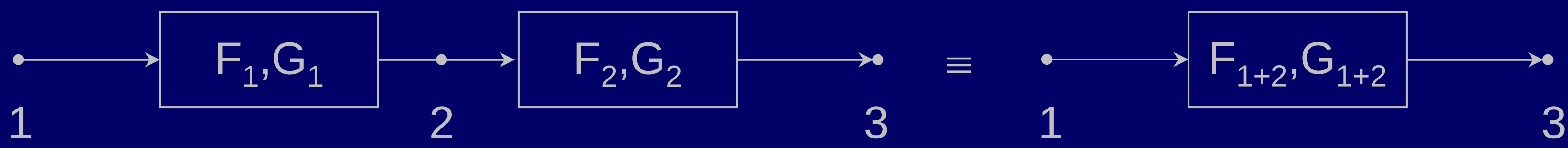
Noise in Cascaded Amplifiers



$$F_{1+2} \triangleq \frac{S_1/N_1}{S_3/N_3} \quad \text{where } S_3 = G_1 G_2 S_1$$

$$N_2 = kT_o F_1 G_1 \quad \left(\text{Recall } F_1 = \frac{S_1/N_1}{S_2/N_2}\right)$$

$$N_3 = \underbrace{G_2 kT_o F_1 G_1}_{\text{amplified from "2"}} + \underbrace{G_2 (F_2 - 1) kT_o}_{\text{excess from "2"}}$$

$$\therefore S_3/N_3 = G_1 G_2 S_1 / \left(G_1 G_2 F_1 + G_2 (F_2 - 1)\right)\left(kT_o\right)$$

$$= \left(S_1/kT_o\right) \Big/ \left(F_1 + \frac{F_2 - 1}{G_1}\right)$$

$$\therefore \quad \boxed{F_{1+2} = \frac{S_1/N_1}{S_3/N_3} = F_1 + \frac{F_2 - 1}{G_1}}$$

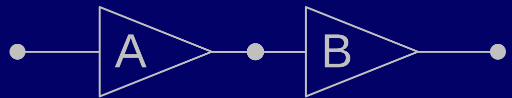
Noise in multiple cascade amplifiers

By extension: $F_{1,2,3} = F_{1+2} + \dfrac{F_3 - 1}{G_1 G_2}$ $\qquad F_{1+2} = F_1 + \dfrac{F_2 - 1}{G_1}$

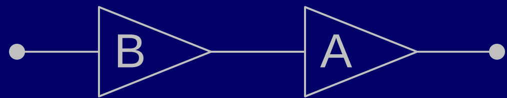
In general,

$$F_{1,2,\ldots} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \ldots \text{cascade noise formula}$$

Note:



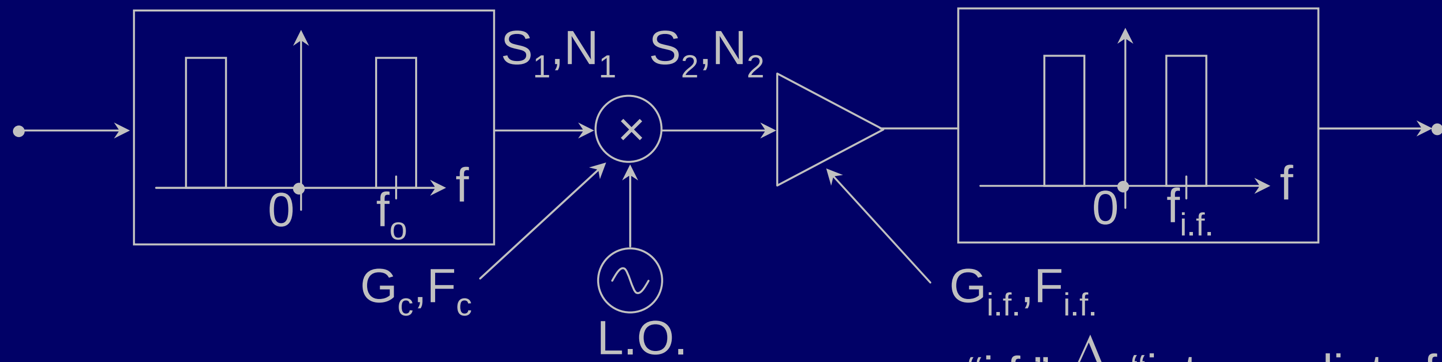
vs.



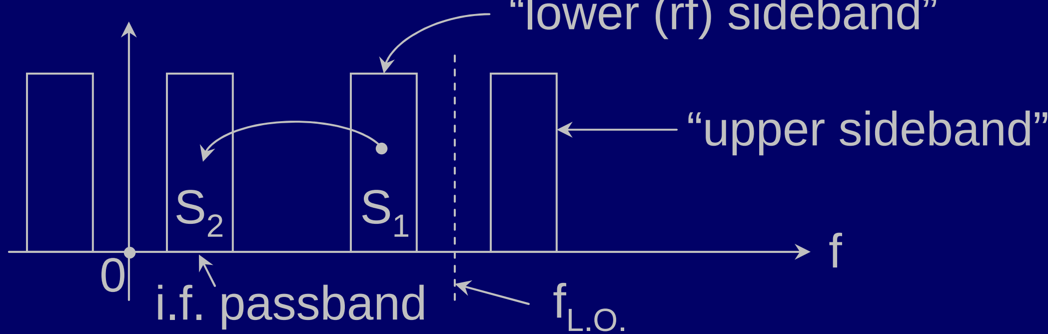
The better choice is not obvious.

