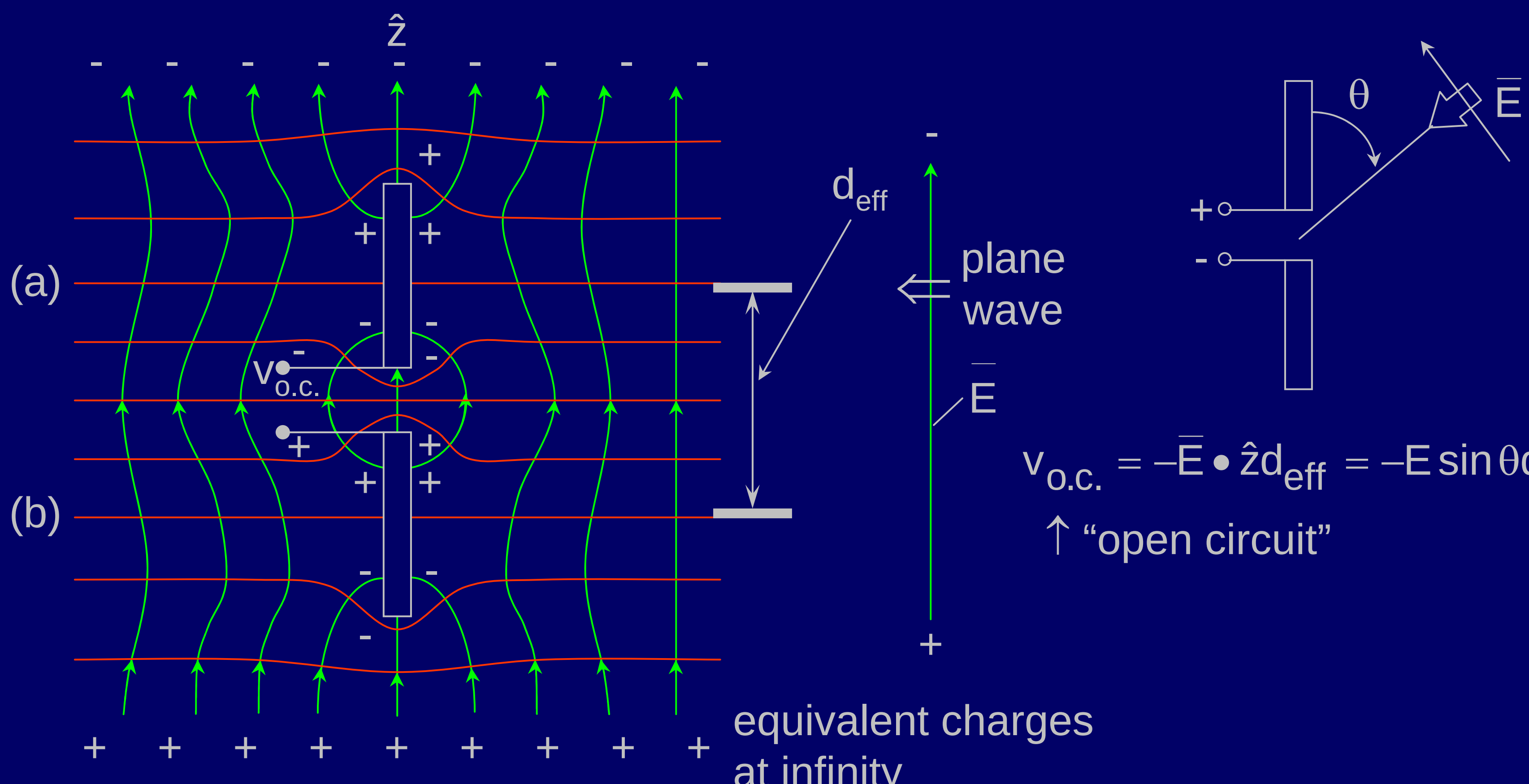
Receiving Properties of Antennas

Open-circuit voltage of short dipole antennas

For d << λ, quasistatic limit

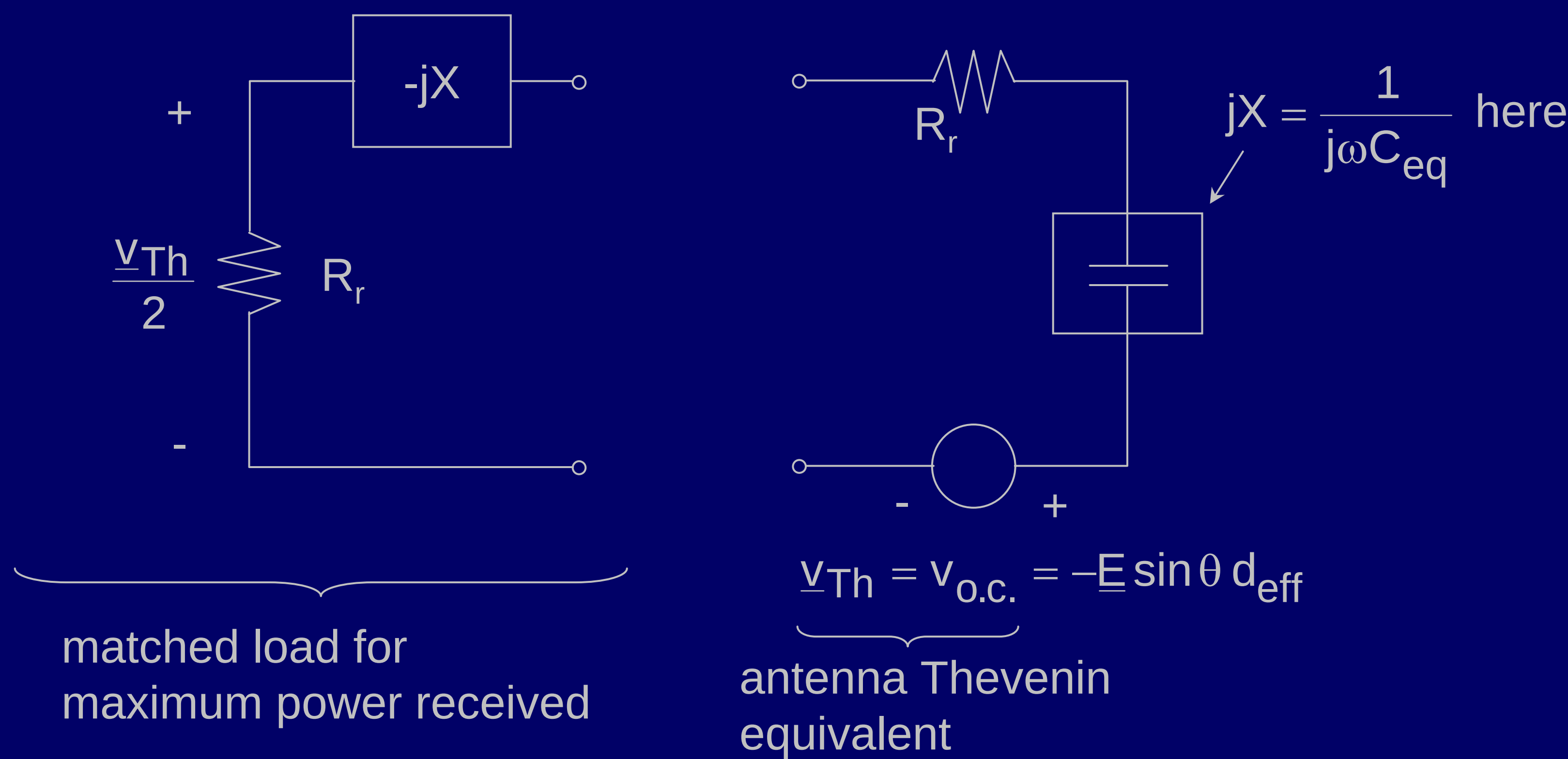
Note that equipotentials (a) and (b) intercept the dipole at the midpoints for $r_{wire} \rightarrow 0$, and are perpendicular.



