Wire Antennas: Maxwell's equations

Maxwell's equations govern radiation variables:

$\bar{E}$ electric field $\left(\text{v m}^{-1}\right)$

$\bar{H}$ magnetic field $\left(\text{a m}^{-1}\right)$

$\bar{D} = \varepsilon\bar{E}$ electric displacement $\left(\text{Coulombs m}^{-2}\right)$

$\bar{B} = \mu\bar{H}$ magnetic flux density (Teslas)

(ε is permittivity; $\varepsilon_o = 8.8542 \times 10^{-12}$ farads/m for vacuum)
(μ is permeability; $\mu_o = 4\pi \bullet 10^{-7}$ henries/m)
(1 Tesla = 1 Weber $m^{-2} = 10^4$ gauss)

Maxwell's Equations: Dynamics and Statics

Maxwell's equations		Statics
$\nabla \times \overline{E} = -\frac{\partial \overline{B}}{\partial t}$	$\rightarrow$	$= 0$
$\nabla \times \overline{H} = \overline{J} + \frac{\partial \overline{D}}{\partial t}$	$\rightarrow$	$= \overline{J}\left(a\,m^{-2}\right)$
$\nabla \bullet \overline{D} = \rho$	$\rightarrow$	$= \rho\ \left(C\ m^{-3}\right)$
$\nabla \bullet \overline{B} = 0$	$\rightarrow$	$= 0$

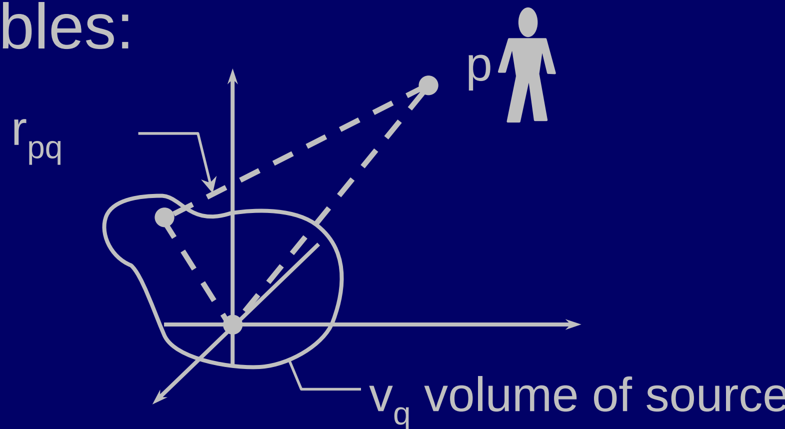
$$\left(\nabla \triangleq \hat{x}\partial/\partial x + \hat{y}\partial/\partial y + \hat{z}\partial/\partial z\ ;\ \nabla \triangleq \hat{r}\frac{\partial}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial}{\partial \theta} + \hat{\phi}\frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\right)$$

Static Solutions to Maxwell's Equations

Maxwell's equations govern variables:

$\overline{E}$ electric field $\left(v\ m^{-1}\right)$

$\overline{H}$ magnetic field $\left(a\ m^{-1}\right)$



$$\overline{E} = -\nabla\phi \text{ since } \nabla \times \overline{E} = 0$$

$$\overline{B} = \nabla \times \overline{A} \text{ since } \nabla \bullet \overline{B} = 0$$

$$\phi_p = \frac{1}{4\pi\varepsilon}\int_{v_q} \frac{\rho_q}{r_{pq}} dv_q$$ volts is electrostatic potential at point p

$$\overline{A}_p = \frac{\mu}{4\pi}\int_{v_q} \frac{\overline{J}_q}{r_{pq}} dv_q$$ is the vector potential at p

Dynamic Solutions to Maxwell's Equations

$\overline{B} = \nabla \times \overline{A}$ since $\nabla \bullet \overline{B} = 0$

$$\overline{A}_p(t) = \frac{\mu}{4\pi}\int_{v_q} \frac{\overline{J}_q\left(t - r_{pq}/c\right)}{r_{pq}} dv_q$$ (static solution, delayed)

$c = 1/\sqrt{\mu_o \varepsilon_o} \cong 3 \times 10^8 ms^{-1}$ (velocity of light)