$F_{1,2}$ also depends on interstage impedance mismatches and gain of the first amplifier, not just on F_1

Noise in superheterodyne receivers



“i.f.” $\triangleq$ “intermediate frequency”



1) “Conversion loss of mixer” $\triangleq L_c \triangleq \dfrac{1}{G_c} = \dfrac{S_2(\text{i.f.})}{S_1(\text{r.f., SSB})}$

2) “Noise temperature ratio of mixer” $\triangleq t_r = N_2(\text{i.f.})/kT_o$

3) $F_{mixer} \triangleq \dfrac{S_1/N_1}{S_2/N_2} = \dfrac{S_1/kT_o}{(S_1/L_c)/t_r kT_o} = L_c t_r$

Noise in superheterodyne receivers

3) $$F_{mixer} \triangleq \frac{S_1/N_1}{S_2/N_2} = \frac{S_1/kT_o}{\left(S_1/L_c\right)/t_r kT_o} = L_c t_r$$

4) $$F_{mixer+i.f.\,amp.} = F_{mixer} + \frac{F_{i.f.} - 1}{G_{mixer}}$$

$$= L_c t_r + L_c\left(F_{i.f.} - 1\right) = L_c\left(F_{i.f.} + t_r - 1\right)$$

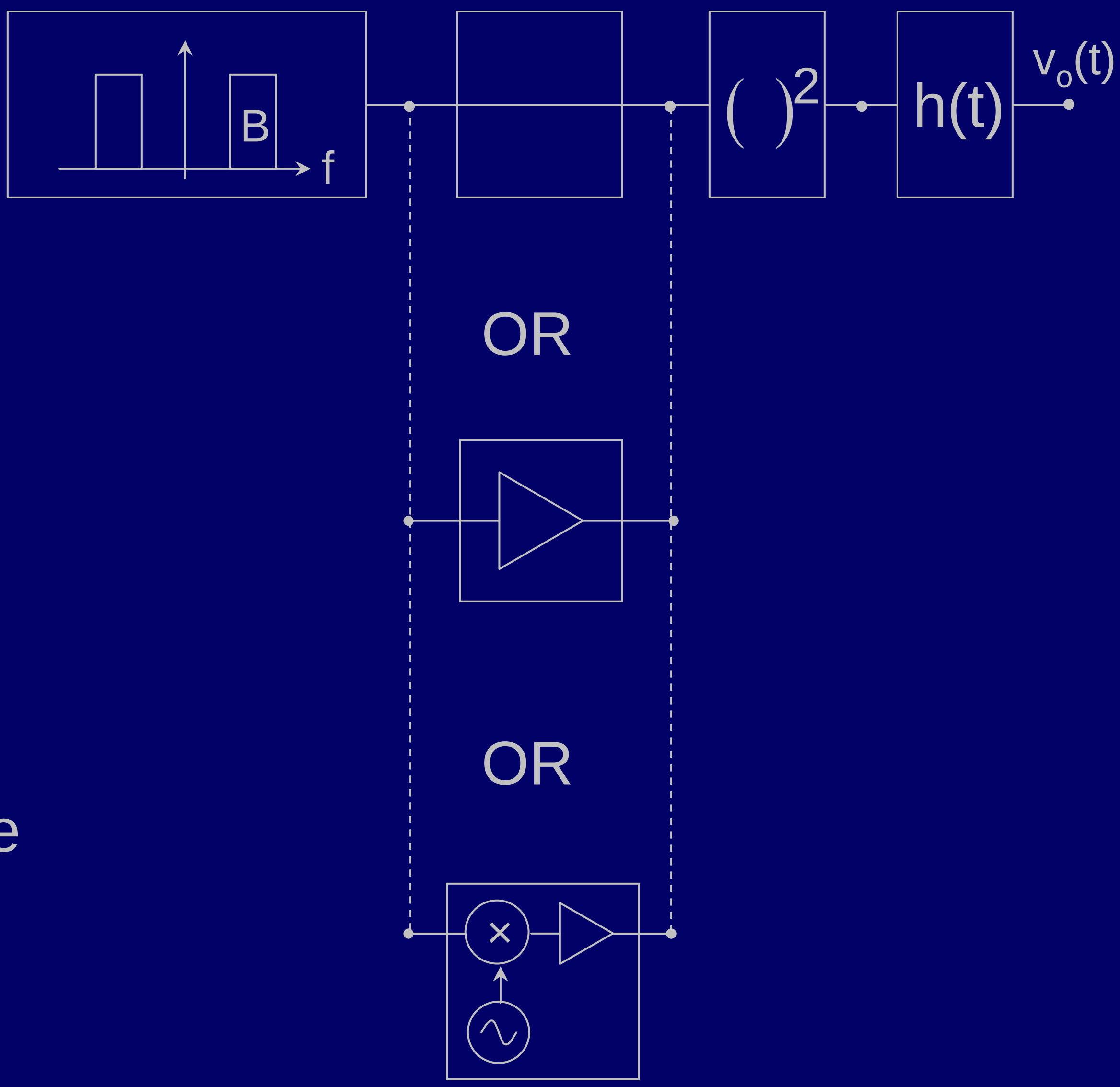
e.g. $F_{mixer+i.f.\,amp.} \cong 2-8\left(\sim 3-9\text{ dB}\right)$

Basic receiver types-Amplification

1) Simple detector

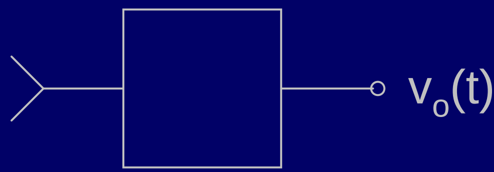
2) RF Amplifier

3) Superheterodyne



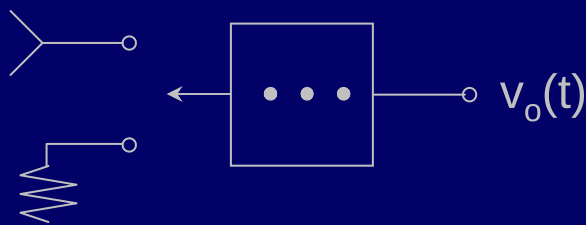
Basic receiver types-Combinors

1)



