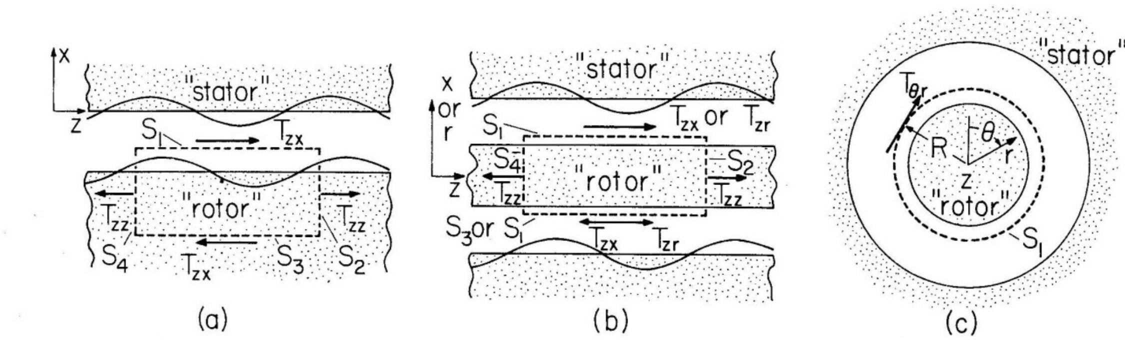
6.642 Continuum Electromechanics
Fall 2008

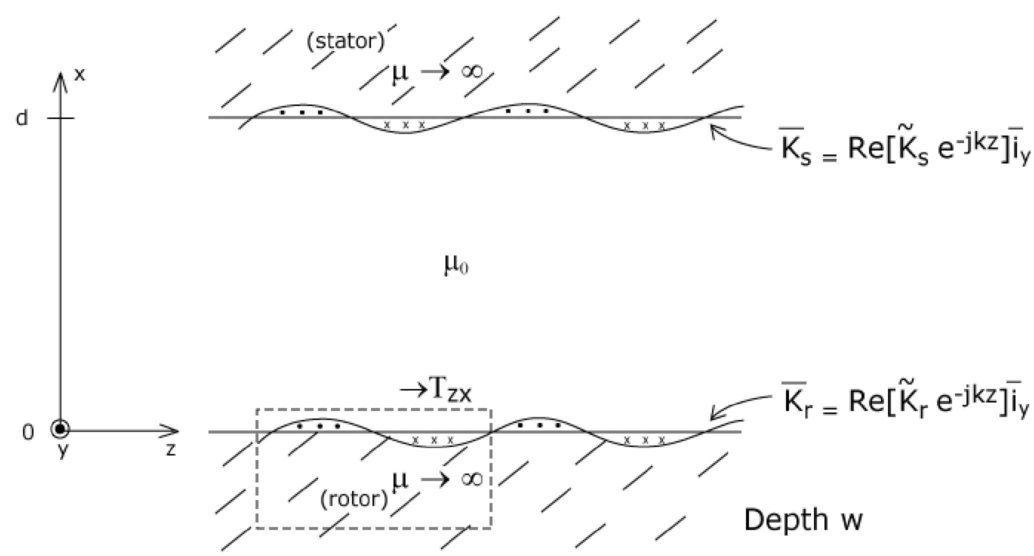
Lecture 3: Continuum Electromechanics (Melcher) – Section 4.1-4.3

I. Air-Gap Magnetic Machines



Typical "air-gap" configurations in which a force or torque on a rigid "rotor" results from spatially periodic sources interacting with spatially periodic excitations on a rigid "stator." Because of the periodicity, the force or torque can be represented in terms of the electric or magnetic stress acting at the air-gap surfaces S_1: (a) planar geometry or developed model; (b) planar or cylindrical beam; (c) cylindrical rotor.