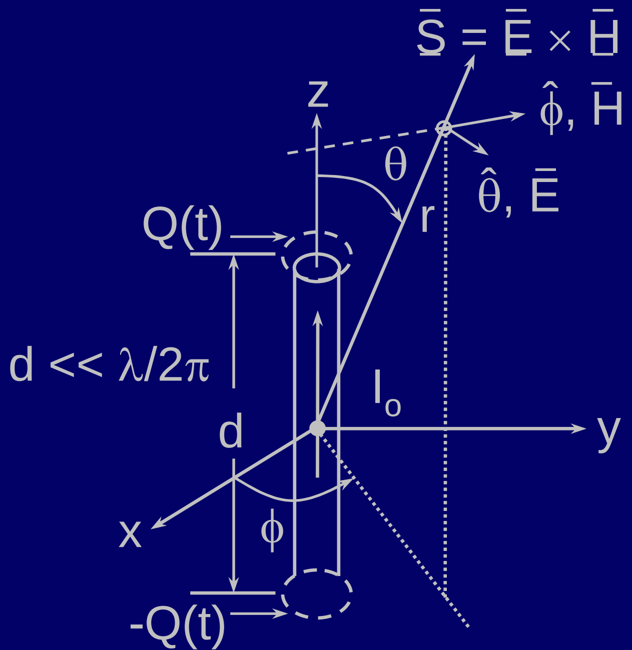
Sinusoidal steady state: $$\overline{A}_p = \frac{\mu}{4\pi}\int_{v_q} \frac{\overline{J}_q e^{-jkr_{pq}}}{r_{pq}} dv_q$$

where propagation constant $k = \omega\sqrt{\mu_o \varepsilon_o} = \omega/c = 2\pi/\lambda$

Solution method : $\overline{J}_q(\overline{r}) \rightarrow \overline{A}(\overline{r}) \rightarrow \overline{B}(\overline{r}) \rightarrow \overline{E}(\overline{r})$

where $\overline{E} = -\left(\nabla \times \overline{H}\right)/j\omega\varepsilon$ from Faraday's law

Elementary Dipole Antenna (Hertzian Dipole)



In "near field" $r << \dfrac{\lambda}{2\pi}$, $r >> d$

and the quasistatic fields are strongest (store active power, $(W_e) >> (W_m)$ here)

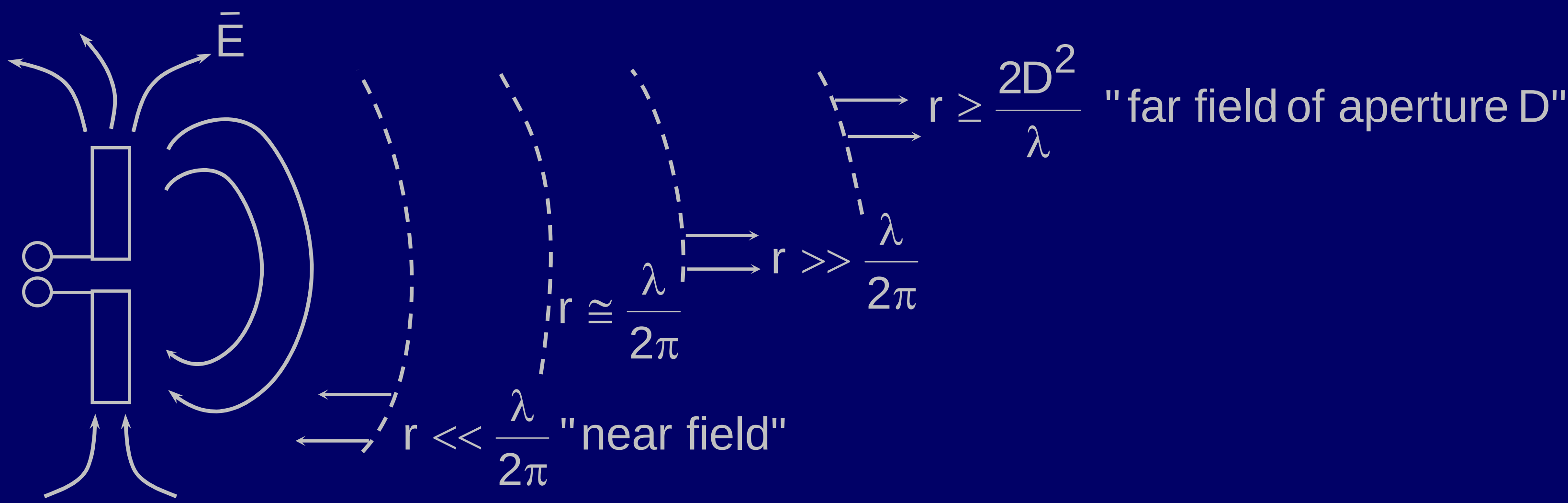
Far field: $r >> \lambda/2\pi$, $\bar{E} \cong \hat{\theta} j \eta_o \dfrac{k I_o d \sin\theta}{4\pi r} e^{-jkr}$

where $k = 2\pi/\lambda$, $\eta_o \equiv \sqrt{\mu_o/\varepsilon_o} = 377\Omega$

characteristic impedance of free space

$$\bar{H} \cong \hat{\phi} \frac{k I_o d \sin\theta}{4\pi j r} e^{-jkr}$$

Elementary Dipole Antenna (Continued)



$$\underline{\overline{S}} \triangleq \underline{\overline{E}} \times \underline{\overline{H}}^* \text{ W m}^{-2} \text{ "Poynting vector"}$$

where average power density $= \langle \overline{S}(t) \rangle = \frac{1}{2} \text{Re}\{\underline{\overline{S}}\} \left(\text{W m}^{-2}\right)$

and $\overline{S}(t) = \overline{E}(t) \times \overline{H}(t) \left(\text{W m}^{-2}\right)$