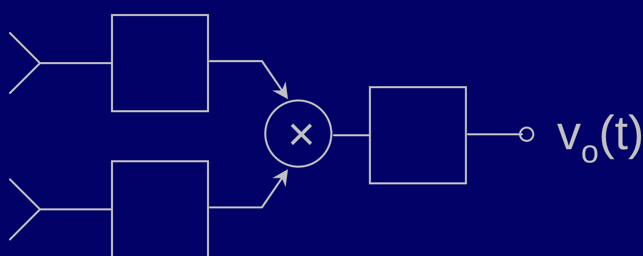
Total power radiometer

2)



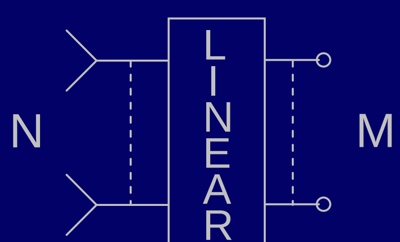
Dicke radiometer

3)



Multiplication (correlation) receiver

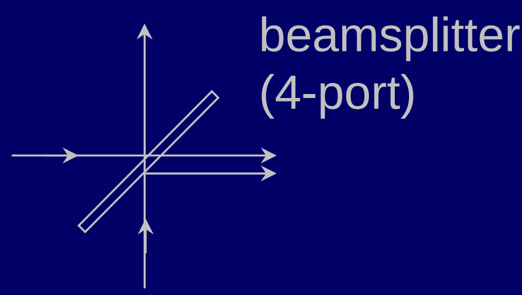
4)



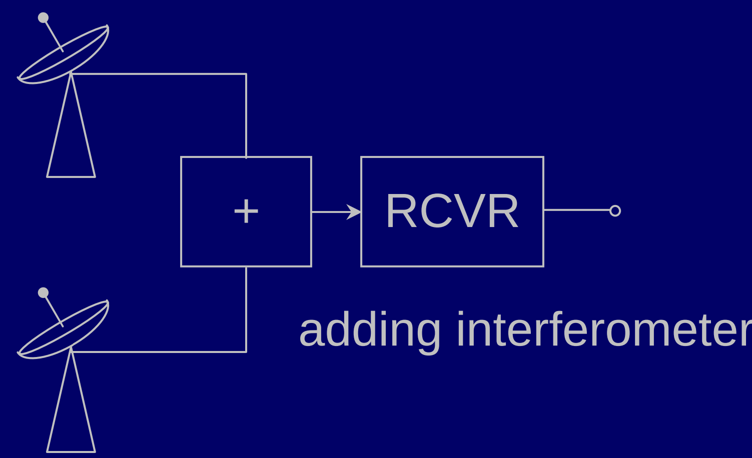
N-way combiner, $N \gtrless M$

Passive Multiport Networks

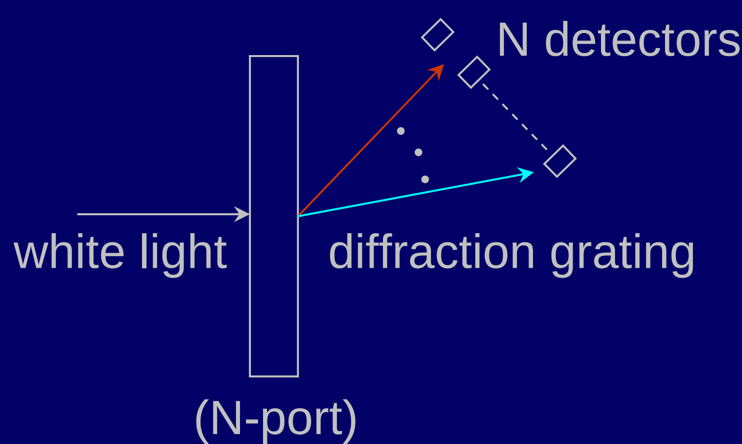
1)



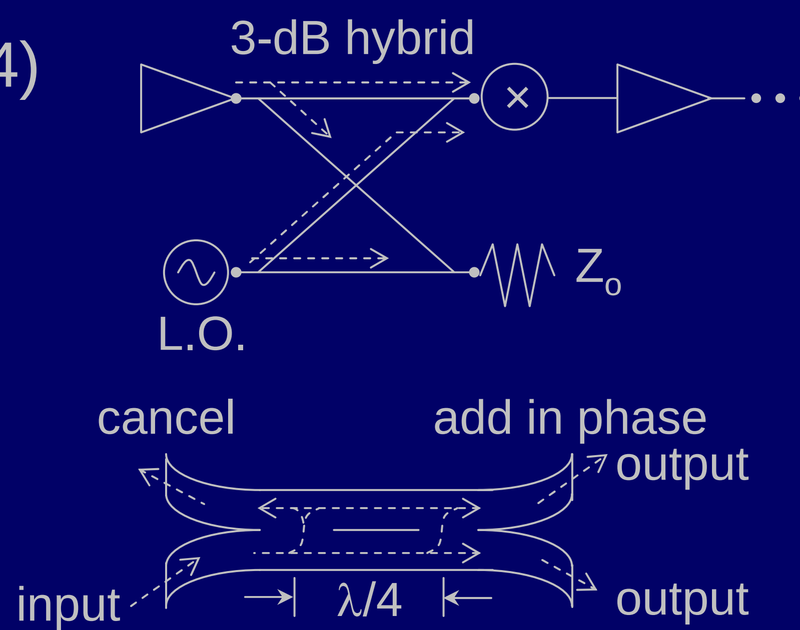
2)



