation magnetic field line that passes through the point $(r = R, \theta = \pi/2)$ find its distance D from $x = 0$ at $(r \to \infty, \theta = \pi)$.
f) Draw the magnetic field lines similar to those shown above.



(a) $H_0 \mathrm{i}_z = H_0(\mathrm{i}_r \cos\theta - \mathrm{i}_\theta \sin\theta)$

Problem 2

The Kelvin force density for charged dielectric media is

$$\bar{F} = (\bar{P}\bullet\nabla)\bar{E} + \rho_f \bar{E}$$

where $\bar{P} = \bar{D} - \varepsilon_0 \bar{E}$ is the polarization field, $\bar{E}$ is the electric field, $\bar{D}$ is the displacement field, and $\rho_f = \nabla\bullet\bar{D}$ is the free charge density. Do not assume that the dielectric has a linear permittivity. Find the stress tensor T_{ij} for this force density in the form

$$F_i = \frac{\partial T_{ij}}{\partial x_j}$$

Problem 3

The Kelvin force density for current carrying magnetic media with magnetization $\bar{M}$ is

$$\bar{F} = \mu_0(\bar{M}\bullet\nabla)\bar{H} + \bar{J}\times\mu_0\bar{H}$$

where $\bar{M} = \dfrac{\bar{B}}{\mu_0} - \bar{H}$ is the magnetization field, $\bar{B}$ is the magnetic flux density, $\bar{H}$ is the magnetic field intensity, and $\bar{J} = \nabla\times\bar{H}$ is the current density. Do not assume that the magnetic media has a linear magnetic permeability. Find the stress tensor T_{ij} for this force density in the form

$$F_i = \frac{\partial T_{ij}}{\partial x_j}$$