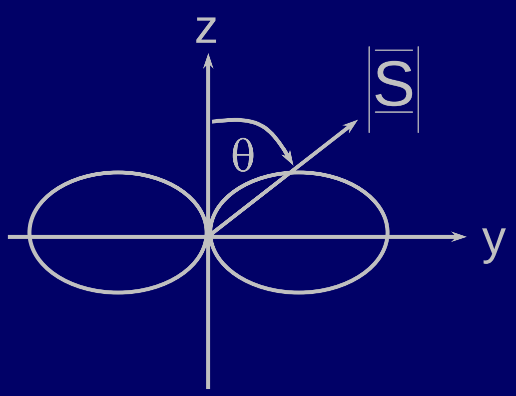
Elementary Dipole Antenna (Continued)

$\underline{\overline{S}} \triangleq \underline{\overline{E}} \times \underline{\overline{H}}^{*}$ W m^{-2} "Poynting vector"

where average power density $= \langle \overline{S}(t) \rangle = \frac{1}{2} \mathrm{Re}\{ \underline{\overline{S}} \}$ $(\mathrm{W\ m^{-2}})$

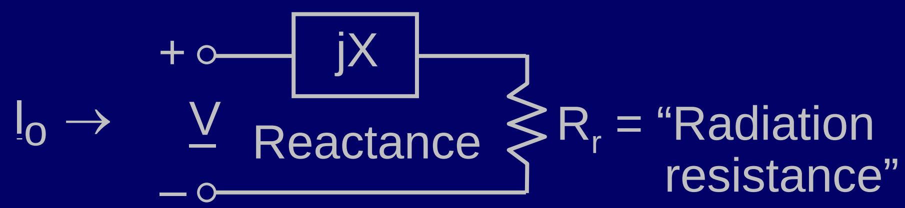
and $\overline{S}(t) = \overline{E}(t) \times \overline{H}(t)$ $(\mathrm{W\ m^{-2}})$

$\langle S(t) \rangle = \hat{r} \frac{\eta_o}{2} \left| \frac{\underline{I}_o d \sin\theta}{2\lambda r} \right|^2$ for a short dipole

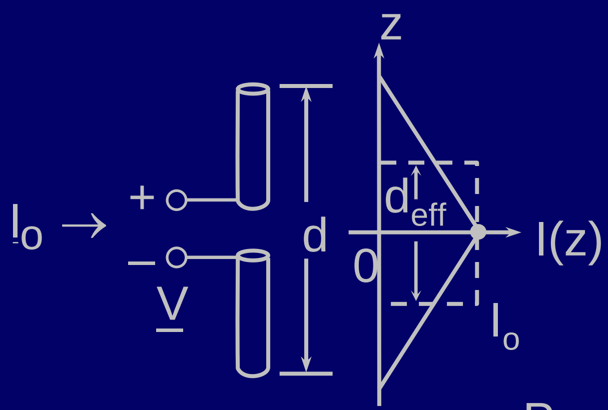


Total power transmitted $P_t = \frac{1}{2} R_e \left\{ \int_{4\pi} \underline{\overline{S}} \bullet \hat{r}\, d\Omega \right\} = \frac{\pi}{3} \eta \left| \frac{\underline{I}_o d}{\lambda} \right|^2$ Watts

Equivalent Circuit for Short Dipole Antenna



Reactance is capacitive for a short dipole antenna and inductive for a small loop antenna



$$\underline{I}_o d \rightarrow \underline{I}_o d_{eff} = \int_{-\infty}^{\infty} \underline{I}(z)\,dz$$