3)

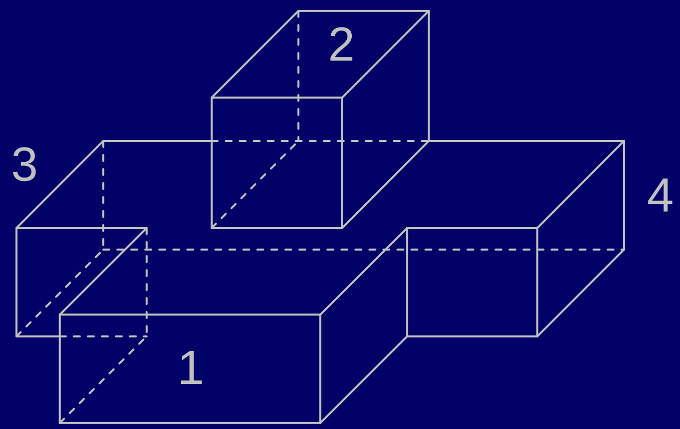


4)



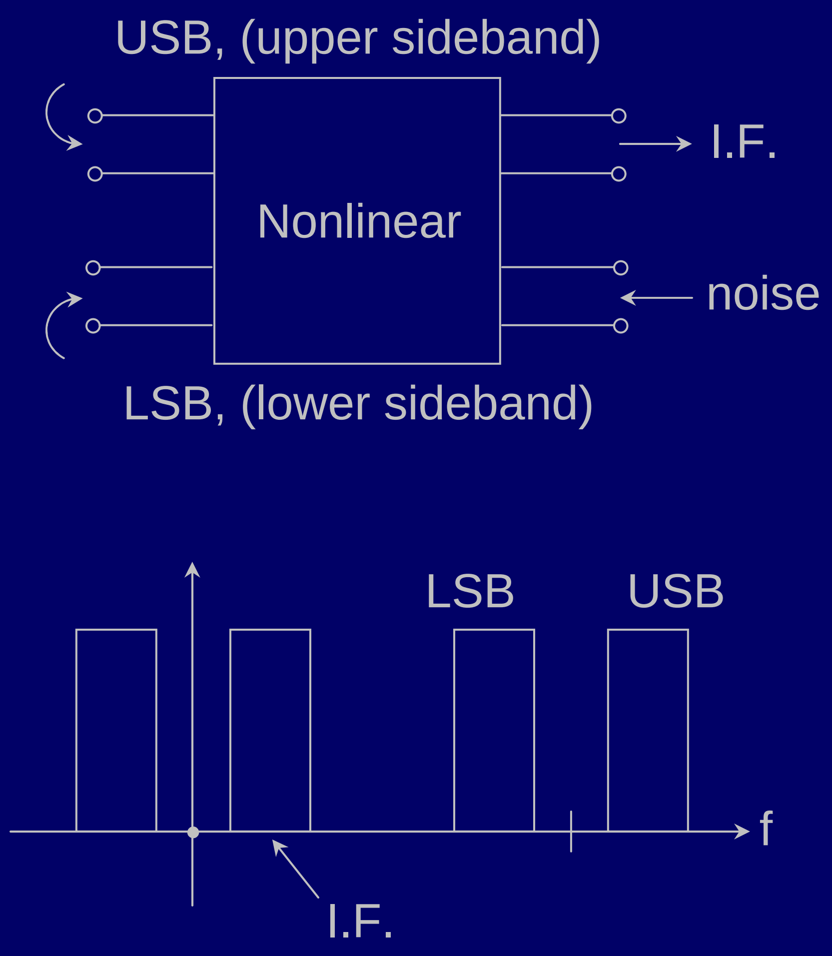
Passive Multiport Networks

5) “Magic tee”



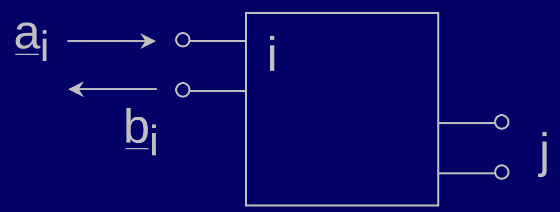
Port 1 orthogonal to Port 2
Port 3 orthogonal to Port 4
All 4 ports can be matched

6) Frequency converter



Linear Passive N-port Networks

$$\text{Define exchangeable power} = \begin{Bmatrix} |\underline{a}_i|^2 \text{ toward port i} \\ |\underline{b}_i|^2 \text{ from port i } (\text{W or WHz}^{-1}) \end{Bmatrix}$$



$$\text{Net power entering port “i”} = |\underline{a}_i|^2 - |\underline{b}_i|^2$$

Scattering matrix equation: $\underline{\overline{b}} = \underline{\overline{\overline{S}}}\,\underline{\overline{a}}$

Note:

1) The scattering matrix $\underline{\overline{\overline{S}}}$ is defined only when the n-port is imbedded in a network
2) The phases of a_i and b_i can be defined as some linear combinations of those for voltage and current

$$\text{e.g. } \underline{a}_i = \underline{V}_+\sqrt{Y_o/2} = (\underline{V} + Z_o\underline{I})/\sqrt{8Z_o}, \quad \underline{b}_i = (\underline{V} - Z_o\underline{I})/\sqrt{8Z_o}$$

Gain definition for N-port networks

"Transducer gain"

$$G_{Tkj} \triangleq |\underline{b}_k|^2 \Big/ |\underline{a}_j|^2 = |\underline{\bar{S}}_{kj}|^2 = P_{Dk} \big/ P_{Aj}$$

"Exchangeable gain" (~"available gain")

$$G_{E_{kj}} \triangleq \frac{P_{E_k}}{P_{E_j}} = \frac{?}{|\underline{a}_j|^2}$$

Note: available power out $\geq |\underline{b}_k|^2$ due to possible port-k mismatch

i.e. fractional power absorbed (FPA) $= \dfrac{|\underline{a}_k|^2 - |\underline{b}_k|^2}{|\underline{a}_k|^2} = 1 - |\underline{S}_{kk}|^2$

But FPA = fractional power emitted, if reciprocity applies.

