6.642 Continuum Electromechanics
Fall 2008

Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
6.642 Continuum Electromechanics

Problem Set #6 Issued: 10/08/08
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Problem 1

Prob. 8.10.1 (Melcher, Continuum Electromechanics)

Problem 2

Prob. 8.12.1 (Melcher, Continuum Electromechanics)

For Section 8.10:

Prob. 8.10.1 A planar layer of insulating liquid having a mass density ρ_s has the equilibrium thickness d. The layer separates infinite half-spaces of perfectly conducting liquid, each half-space having the same mass density ρ. The interfaces between insulating and conducting liquids each have a surface tension γ, but ρ_s is sufficiently close to ρ so that gravity effects can be ignored. Voltage applied between the conducting fluids results in an electric field in the insulating layer. In static equilibrium, this field is E_o. Determine the dispersion equations for kinking and sausage modes on the interfaces. Show that in the long-wave limit kd << 1, the effect of the field on the kinking motions is described by a voltage-dependent surface tension. In this long-wave limit, what is the condition for incipient instability?

For Section 8.11:

Prob. 8.11.1 A vertical wire carries a current I so that there is a surrounding magnetic field

$$\vec{H} = \vec{i}_\theta H_o(R/r), \quad H_o \equiv I/2\pi R$$

(a) In the absence of gravity, a static equilibrium exists in which a ferrofluid having permeability μ forms a column of radius R coaxial with the wire. (The equilibrium shown in Fig. 8.3.2b approaches this circular cylindrical geometry.) Show that conditions for a static equilibrium are satisfied.

(b) Assume that the wire is so thin that its presence has a negligible effect on the fluid mechanics and on the magnetic field. The ferrofluid has a surface tension γ and a mass density much greater than that of the surrounding medium. Find the dispersion equation for perturbations from this equilibrium.

(c) Show that the equilibrium is stable provided the magnetic field is large enough to prevent capillary instability. How large must H_o be made for the equilibrium to be stable?

(d) To generate a significant magnetic field using an isolated wire requires a substantial current. A configuration that makes it easy to demonstrate the electromechanics takes advantage of the magnet from a conventional loudspeaker. A cross section of such a magnet is shown in Fig. P8.11.1. In the region above the magnet, the fringing field has the form H_oR/r. Ferrofluid placed over the gap will form an equilibrium figure that is roughly hemispherical with radius R. Viewed from the top, each half-cylindrical segment of the hemisphere closes on itself with a total length ℓ. For present purposes, the curvature introduced by this closure is ignored so that the axial distance is approximated by z with the understanding that z = 0 and z = ℓ are the same position. Effects of surface tension and gravity are ignored. Argue that the m = 0 mode represented by the dispersion equation from (b) is mechanically and magnetically consistent with this revised configuration.

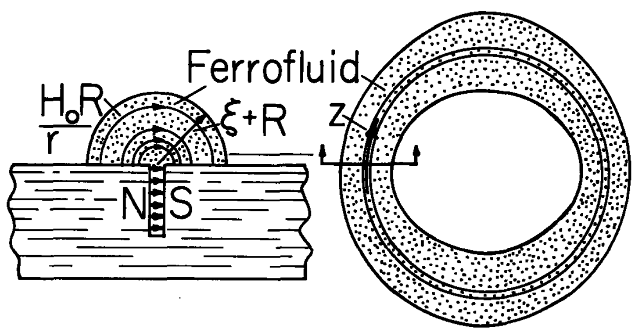

Fig. P8.11.1

(e) Show that, in the long-wave limit kR << 1, the m = 0 waves that propagate in the z direction (around the closed loop of ferrofluid) do so without dispersion. What is the dispersion equation?

(f) One way to observe these waves exploits the fact that the fluid is closed in the z direction, and therefore displays resonances. Again using the long-wave approximations, what are the resonant frequencies? How would you excite these modes?

For Section 8.12:

Prob. 8.12.1 The planar analog of the axial pinch is the sheet pinch shown in Fig. P8.12.1. A layer of perfectly conducting fluid (which models a plasma as an incompressible inviscid fluid), is in equilibrium with planar interfaces at x = $\pm$ d/2. At distances a to the left and right of the interfaces are perfectly conducting electrodes that provide a return path for surface currents which pass vertically through the fluid interfaces. The equilibrium magnetic field intensity to right and left is H_o, directed as shown. Regions a and b are occupied by fluids having negligible density.