$$v_{o.c.} = -\overline{E} \bullet \hat{z} d_{eff} = -E \sin\theta d_{eff}$$

↑ "open circuit"

Equivalent circuit for short dipole antennas



Available power from a short dipole antenna

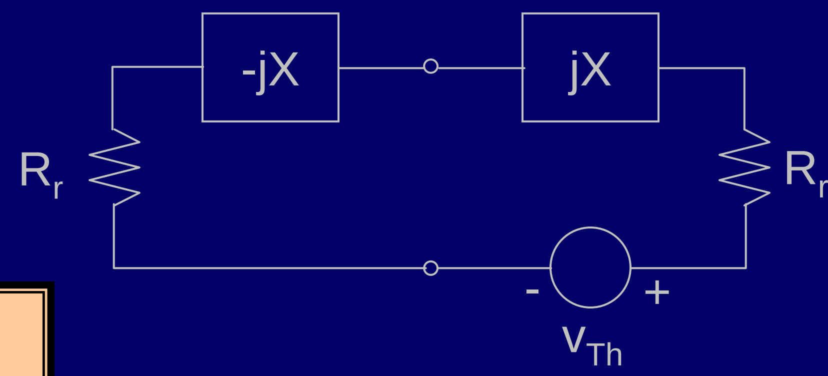
$$P_{rec} = \left(\frac{\underline{V}_{Th}}{2}\right)^2 \Big/ 2R_r = \frac{|\underline{E}|^2 d_{eff}^2}{8R_r}\sin^2\theta$$