Therefore available power from port K $= |\underline{b}_k|^2 \Big/ \left(1 - |\underline{S}_{kk}|^2\right)$

Therefore $G_{E_{kj}} = \left[|\underline{b}_k|^2\right] \Big/ \left(1 - |\underline{S}_{kk}|^2\right) \Big/ |\underline{a}_j|^2 = \dfrac{|\underline{S}_{kj}|^2}{1 - |\underline{S}_{kk}|^2} = G_{E_{kj}}$

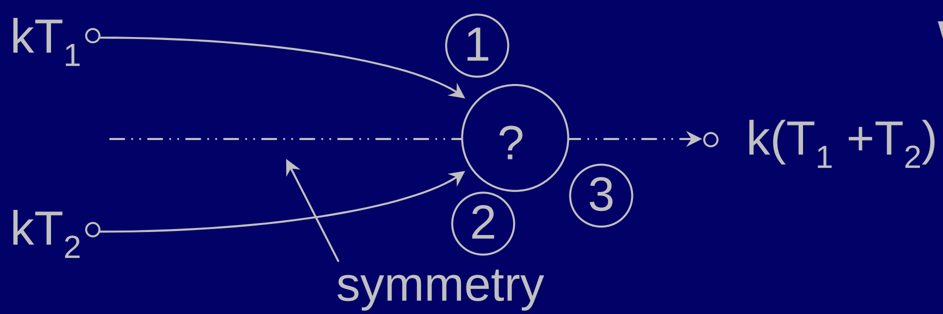
Constraints on N-port networks

Lossless passive networks $\Rightarrow \sum_{i=1}^{N} \left|\underline{a}_i\right|^2 = \sum_{i=1}^{N} \left|\underline{b}_i\right|^2$

Reciprocity $\Rightarrow \overline{\overline{S}} = \overline{\overline{S}}^t$

Example of constrained N-port networks

Does an ideal power combiner exist?



We want $\underline{\underline{S}} = \begin{bmatrix} 0 & 0 & \underline{\alpha} \\ 0 & 0 & \underline{\alpha} \\ \underline{\alpha} & \underline{\alpha} & \underline{\beta} \end{bmatrix}$

$$\text{Losslessness} \Rightarrow \underline{a}^{t^*} \bullet \underline{a} = \underline{b}^{t^*} \underline{b}$$

$$\underline{b}^{t^*} \bullet \underline{b} = \left(\underline{\underline{S}}\,\underline{a}\right)^{t^*} \bullet \left(\underline{\underline{S}} \bullet \underline{a}\right) = \underline{a}^{t^*} \underline{\underline{S}}^{t^*} \underline{\underline{S}}\,\underline{a}$$

Therefore $\underline{\underline{S}}^{t^*} \underline{\underline{S}} = \underline{\underline{I}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$; if the combiner is lossless

Example of constrained N-port networks

Therefore $\underline{\underline{S}}^{t*}\,\underline{\underline{S}} = \underline{\underline{I}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$; if the container is lossless

$$\underline{\underline{S}} = \begin{bmatrix} 0 & 0 & \underline{\alpha} \\ 0 & 0 & \underline{\alpha} \\ \underline{\alpha} & \underline{\alpha} & \underline{\beta} \end{bmatrix}$$

Test: $\underline{\underline{S}}^{t*}\,\underline{\underline{S}} = \begin{bmatrix} |\underline{\alpha}|^2 & |\underline{\alpha}|^2 & \underline{\alpha}^*\underline{\beta} \\ |\underline{\alpha}|^2 & |\underline{\alpha}|^2 & \underline{\alpha}^*\underline{\beta} \\ \underline{\alpha}\underline{\beta}^* & \underline{\alpha}\underline{\beta}^* & 2|\underline{\alpha}|^2 + |\underline{\beta}|^2 \end{bmatrix}$

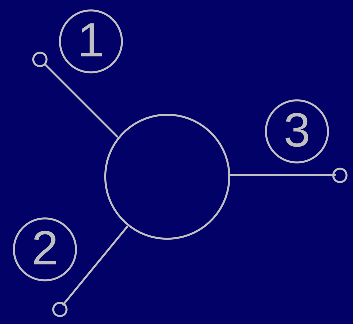
If $|\underline{\alpha}|^2 = 1$, then $2|\underline{\alpha}|^2 + |\underline{\beta}|^2 \neq 1$

Therefore constraints (8-14) and (8-15) cannot be satisfied simultaneously for this system

Ideal matched 2-input-port combiners are impossible

Another N-port network example

Can we match all 3 ports simultaneously?