A. Generalized Description



B.C.: $\tilde{H}_z(x=\Delta) = \tilde{K}^s$

$\tilde{H}_z(x=0) = -\tilde{K}_r$

$$f_z = \oint_S T_{zx} n_x dz dy = w \int_0^{2\pi/k} \mu_0 H_z H_x \Big|_{x=0} dz = w \int_0^{2\pi/k} \mu_0 H_z^r H_x^r dz$$

force on a wavelength

$$a(z) = \mathrm{Re}\left[\tilde{A} e^{-jkz}\right]$$

$$b(z) = \mathrm{Re}\left[\tilde{B} e^{-jkz}\right]$$

$$\frac{k}{2\pi} \int_0^{2\pi/k} a(z)\, b(z)\, dz = \frac{1}{2} \mathrm{Re}\left[\tilde{A}\tilde{B}^*\right] = \frac{1}{2} \mathrm{Re}\left[\tilde{A}^* \tilde{B}\right]$$

$$f_z = \frac{2\pi w}{k} \frac{\mu_0}{2} \mathrm{Re}\left[\tilde{H}_z^r \; \tilde{H}_x^{r\,*}\right]$$

$$= \frac{\pi w \mu_0}{k} \mathrm{Re}\left[-\tilde{K}_r \; \tilde{H}_x^{r\,*}\right]$$

$$\begin{bmatrix} \tilde{B}_x^s \\ \tilde{B}_x^r \end{bmatrix} = \mu_0 k \begin{bmatrix} -\coth kd & \dfrac{1}{\sinh kd} \\ -\dfrac{1}{\sinh kd} & \coth kd \end{bmatrix} \begin{bmatrix} \tilde{\chi}^s \\ \tilde{\chi}^r \end{bmatrix}$$

$$\tilde{H}_z = +jk\tilde{\chi} \Rightarrow \tilde{\chi}^s = \frac{1}{jk} \tilde{H}_z^s = \frac{\tilde{K}^s}{jk}$$

$$\tilde{\chi}^r = \frac{\tilde{H}_z^r}{jk} = -\frac{\tilde{K}_r}{jk}$$

$$\mu_0 \tilde{H}_x^r = \mu_0 k \left[\frac{-\tilde{\chi}^s}{\sinh kd} + \tilde{\chi}^r \coth kd\right]$$

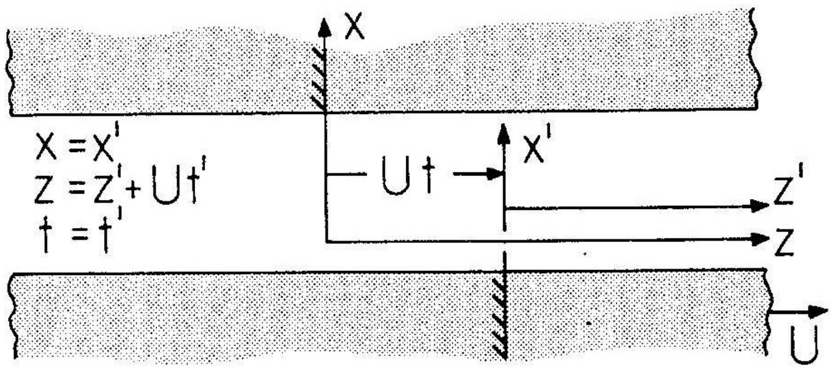
$$= \mu_0 k \left[\frac{-\tilde{K}^s}{jk \sinh kd} - \frac{\tilde{K}^r}{jk} \coth kd\right]$$

$$\mathrm{Re}\left[-\tilde{K}_r^* \tilde{H}_x^r \mu_0\right] = -\mathrm{Re}\left[+j\mu_0 \left(\frac{\tilde{K}_r^* \tilde{K}^s}{\sinh kd} + \tilde{K}_r^* \tilde{K}_r \coth kd\right)\right]$$

$$= \mathrm{Re}\left[-\mu_0 j \tilde{K}_r^* \tilde{K}^s \big/ \sinh kd\right]$$

$$f_z = -\frac{\pi w}{k} \frac{\mu_0}{\sinh kd} \mathrm{Re}\left[j \tilde{K}_r^* \tilde{K}_s\right] \text{ (force on each wavelength)}$$

B. Synchronous Interaction



Rotor and stator reference frames z' and z.