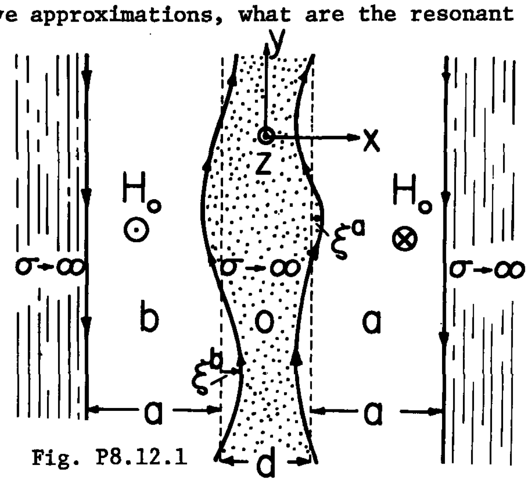

Fig. P8.12.1

Prob. 8.12.1 (continued)

(a) Determine the equilibrium difference in pressure between the regions a and b and the fluid o.

(b) Show that deflections of the interfaces can be divided into kink modes $[\xi^a(y,z,t) = \xi^b(y,z,t)]$, and sausage modes $[\xi^a(y,z,t) = -\xi^b(y,z,t)]$.

(c) Show that the dispersion equation for the kink modes is*, with $k \equiv \sqrt{k_y^2 + k_z^2}$,

$$\frac{\rho\omega^2}{k}\tanh\left(\frac{kd}{2}\right) = \mu_o H_o^2 \frac{k_z^2}{k}\coth(ka)$$

while the dispersion equation for the sausage modes is

$$\frac{\rho\omega^2}{k}\coth\left(\frac{kd}{2}\right) = \mu_o H_o^2 \frac{k_z^2}{k}\coth(ka)$$

(d) Is the equilibrium, as modeled, stable? The same conclusion should follow from both the analytical results and intuitive arguments.

Prob. 8.12.2 At equilibrium, a perfectly conducting fluid (plasma) occupies the annular region $R < r < a$ (Fig. P8.12.2.) It is bounded on the outside by a rigid wall at $r = a$ and on the inside by free space. Coaxial with the annulus is a "perfectly" conducting rod of radius b. Current passing in the z direction on this inner rod is returned on the plasma interface in the -z direction. Hence, so long as the interface is in equilibrium, the magnetic field in the free-space annulus $b < r < R$ is

$$\vec{H} = H_o \frac{R}{r}\vec{i}_\theta$$

(a) Define the pressure in the region occupied by the magnetic field as zero. What is the equilibrium pressure Π in the plasma?

(b) Find the dispersion equation for small-amplitude perturbations of the fluid interface. (Write the equation in terms of the functions $F(\alpha,\beta)$ and $G(\alpha,\beta)$.)

(c) Show that the equilibrium is stable.

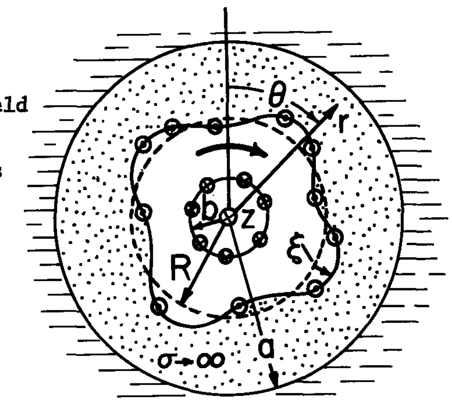

Fig. P8.12.2

Prob. 8.12.3 A "perfectly" conducting incompressible inviscid liquid layer rests on a rigid support at $x = -b$ and has a free surface at $x = \xi$. At a distance a above the equilibrium interface $\xi=0$ is a thin conducting sheet having surface conductivity σ_s. This sheet is backed by "infinitely" permeable material. The sheet and backing move in the y direction with the imposed velocity U. With the liquid in static equilibrium, there is a surface current $K_z = -H_o$ in the conducting sheet that is returned on the interface of the liquid. Thus, there is an equilibrium magnetic field intensity $\vec{H} = H_o\vec{i}_y$ in the gap between liquid and sheet. Include in the model gravity acting in the -x direction and surface tension. Determine the dispersion equation for temporal or spatial modes.

Prob. 8.12.4 In the pinch configuration of Fig. 8.12.1, the wall at r=a consists of a thin conducting shell of surface conductivity σ_s (as described in Sec. 6.3) surrounded by free space.

(a) Find the dispersion equation for the plasma column coupled to this lossy wall.

(b) Suppose that the frequencies of modes have been found under the assumption that the wall is perfectly conducting. Under what condition would these frequencies be valid for the wall of finite conductivity?

(c) Now suppose that the wall is very lossy. Show that the dispersion equation reduces to a quadratic expression in $(j\omega)$ and show that the wall tends to induce damping.

For Section 8.13:

Prob. 8.13.1 A cylindrical column of liquid, perhaps water, of equilibrium radius R, moves with uniform equilibrium velocity U in the z direction, as shown in Fig. P8.13.1. A coaxial cylindrical electrode is used to impose a radially symmetric electric field intensity

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* $\coth kd - \frac{1}{\sinh kd} \equiv \tanh\left(\frac{kd}{2}\right)$; $\coth kd + \frac{1}{\sinh kd} \equiv \coth\left(\frac{kd}{2}\right)$