$$= \underbrace{\frac{|\underline{E}|^2}{2\eta_o}}_{\substack{S(Wm^{-2})\\ \text{incident}}} \bullet \underbrace{\frac{\lambda^2}{4\pi} \bullet \frac{3}{2}\sin^2\theta}$$

$d_{eff_r} = d_{eff_t}$,

thin - wire,

short - dipole limit



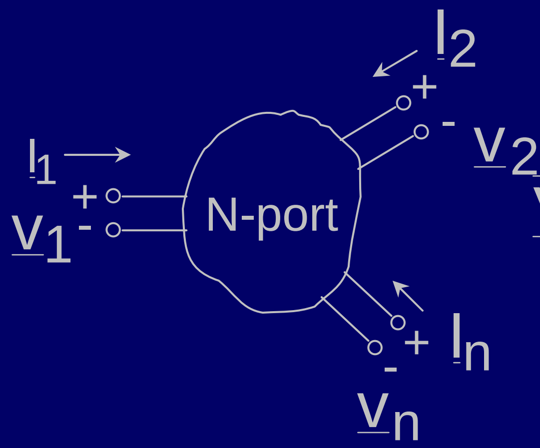
$$\underbrace{A_e(\theta,\phi)}_{\text{“effective area” (m}^2\text{)}} = \frac{\lambda^2}{4\pi}G(\theta,\phi)$$

↑ short dipole

$$R_r = (2\pi\eta_o/3)(d_{eff}/\lambda)^2$$

$$\underline{V}_{Th} = \underline{E}d_{eff}\sin\theta$$

Proof that A = Gλ²/4π for all reciprocal antennas



$$\underline{V} = \underline{\underline{Z}}\,\underline{I}, \text{ where } \underline{\underline{Z}} \leftrightarrow \underline{\underline{S}}$$

Impedance matrix for imbedded N-port

If reciprocity applies: $\underline{\underline{Z}}^t = \underline{\underline{Z}}, \ \underline{\underline{S}}^t = \underline{\underline{S}}$ (scattering matrix)

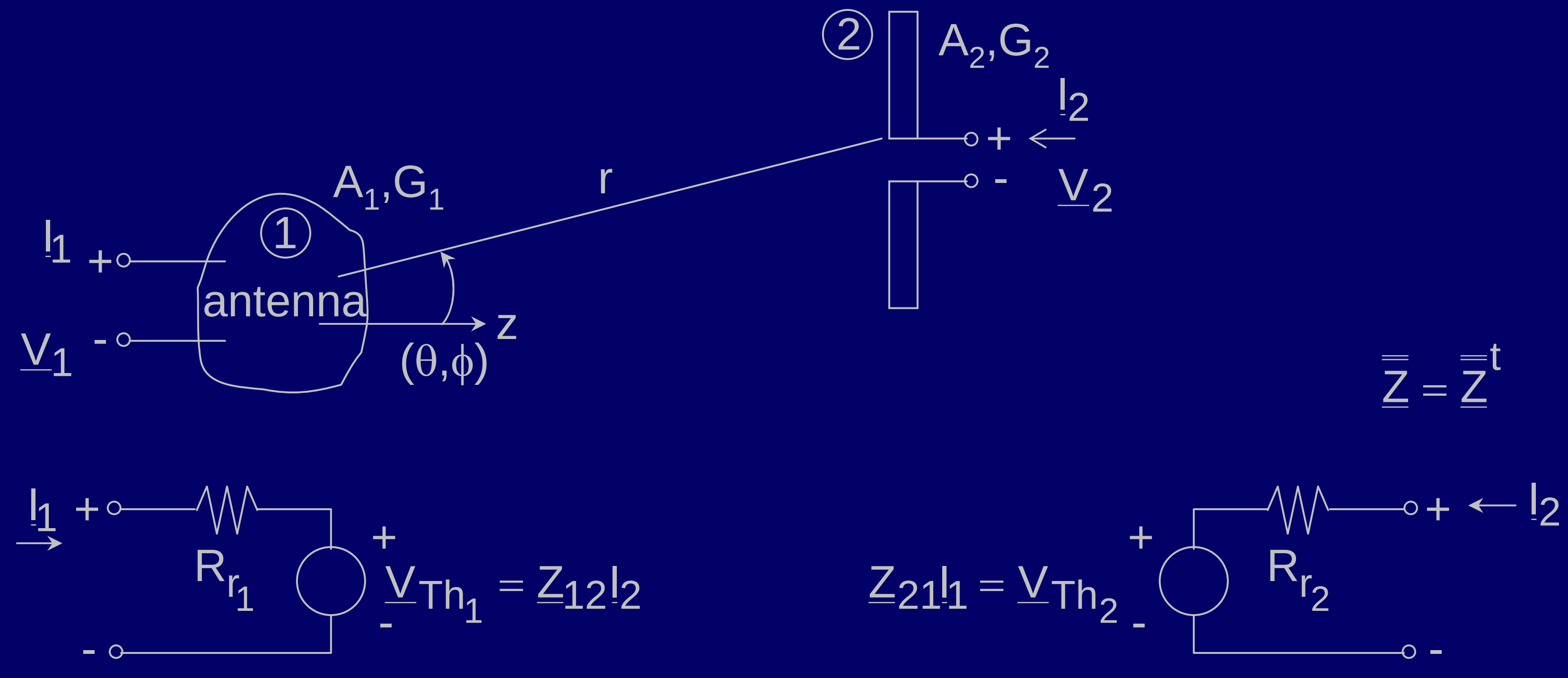
(Reference: Electromagnetic Waves, Staelin, Morgenthaler, and Kong, p. 459)