$$K^s = K_0^s \sin\left[\omega_s t - kz\right] = \text{Re}\left[-jK_0^s e^{j(\omega_s t - kz)}\right]$$

$$\begin{aligned} K^r &= K_0^r \sin\left[\omega_r t - k\left(z' - \delta\right)\right];\ z' = z - Ut \\ &= K_0^r \sin\left[\left(\omega_r + kU\right)t - k\left(z - \delta\right)\right] \\ &= \text{Re}\left[-jK_0^r e^{j(\omega_r + kU)t} e^{-jk(z-\delta)}\right] \end{aligned}$$

$$\tilde{K}^s = -jK_0^s e^{j\omega_s t}$$

$$\tilde{K}^r = -jK_0^r e^{jk\delta} e^{j(\omega_r + kU)t}$$

$$\begin{aligned} f_z &= -\frac{\pi W}{k}\frac{\mu_0}{\sinh kd}\text{Re}\left[j\left(-jK_0^s\right)e^{j\omega_s t}\left(jK_0^r e^{-jk\delta}\right)e^{-j(\omega_r + kU)t}\right] \\ &= -\frac{\pi W}{k}\frac{\mu_0}{\sinh kd}K_0^s K_0^r \text{Re}\left[je^{-jk\delta}e^{j(\omega_s - \omega_r - kU)t}\right] \end{aligned}$$

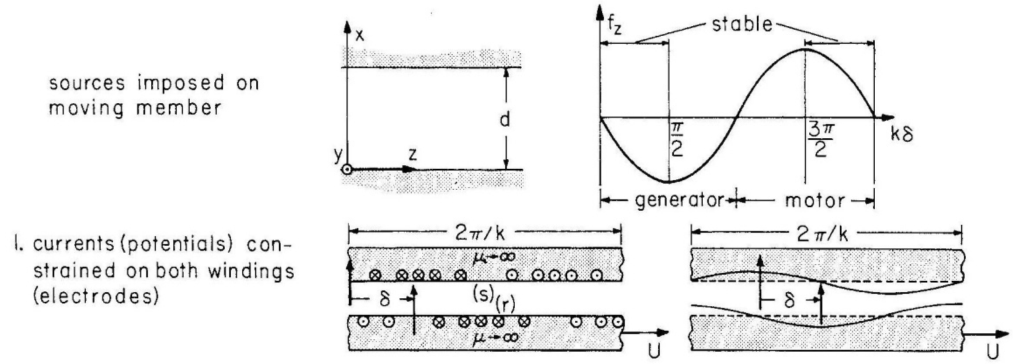
For time average force

$\Rightarrow \omega_s = \omega_r + kU$ (synchronous condition)

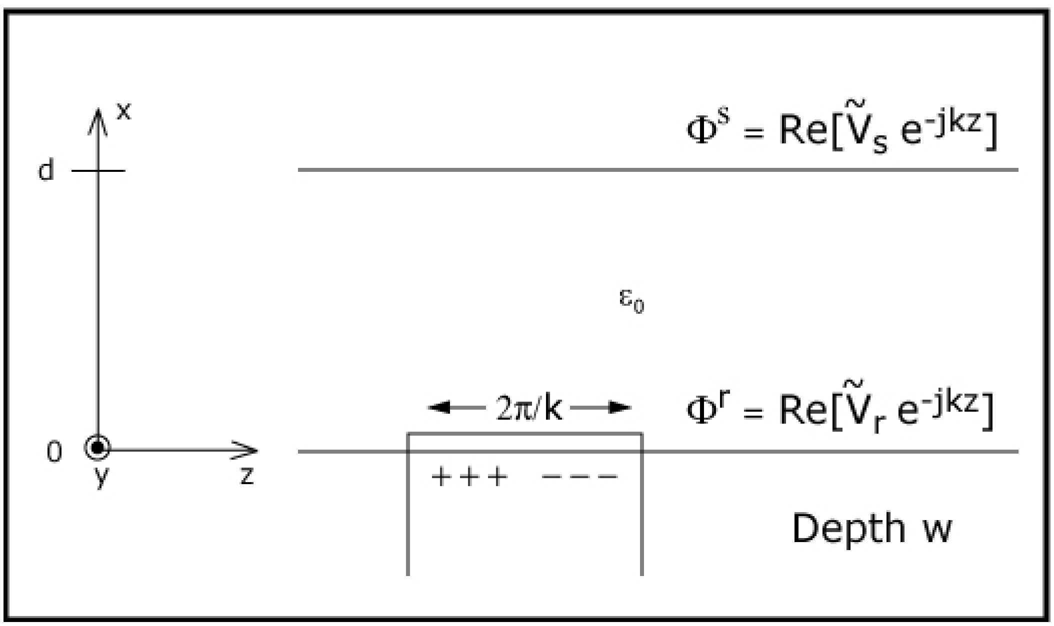
Usually $\omega_r = 0 \Rightarrow \omega_s = kU$