(Note: $\underline{\underline{S}}$ is defined only when imbedded in network)

For 3 matched ports: $$\underline{\underline{S}} = \begin{bmatrix} 0 & \underline{\alpha} & \underline{\beta} \\ \underline{\alpha} & 0 & \underline{\gamma} \\ \underline{\beta} & \underline{\gamma} & 0 \end{bmatrix}$$

Does $\underline{\underline{S}}^{t*}\,\underline{\underline{S}} = \underline{\underline{I}}$? (lossless passive constraint)

Matched 3-port example

For 3 matched ports: $$\underline{\underline{S}} = \begin{bmatrix} 0 & \underline{\alpha} & \underline{\beta} \\ \underline{\alpha} & 0 & \underline{\gamma} \\ \underline{\beta} & \underline{\gamma} & 0 \end{bmatrix}$$

Does $\underline{\underline{S}}^{t*}\,\underline{\underline{S}} = \overline{\overline{I}}$? (lossless passive constraint)

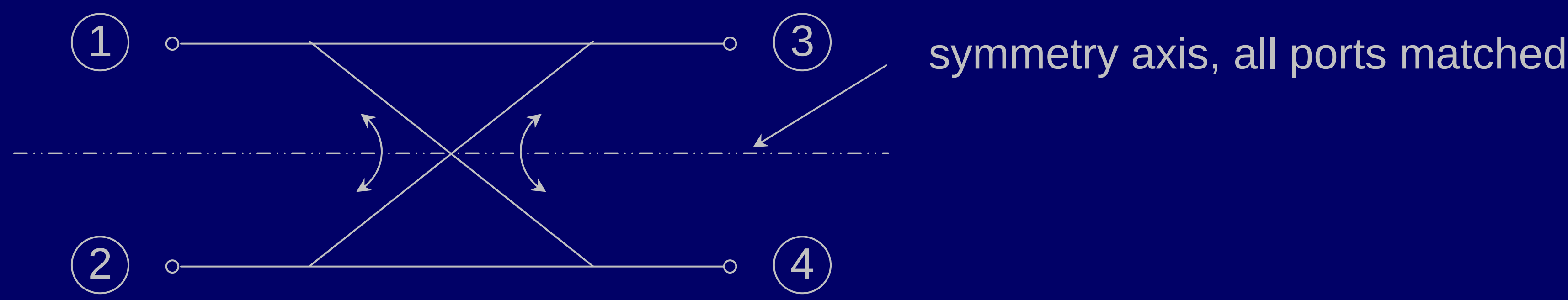
$$\underline{\underline{S}}^{t*}\,\underline{\underline{S}} = \begin{bmatrix} |\underline{\alpha}|^2 + |\underline{\beta}|^2 & \underline{\beta}^*\underline{\gamma} & \underline{\alpha}^*\underline{\gamma} \\ \underline{\beta}\underline{\gamma}^* & |\underline{\alpha}|^2 + |\underline{\gamma}|^2 & \underline{\alpha}^*\underline{\beta} \\ \underline{\alpha}\underline{\gamma}^* & \underline{\alpha}\underline{\beta}^* & |\underline{\beta}|^2 + |\underline{\gamma}|^2 \end{bmatrix} \overset{?}{=} \overline{\overline{I}}$$

If $\underline{\beta}^*\underline{\gamma} = \underline{\alpha}^*\underline{\gamma} = \underline{\alpha}^*\underline{\beta} = 0$, then 2 of (α,β,γ) and at least one diagonal element of $\underline{\underline{S}}^{t*}\,\underline{\underline{S}} = 0$

Therefore not possible to match all 3 ports simultaneously

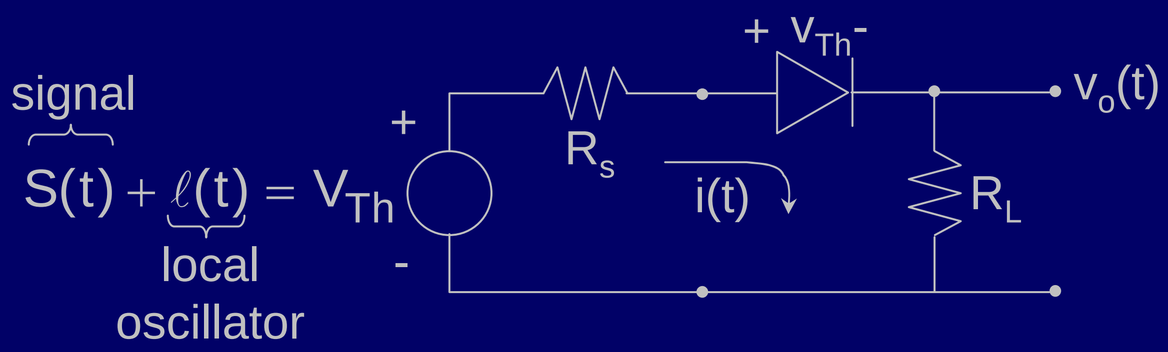
Lossless passive reciprocal symmetric 4-port network