Reciprocity applies if $\underline{\underline{u}}^t = \underline{\underline{u}}, \ \underline{\underline{\varepsilon}}^t = \underline{\underline{\varepsilon}}, \ \underline{\underline{\sigma}}^t = \underline{\underline{\sigma}}$

[Excludes ferrites, magnetized plasmas, etc.]

(Reference: Op. Cit., p.454)

Proof that A = Gλ²/4π for all reciprocal antennas



Power received by antennas 1 and 2:

$$P_{r_1} = \left|\underline{Z}_{12}\underline{I}_2\right|^2 / 8R_{r_1} = P_{t_2}\frac{G_2}{4\pi r^2} \bullet A_1$$

$$P_{r_2} = \left|\underline{Z}_{21}\underline{I}_1\right|^2 / 8R_{r_2} = P_{t_1}\frac{G_1}{4\pi r^2} \bullet A_2$$

Proof that $A = G\lambda^2/4\pi$ for all reciprocal antennas

Power received by antennas 1 and 2:

$$P_{r_1} = \left|\underline{Z}_{12}\underline{I}_2\right|^2 / 8R_{r_1} = P_{t_2}\frac{G_2}{4\pi r^2} \bullet A_1$$

$$P_{r_2} = \left|\underline{Z}_{21}\underline{I}_1\right|^2 / 8R_{r_2} = P_{t_1}\frac{G_1}{4\pi r^2} \bullet A_2$$

Thus $P_{r_2} / P_{r_1} = \dfrac{G_1 A_2 P_{t_1}}{G_2 A_1 P_{t_2}}$ Therefore $\dfrac{A_1}{G_1} = \dfrac{A_2}{G_2} \bullet \dfrac{P_{t_1}}{P_{t_2}} \bullet \dfrac{P_{r_1}}{P_{r_2}}$

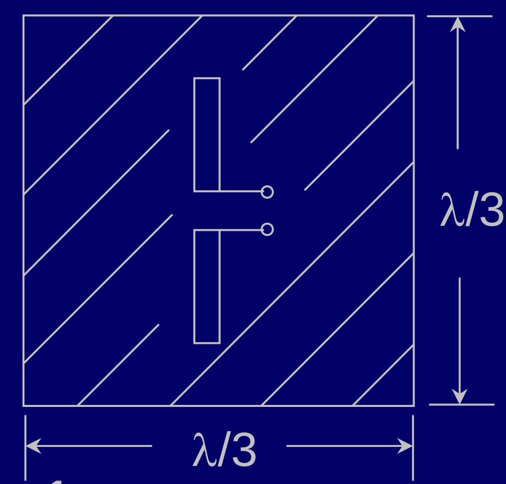
But $\dfrac{P_{r_1}}{P_{r_2}} = \dfrac{\left|\underline{Z}_{12}\underline{I}_2\right|^2 R_{r_2}}{\left|\underline{Z}_{21}\underline{I}_1\right|^2 R_{r_1}} = \dfrac{P_{t_2}}{P_{t_1}}$ if $\underline{Z}_{12} = \underline{Z}_{21}$