Here $d_{eff} = d/2$

Since $d << \lambda/2\pi$

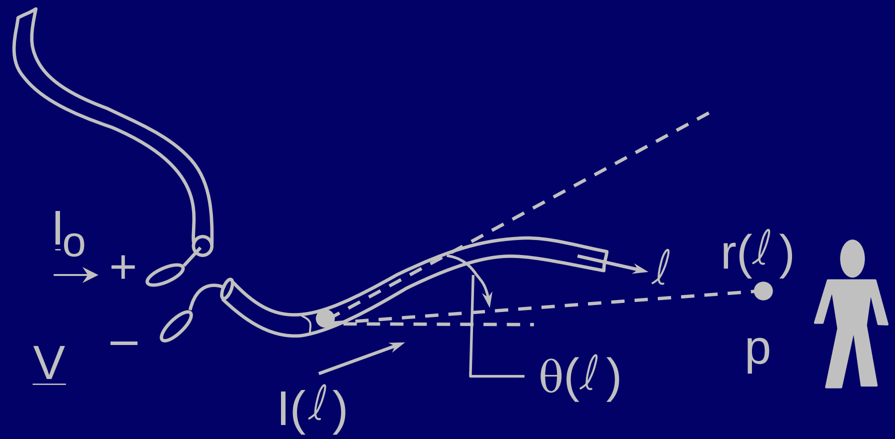
$$P_t = \frac{1}{2}\left|\underline{I}_o\right|^2 R_r = \frac{\pi\eta}{3}\left|\frac{\underline{I}_o d_{eff}}{\lambda}\right|^2$$

$$R_r = \frac{2\pi}{3}\eta_o \left(d_{eff}/\lambda\right)^2 \text{ ohms for a short dipole antenna}$$

Example: λ = 300m(1 MHz), d_{eff} = 1m $\Rightarrow R_r \cong 0.01\Omega$
such a mismatch requires transformers or hi-Q resonators for a good coupling to receivers or transmitters

Wire Antennas of Arbitrary Shape

Finding self-consistent solution is difficult
(matches all boundary conditions)
Approximate solutions are often adequate



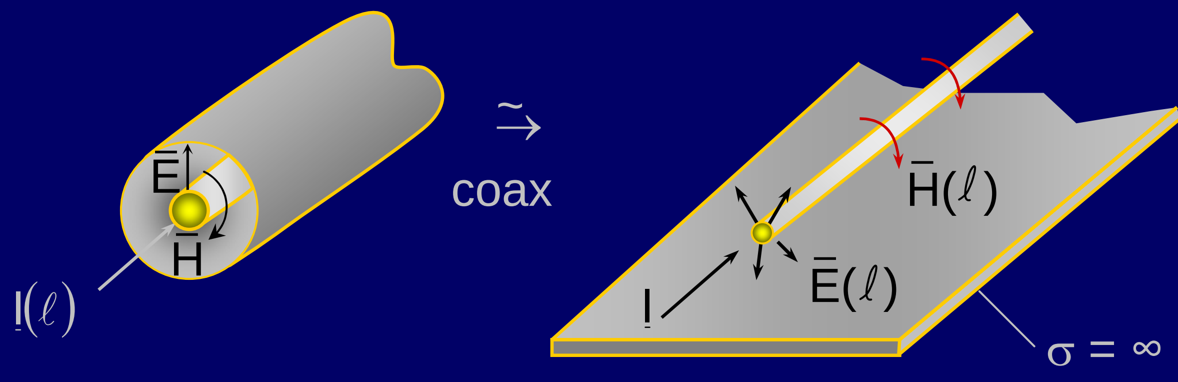
Approach

1) Use TEM - line reasoning to guess $\underline{I}(\bar{r})$

2) $\underline{I}(\bar{r}) \rightarrow \underline{\bar{A}}_{farfield} \rightarrow \underline{\bar{H}}_{ff} \rightarrow \underline{\bar{E}}_{ff}$

3) $$\underline{\bar{E}}_{ff} \cong \frac{jk\eta}{4\pi r} \int_L \hat{\theta}(\ell)\, \underline{I}(\ell) e^{-jkr(\ell)} \sin\theta(\ell)\, d\ell$$

Estimating Current Distribution $\underline{I}(\ell)$ on Wire Antennas



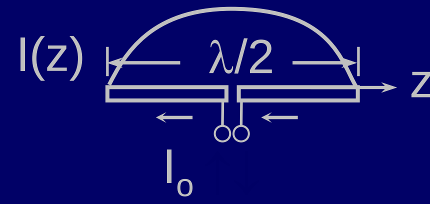
$$W_e = \frac{1}{4}\varepsilon_o|\overline{\underline{E}}|^2, \; W_m = \frac{1}{4}\mu_o|\overline{\underline{H}}|^2 \left[Jm^{-3}\right], \; W_{e,m} \propto \frac{1}{r^2}$$

Most energy stored within 1–2 wire radii.