$$\left\langle f_z \right\rangle = -\frac{\pi W}{k}\frac{\mu_0}{\sinh kd}K_0^s K_0^r \sin k\delta$$

Basic configurations illustrating classes of electromechanical interactions and devices. MQS and EQS systems respectively in left and right columns.



II. <u>Electrostatic Machine</u>



$$f_z = w\frac{2\pi}{k}\int_0^{2\pi/k} T_{zx}\Big|_{x=0} dz = \frac{2\pi w}{k}\int_0^{2\pi/k} \varepsilon_0 E_z E_x\Big|_{x=0} dz$$

$$\tilde{E}_z^r = jk\tilde{V}_r$$

$$f_z = \frac{\pi w}{k} Re\left[\varepsilon_0 \tilde{E}_z^{r\,*} \tilde{E}_x^r\right]$$

$$= \frac{\pi w}{k} Re\left[\varepsilon_0 \left(-jk\tilde{V}_r^*\right)\tilde{E}_x^r\right]$$

$$\begin{bmatrix} \tilde{D}_x^s \\ \tilde{D}_x^r \end{bmatrix} = \varepsilon_0 k \begin{bmatrix} -\coth kd & \dfrac{1}{\sinh kd} \\ -\dfrac{1}{\sinh kd} & \coth kd \end{bmatrix} \begin{bmatrix} \tilde{V}_s \\ \tilde{V}_r \end{bmatrix}$$

$$\varepsilon_0 \tilde{E}_x^r = \varepsilon_0 k \left[\frac{-\tilde{V}_s}{\sinh kd} + \tilde{V}_r \coth kd \right]$$

$$\mathrm{Re}\left[-jk\varepsilon_0 \tilde{V}_r^* \tilde{E}_x^r\right] = \mathrm{Re}\left[-jk^2\varepsilon_0 \tilde{V}_r^* \left(\frac{-\tilde{V}_s}{\sinh kd} + \tilde{V}_r \coth kd\right)\right]$$

$$= \mathrm{Re}\left[+jk^2\varepsilon_0 \tilde{V}_s \tilde{V}_r^* \big/ \sinh kd\right]$$

$$f_z = \pi w \frac{k\varepsilon_0}{\sinh kd} \mathrm{Re}\left[j\tilde{V}_s \tilde{V}_r^*\right]$$

$$V_s = V_0^s \cos\left(\omega_s t - kz\right)$$

$$V_r = -V_0^r \cos\left(\omega_r t - k\left(z' - \delta\right)\right);\ z' = z - Ut$$

$$\tilde{V}^r = -V_0^r e^{j(\omega_r + kU)t} e^{jk\delta}$$

$$\tilde{V}^s = V_0^s e^{j\omega_s t}$$

$$\langle f_z \rangle = \frac{\pi w k \varepsilon_0}{\sinh kd} \mathrm{Re}\left[-j\, V_0^s\ V_0^r\ e^{-jk\delta}\ e^{j(\omega_s - \omega_r - kU)t}\right]$$

$$\omega_s = \omega_r + kU$$

$$\langle f_z \rangle = -\frac{\pi w k \varepsilon_0}{\sinh kd} V_0^s\ V_0^r \sin k\delta$$