Therefore $\dfrac{A_1}{G_1} =$ $\dfrac{A_2}{G_2} = \dfrac{\lambda^2}{4\pi}$ for all antennas if $\underline{\overline{\overline{\mu}}}^t = \underline{\overline{\overline{\mu}}}, \underline{\overline{\overline{\varepsilon}}}^t = \underline{\overline{\overline{\varepsilon}}}$

Example: A_{eff} for short dipole

$$A = G\frac{\lambda^2}{4\pi} \le \underbrace{\frac{3\lambda^2}{8\pi}}_{\text{max if matched}} \cong \left(\frac{\lambda}{3}\right)^2 \ne f(d_{eff})$$

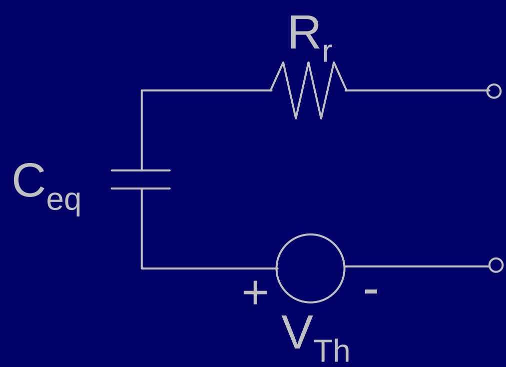
(G: $\le 3/2$)



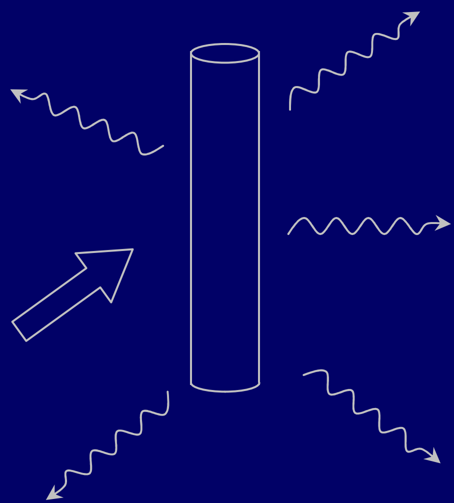
e.g. λ = 300 m @ 1 MHz, yet d ≅ 1 m on car

e.g. cell phone @ 900 MHz → λ ≅ 30 cm, d ≅ 15 cm

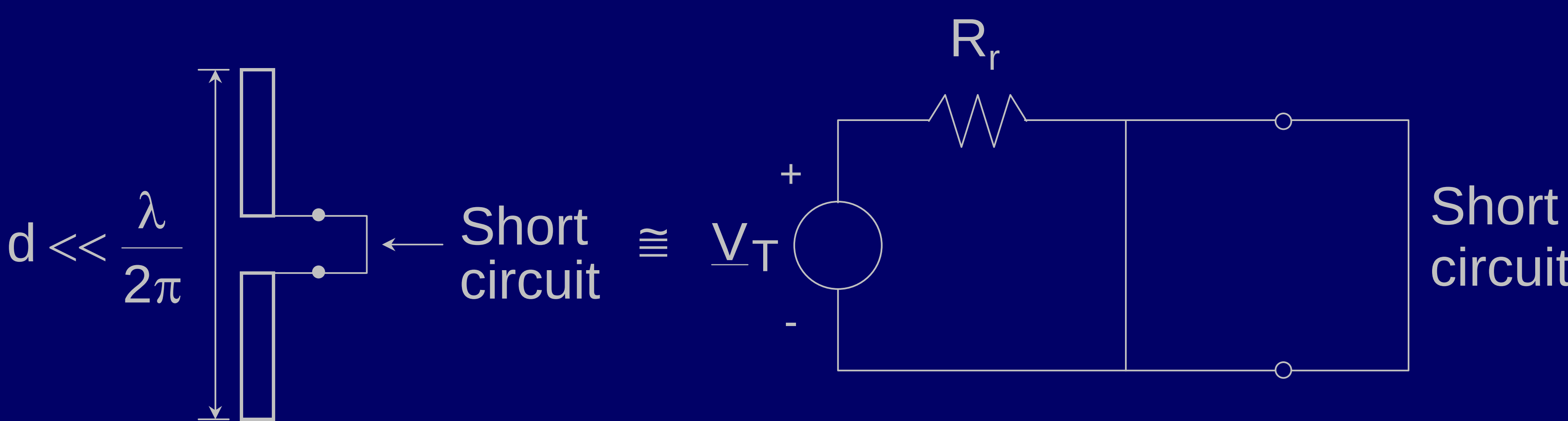
Note: A_{eff} can be much larger than physical antenna when the load is roughly impedance matched, but this match may provide excessively narrow bandwidth $\Delta\omega \cong 1/R_rC_{eq}$