Therefore $\underline{V}, \underline{I}$ on thin wires is TEM-like

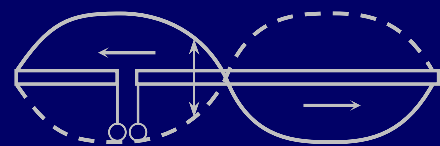
Examples: Current Distributions and Patterns

"Half-wave dipole"



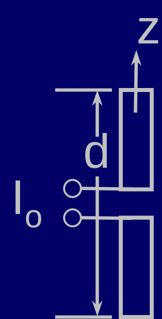
$R_r \cong 73\Omega$, $jX = 0$

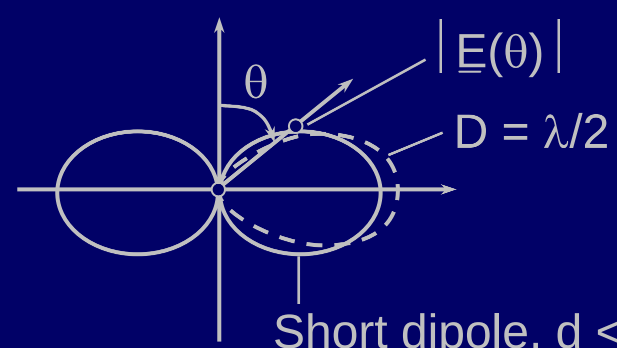
"Full-wave antenna"



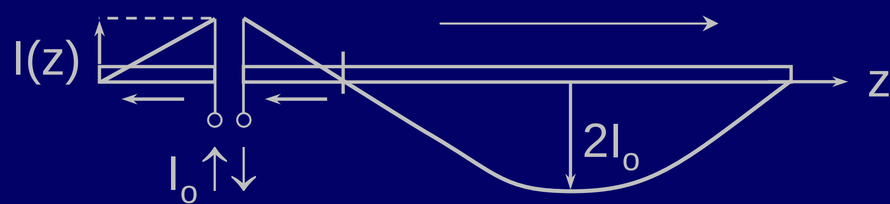
$$\underline{\overline{E}}_{ff} \cong \hat{\theta}\, \frac{j\eta I_o e^{-jkr}}{2\pi r \sin\theta} \underbrace{\left[\cos\left(\frac{k\ell}{2}\cos\theta\right) - \cos\frac{k\ell}{2}\right]}_{= \cos\left(\frac{\pi}{2}\cos\theta\right) \text{ if } \ell = \frac{\lambda}{2}}$$

for center-fed wire of length ℓ



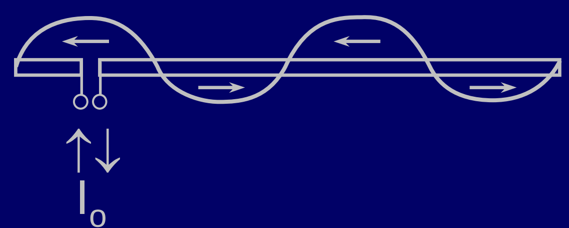


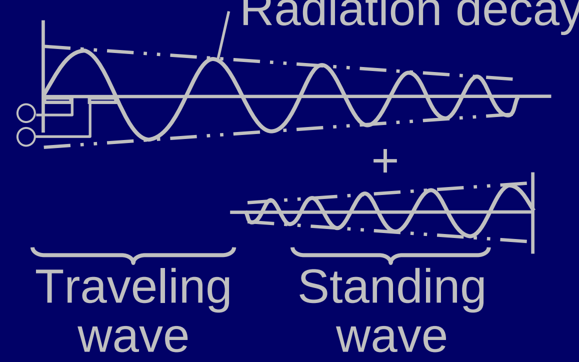
Examples: Current Distributions and Patterns