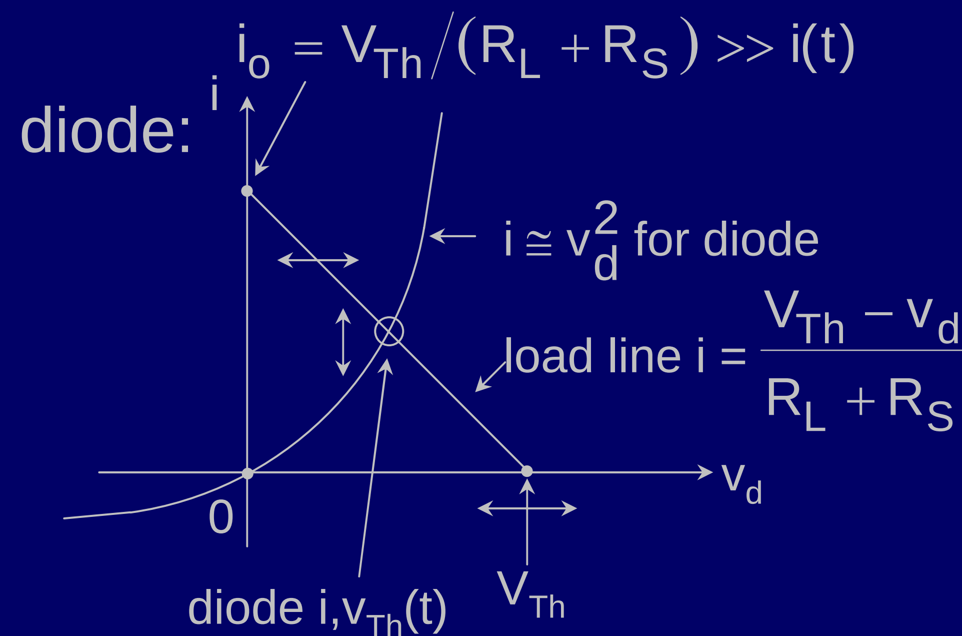
Ports 1, 2 are isolated; also 3, 4



We can show $\Delta\phi$ for ① ⟶ ③ versus ① ⟶ ④ paths is unique using losslessness, reciprocity, and symmetry

Mixer





$$v_d(t) \cong v_\ell \sin\omega_o t + v_s \sin\omega_s t$$

$$+ \text{small high-order terms}$$

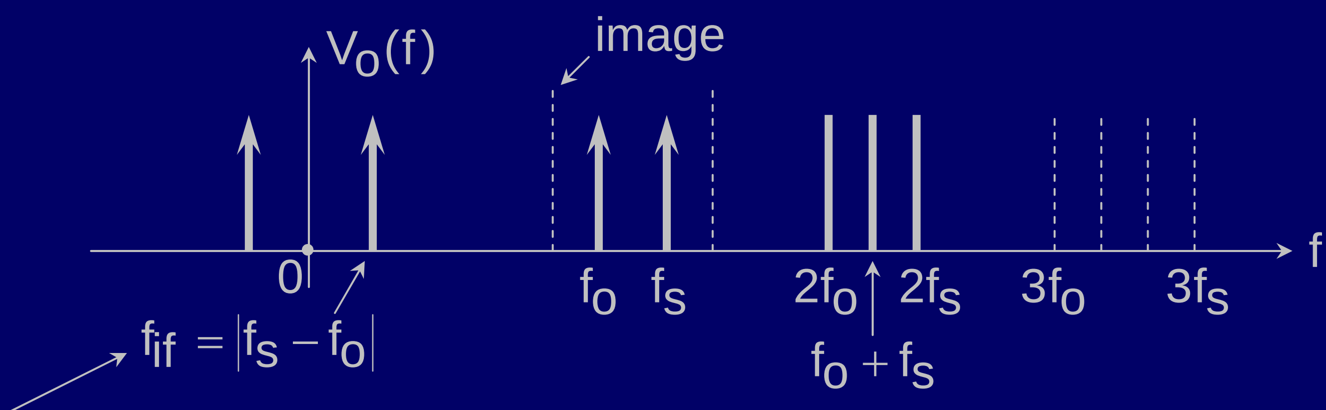
$$v_o(t) = iR_L\left(\propto v_{Th}^2 R_L\right)$$

$$= k_o + k_1 v_{Th} + \underbrace{k_2 v_{Th}^2}_{\text{dominant}} + k_3 v_{Th}^3 + \ldots$$

Mixer

$$v_o(t) = iR_L = k_o + k_1 v_{Th} + \underbrace{k_2 v_{Th}^2}_{\text{dominant}} + k_3 v_{Th}^3 + \ldots \qquad \left(\tilde{\propto}\ v_d^2 R_L\right)$$

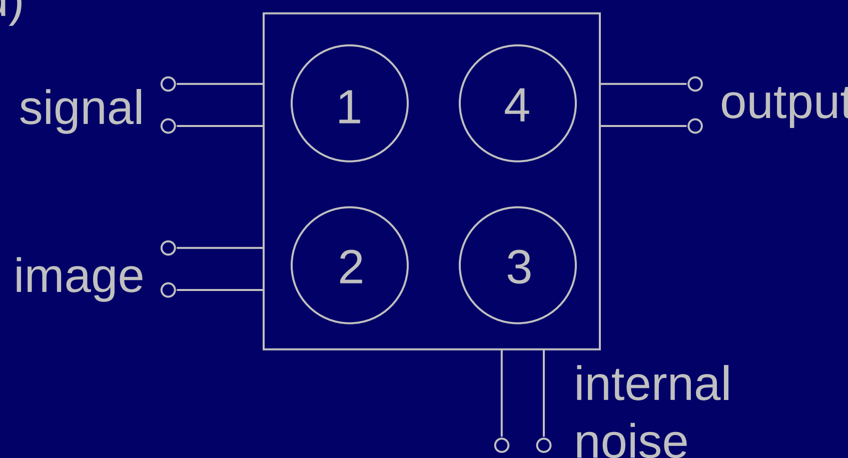
Spectral content of detector output:



Pure product (frequency component for pure s(t) • ℓ(t))

(Note: $v_d^4 \Rightarrow 2f_{i.f.}$, can be in i.f. passband)