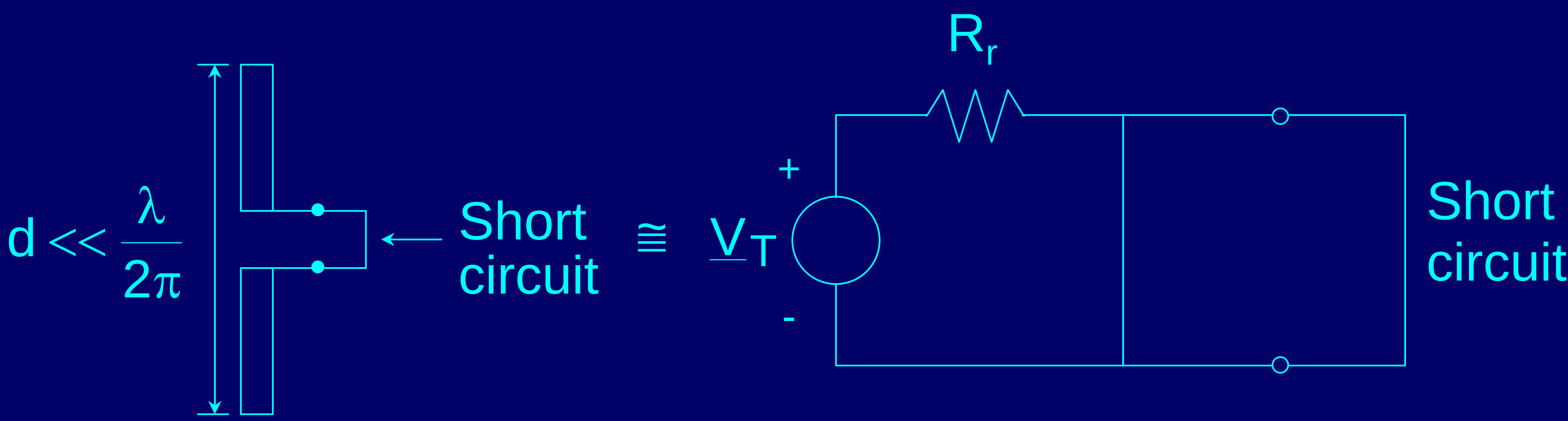
Multi-conductor wire antennas



Short dipoles scatter.
How much?



Multi-conductor wire antennas

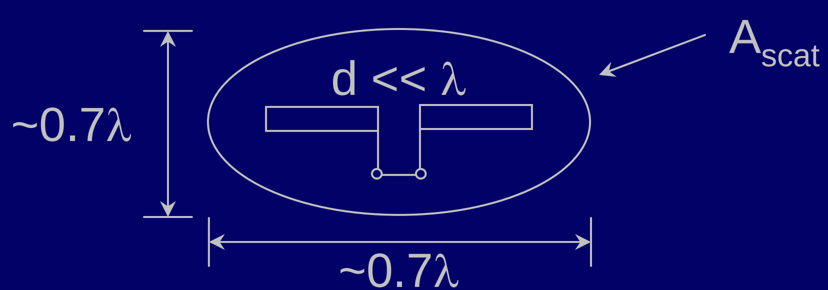


Recall:

$$P_{rec} = \left|\underline{V}_{Th}\right|^2 / 8R_r \cong S_{inc} \bullet A_{matched} = \left(\frac{\left|\underline{E}\right|^2 d_{eff}^2}{8R_r} \sin^2\theta\right)$$

$$P_{scat} = \left|\underline{V}_{Th}\right|^2 / 2R_r \cong S_{inc} \bullet A_{scat} \qquad S_{inc} = \left|\underline{E}\right|^2 / 2\eta_o \; \left(Wm^{-2}\right)$$

Therefore $A_{scat} \cong 4A_{matched} \leq 3\lambda^2/2\pi$ (for short dipoles)