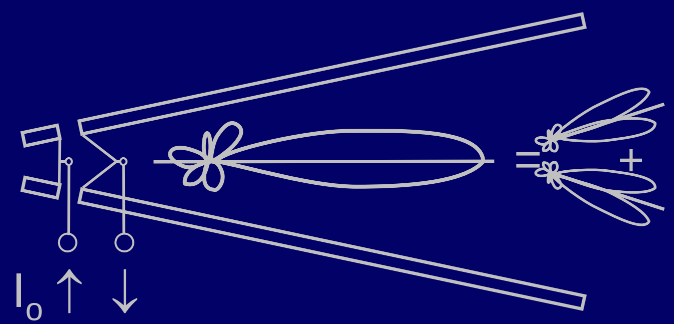
Current transformer, jX ≠ 0

Long-wire antenna



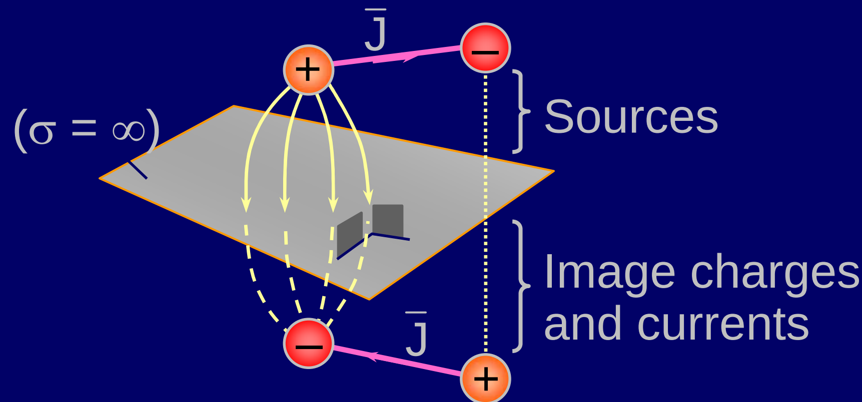


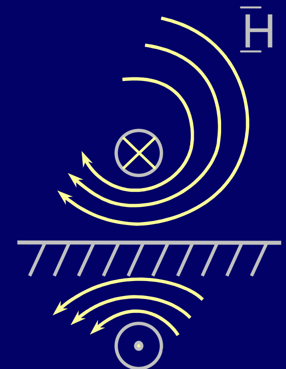
Vee (broadband)



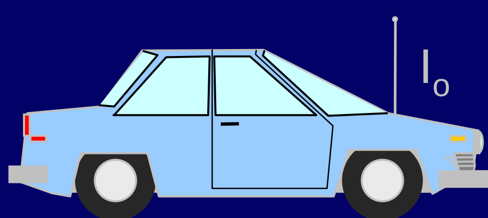
Mirrors, Image Charges and Currents

We can replace planar mirrors ($\sigma = \infty$) with image charges and currents; $\overline{E}$, $\overline{H}$ solution unchanged.

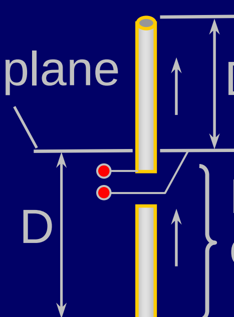


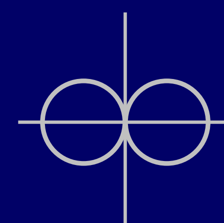


Note: Anti-symmetry of image charges and currents guarantees $\overline{E}\perp(\sigma = \infty)$, $\overline{H}//(\sigma = \infty)$, matching boundary conditions. Also, the uniqueness theorem says any solution is the valid one.



≅ Ground plane

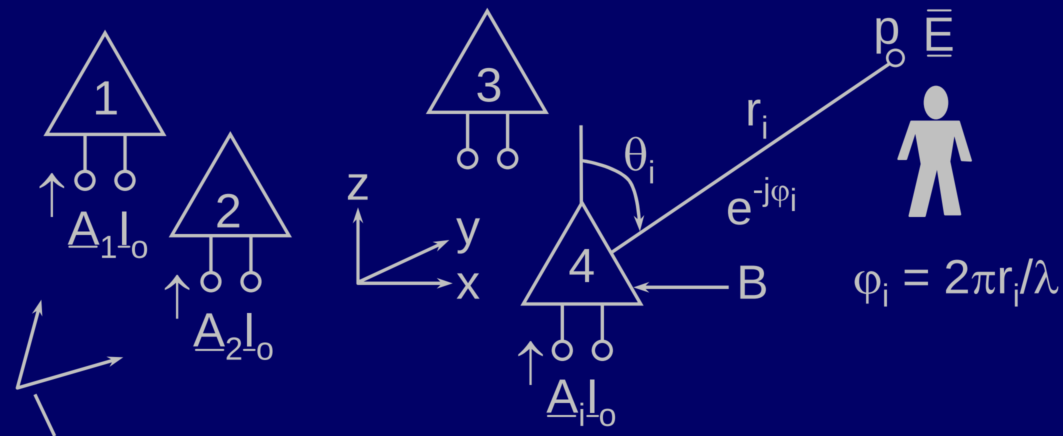




Antenna pattern

Say 1 MHz (λ = 300m)
D ≅ 1m (short dipole); D << λ
$D_{eff} \cong 2D/2 = D$; X is capacitive

