MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

Receivers, Antennas, and Signals – 6.661

Solutions to Problem Set 10 Due: 4/24/03

Problem 10.1

The received power must support the desired output SNR, which is 20 + 40 dB ($\Rightarrow 10^6$). But for SSBSC, which performs the same as DSBSC, the necessary S/N_{out} is $<s^2(t)>(P_c/2N_oW)$ where $N_o = kT_s/2$, $T_s = 4000K$, $k = 1.38\times10^{-23}$, and $W = 10^4$ Hz. $<s^2(t)> = 0.5$ for pure sine waves at maximum amplitude, and 1 for square waves. The maximum average received power is then $P_c<s^2(t)> = 2N_oW\times10^6$ $= 2\times1.38\times10^{-23}\times4000\times10^4\times10^6 = 1.1\times10^{-9}$ [W]. If we allow for a 70-dB path loss, then the average transmitter power is $\sim1.1\times10^{-2}$W, or ~10 milliwatts, which is reasonable.

Problem 10.2

a) *Referring to Figure 4.7-10 and associated text, the FM threshold S/N for $\beta^* = 10$ is approximately 18 dB, so $P_c/BkT = 10^{1.8}$ and $P_r = P_c > 63\times2W(1+\beta^*)kT$ $= 63\times2\times10^4(1+10)1.38\times10^{-23}\times4000 =$ 7.7×10^{-13} W received and 7.7×10^{-6} W transmitted*

b) *The output SNR requirement is 20+40 = 60 dB, where $S_{out}/N_{out} = P_c<s^2>3\beta^{*2}/2N_oW$ $= 10^6$, so $P_c \cong 10^6\times2\times1.38\times10^{-23}\times4000\times10^4/(0.5\times3\times100)$ $= 7.36\times10^{-12}$W; $P_t \cong 7.4\times10^{-5}$W.*

c) *The requirements for P_t are a factor of ~9.6 greater than the FM threshold, so we could reduce β^* slightly to x, where $10^2/x^2 = 9.6$, so the new x = β^* could be 3.23, but then the only margin left would be due to the fact that the FM threshold drops slightly with β^*.*

Problem 10.3

a) *FM broadcast signals are often of constant amplitude even as their carrier frequency drifts back and forth. Pre-emphasis would boost the amplitude of this slowly drifting constant-amplitude carrier so that it has a larger amplitude at larger frequency deviations from the central frequency, where the larger frequency deviations correspond to peak positive or negative values in the original signal of interest, s(t). For most program materials the probability distribution of signal amplitudes drops perhaps linearly toward zero for the largest values of |s(t)|, and the signal feeding the final amplifier stage is adjusted so that the peak output power doesn't exceed either the capacity of that amplifier or any applicable regulations. Independent of any signal probability distribution, however, if the peak transmitted power is limited, the* *gain cannot be increased for the peaks of |s(t)|, and so we must instead implement any pre-emphasis by reducing the gain for values of |s(t)| near zero, thus using the amplifier less efficiently. Moreover, this reduces the average signal power and thus risks dropping below the FM threshold for the more distant listeners. Maximizing the number of listeners might reasonably be considered more important than improving further the output SNR of FM broadcasts, which is already quite good.*

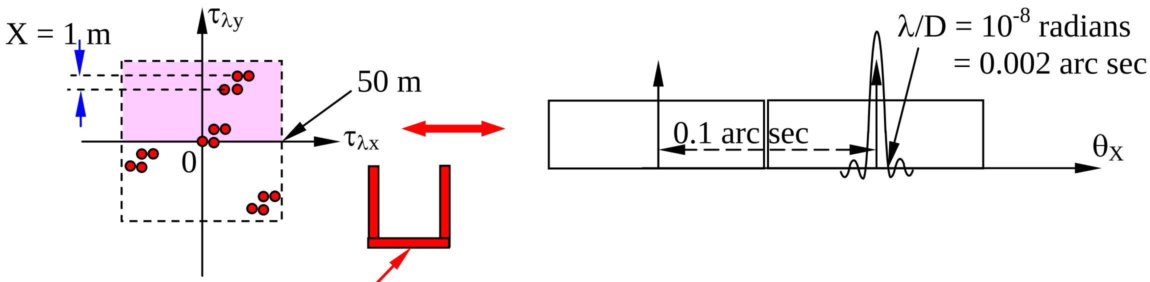
b) *If the average power is limited instead, the only disadvantage of pre-emphasis is a slight increase in system complexity and the possible challenge of achieving agreement in some standards process concerning the appropriate degree of pre-emphasis.*

Problem 10.4

a) Aliasing can occur when either the stars beyond 1 arc sec or unknown objects within the 0.1-arc-sec radius are aliased into the reconstructed image. One sensible strategy is to set the minimum telescope diameter to avoid aliasing of those stars beyond ~1 arc second, i.e. beyond ~5×10^{-6} radians, and to avoid aliasing of objects closer than 0.1 arc sec by imaging that region with sufficiently dense observations in $\underline{\tau}_\lambda$ space, as discussed below in parts (c) and (d). Basically, we want all distant stars to be beyond the first null in the diffraction pattern of the individual telescopes, or more than ~λ/D away. Setting $\lambda/D \cong 5\times10^{-6} \Rightarrow D \cong 10^{-6}/5\times10^{-6} \cong$ 20 cm. Although telescope sidelobes do result in some aliasing, these sidelobes can be reduced by suitably apodizing the telescope aperture. Alternatively smaller telescopes could be used, but then a larger area must be mapped near the target star, enabling these neighboring stars to be identified and subtracted from the synthetic image. Also, the answer to part (c) below would increase as the square of the diameter of the viewed region, and the reduced photon flux for smaller mirrors would also increase the required integration time.

b) The angular resolution of a synthesized image is $\lambda./2L$, where L is the maximum separation needed in the synthesis procedure, and $2L/\lambda$ is the full width of the field autocorrelation function $\phi_E(\tau_\lambda)$. Therefore $2L/\lambda$ is also the maximum angular frequency that can be resolved, given as 10^8 periods/radian. Therefore $L = 10^8\times10^{-6}/2 = 50$ m.

c) The field autocorrelation function must be sampled sufficiently often that the unknown region of diameter 0.2 arc sec is not aliased on top of itself. Fortunately the region between 0.1 and 1 arc sec from center is known to be empty, and the stars beyond that are beyond the diffraction limit of the telescopes that are being used (part (a)). The aliased images are spaced λ/X *apart, where X is the separation between the observed samples of* $\underline{\phi}_E(\tau_\lambda)$. *Here we require* $\lambda/X > 0.2$ *arc sec* $= 10^{-6}$ *radians to avoid aliasing of the 0.1-arc sec region. Therefore* $X \leq 10^6\lambda \cong$ *1 m. Now we can compute the required number of observations of* $\underline{\phi}_E(\tau_\lambda)$. $L/X = 50/1 = 50$, *so we need* $2\times 50^2 =$ *5000 observations of* $\underline{\phi}_E(\tau_\lambda)$. *The factor of two is needed because we need to observe two quadrants of* $\underline{\phi}_E(\tau_\lambda)$*, before we use* $\underline{\phi}_E(\tau_\lambda) = \underline{\phi}_E^*(-\tau_\lambda)$ *to fill in the other two quadrants.*



d) If the telescope apertures were square, we would need three rows of 50 telescopes arranged along three of the four sides of a field 50 m square, or perhaps in the shape of a tee, i.e., 150 telescopes. Since we probably are using circular apertures, the telescopes might be arranged more in the form of a circle, and the numbers might be reduced slightly.