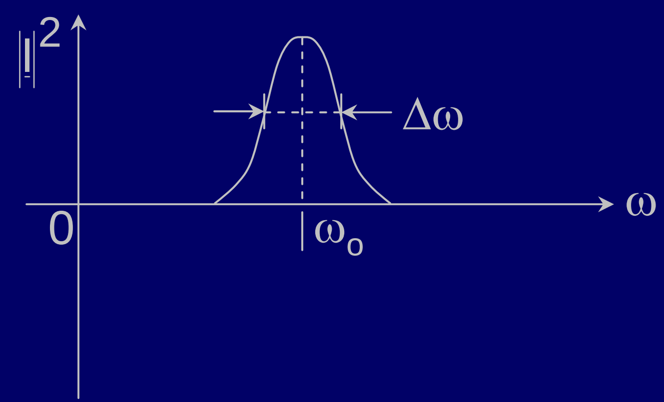
Scattering from a half-wave dipole

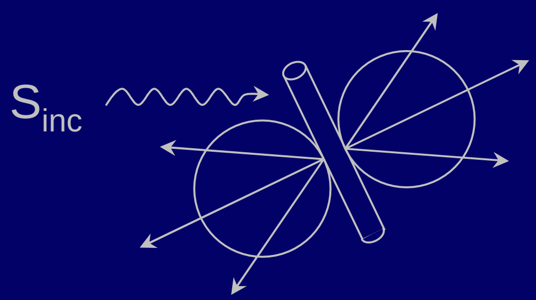
$R_r \cong 73\Omega$, $G \leq 1.64$, $X \cong 0$ because $W_e \cong W_m$

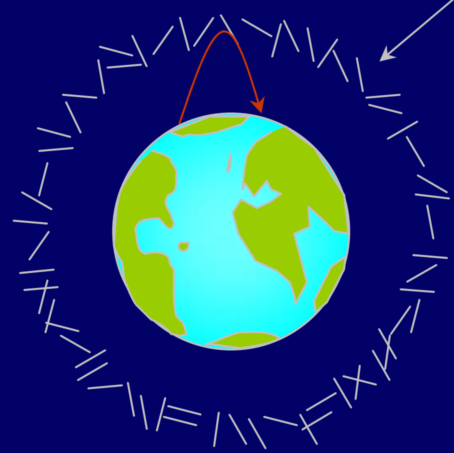
Most EM energy ($W_T = W_e + W_m \cong 2W_m$) is stored within a few wire radii

$$\text{Resonance } Q = \frac{\omega_o W_T}{P_d} = \omega/\Delta\omega \cong 10$$