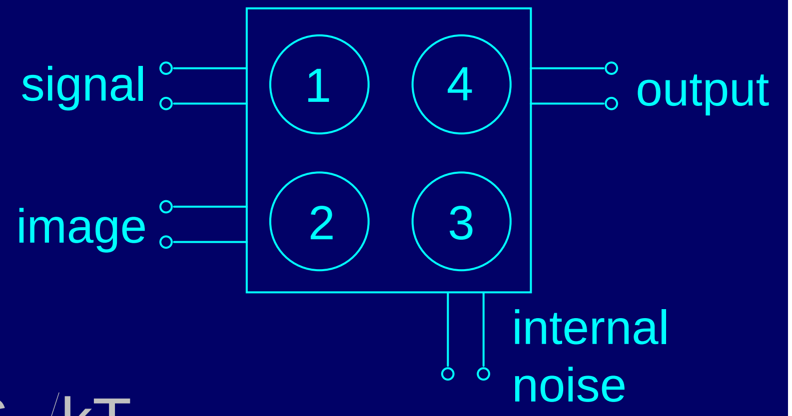
Normally:
a 4-port model suffices
to find F, T_{SSB}, "T_R":



Mixer Noise Figure Using 4-port Model

$$T_R = T_{SSB} = (F-1)T_o = T_o(L_c t_r - 1)$$

for single sideband (SSB) operation



$$F_{SBB} = \frac{S_1/N_1}{S_4/N_4} = \frac{S_1/kT_o}{\left[\dfrac{S_1|S_{41}|^2}{1-|S_{44}|^2}\right] \Big/ \left[\dfrac{kT_o|S_{41}|^2 + kT_2|S_{42}|^2 + kT_3|S_{43}|^2}{1-|S_{44}|^2}\right]}$$

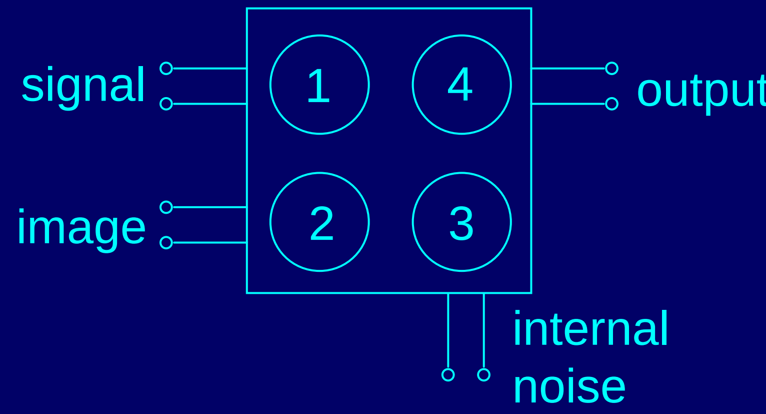
[Note: "S", signifies signal in port 1, S_{kj} is a scattering matrix element]

Simplifying, $F_{SSB} = 1 + \dfrac{T_2}{T_o}\dfrac{|S_{42}|^2}{|S_{41}|^2} + \dfrac{T_3}{T_o}\dfrac{|S_{43}|^2}{|S_{41}|^2} = 1 + \dfrac{T_{SSB}}{T_o}$

Therefore $T_{SSB} = T_2\dfrac{|S_{42}|^2}{|S_{41}|^2} + T_3\dfrac{|S_{43}|^2}{|S_{41}|^2}$

Double-sideband Receiver

Therefore $T_{SSB} = T_2 \frac{|S_{42}|^2}{|S_{41}|^2} + T_3 \frac{|S_{43}|^2}{|S_{41}|^2}$



Both ports 1 and 2 are signal, so

$$S_4 = kT_o \left(|S_{41}|^2 + |S_{42}|^2 \right) / \left(1 - |S_{44}|^2 \right)$$

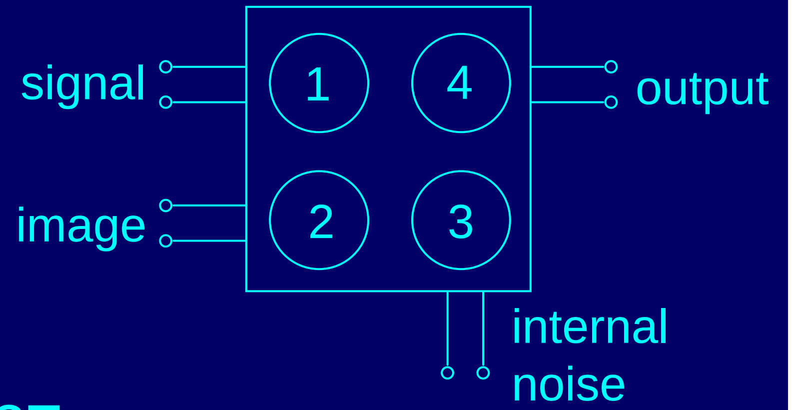
It follows that

$$T_{DSB} = T_3 |S_{43}|^2 / \left[|S_{41}|^2 + |S_{42}|^2 \right]$$

Often suggested: $F_{SSB} \cong 2F_{DSB}$

Double-sideband Receiver

Therefore $T_{SSB} = T_2 \frac{|S_{42}|^2}{|S_{41}|^2} + T_3 \frac{|S_{43}|^2}{|S_{41}|^2}$



Often suggested: $F_{SSB} \cong 2F_{DSB}$

Let $|S_{41}|^2 = |S_{42}|^2$ and $T_o = T_1 = T_2$; then

$$F_{DSB} = 1 + \frac{T_{DSB}}{T_o} = 1 + \frac{1}{2}\frac{T_3}{T_o}\frac{|S_{43}|^2}{|S_{41}|^2}$$

$$F_{SSB} = 1 + \frac{T_{SSB}}{T_o} = 2 + \frac{T_3}{T_o}\frac{|S_{43}|^2}{|S_{41}|^2} = 2F_{DSB}$$