Antenna Arrays



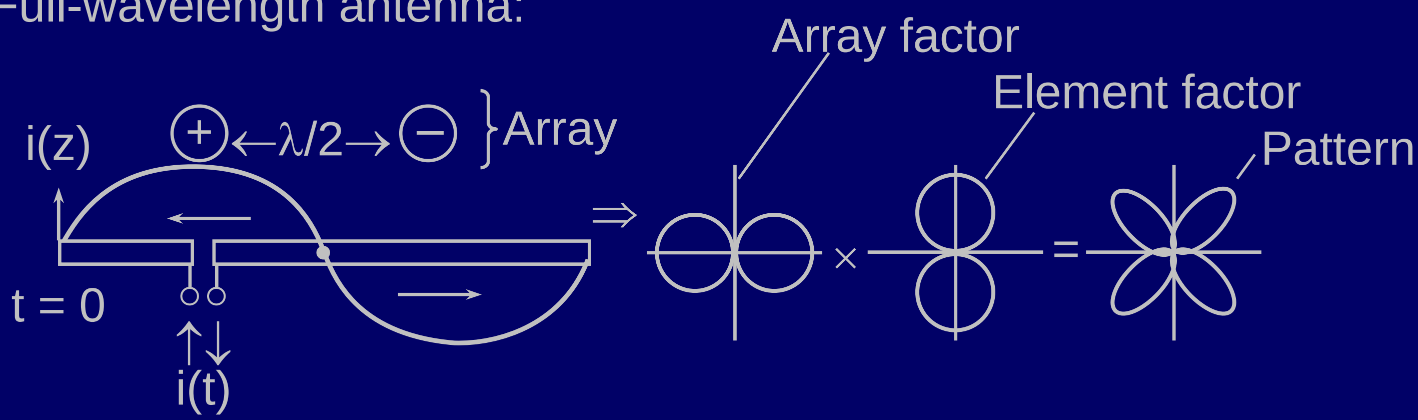
N identically oriented antenna elements

$$\underline{\overline{E}}_p \cong \underbrace{\underline{\overline{E}}(\theta,\phi)e}_{\text{from } \underline{I}_o \text{ at } x = y = z = 0}{}^{j\varphi_o} \sum_{i=1}^{N} \underline{A}_i e^{-j\varphi_i(\theta,\phi)}$$

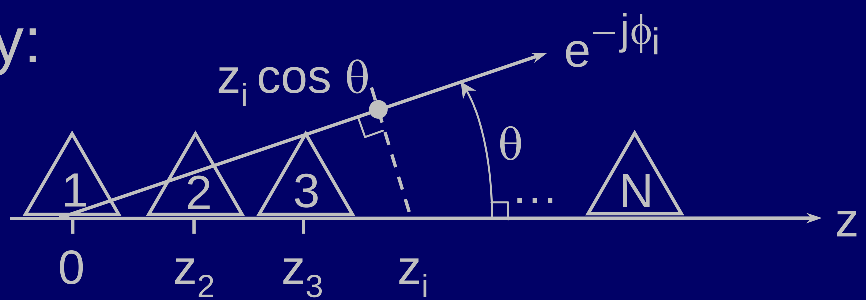
$$G(\theta,\phi) \propto \left|\underline{\overline{E}}\right|^2 = \underbrace{\left|\underline{\overline{E}}_o\{\theta,\phi\}\right|^2}_{\text{element factor}} \bullet \underbrace{\left|\sum_i \underline{A}_i e^{-j\varphi_i(\theta,\phi)}\right|^2}_{\text{array factor}}$$

Examples of Antenna Arrays

Full-wavelength antenna:



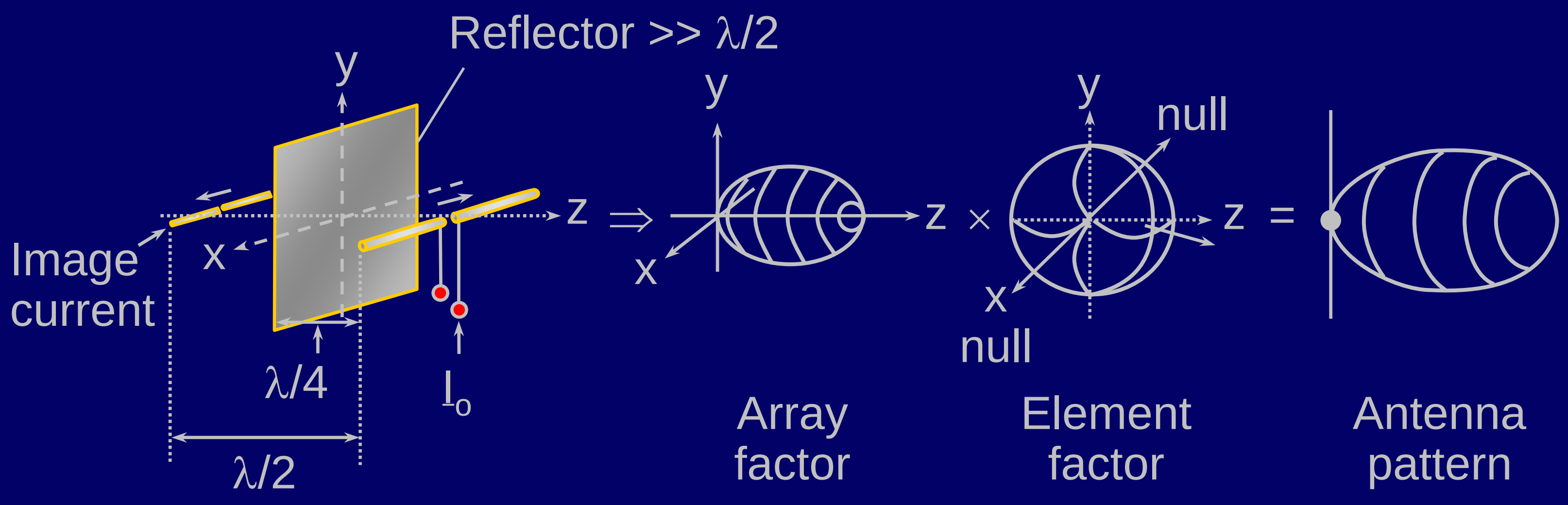
Linear array:



Let $\varphi_i = \alpha_i + \frac{2\pi}{\lambda} z_i \cos\theta = 0$ at $z = 0$;

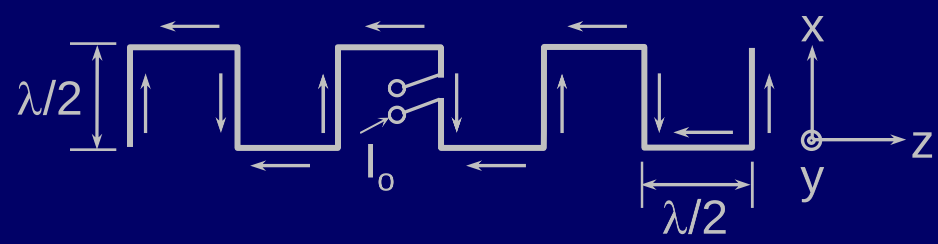
α_i is phase of ith element current

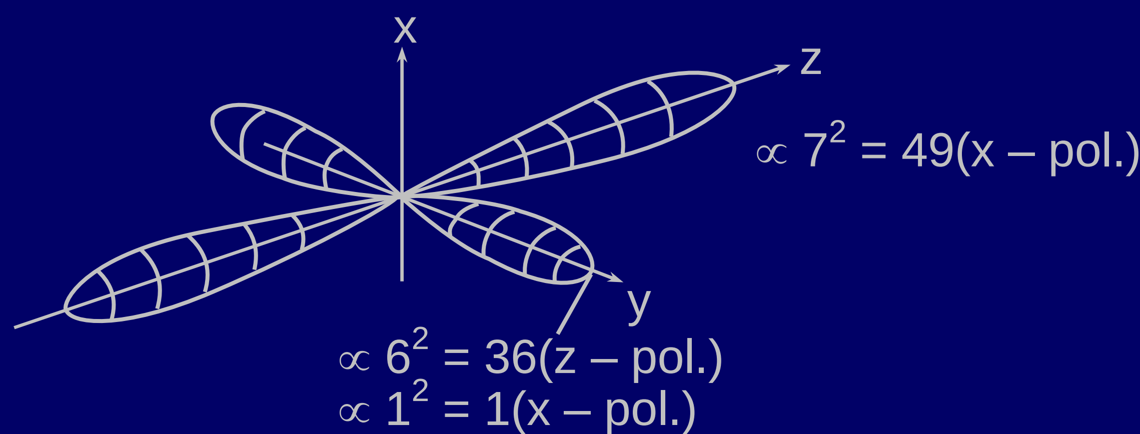
Linear Array Example:
Half-Wave Dipole Plus Reflector



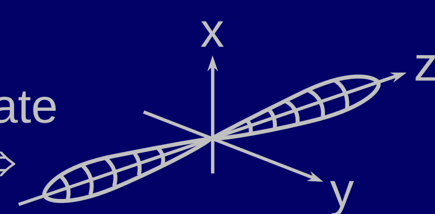
Linear Array Example: Jansky Antenna

Used in 1927 to discover galactic radiation at 27 MHz while seeking radio interference on AT&T transatlantic radio telephone circuits.





To cancel y-direction lobes, drive two duplicate antennas in phase, $\lambda/2$ apart in y-direction $\Rightarrow$



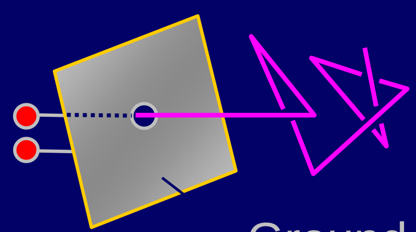
Genetic Algorithms for Designing Wire Antennas

1. Need performance metric, e.g. target gain or pattern plus cost function.

2. Need software tool to compute that metric for wire antennas.

3. Need vector description to represent each possible antenna.

4. Need genetic algorithm to randomly vary vector so its metric can be computed. The algorithm hill-climbs efficiently toward optimum design, especially if the antenna is not too reactive.
   Yields rats-nest configurations that perform well.
   e.g.



Ground plane

5. Metrics and vector descriptions favoring simplicity bias the design accordingly.