where $\omega_o = 2\pi c/\lambda$, $P_d \equiv P_r$

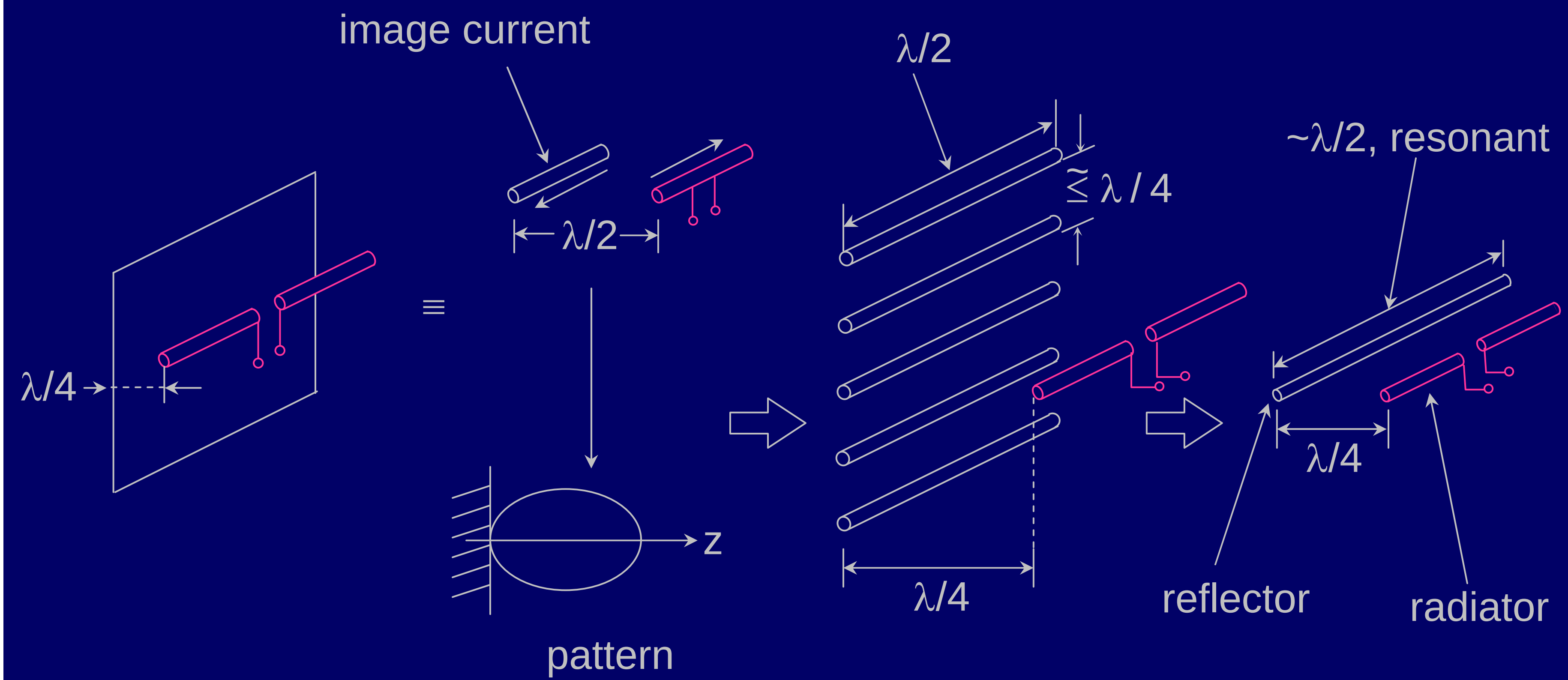




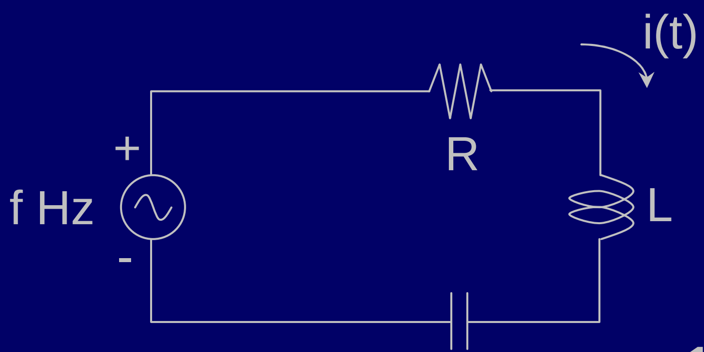


Orbiting $\lambda/2$ needles for passive satellite communications link (artificial ionosphere)

Scattering from parasitic antenna elements



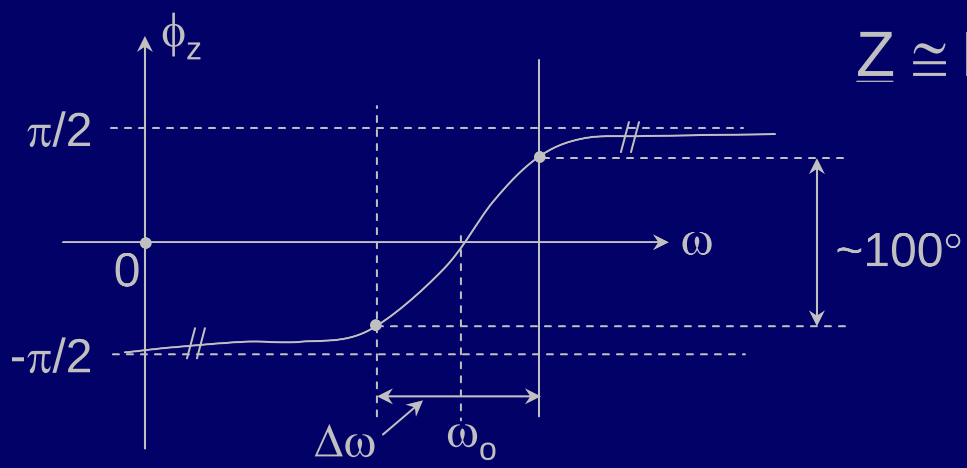
Phase control in isolated wires



s = jω here

$$\underline{V} = \underline{I}\,\underline{Z} = \underline{I}\left(R + Ls + 1/Cs\right) = \underline{I}\left|\underline{Z}\right|e^{j\phi_z}$$

$$\omega_o = 1/\sqrt{LC}$$



$$\underline{Z} \cong R + jX_o\left(\omega - \omega_o\right) \text{ for } \omega \cong \omega_o$$

$$Q = \frac{\omega_o}{\Delta\omega}$$

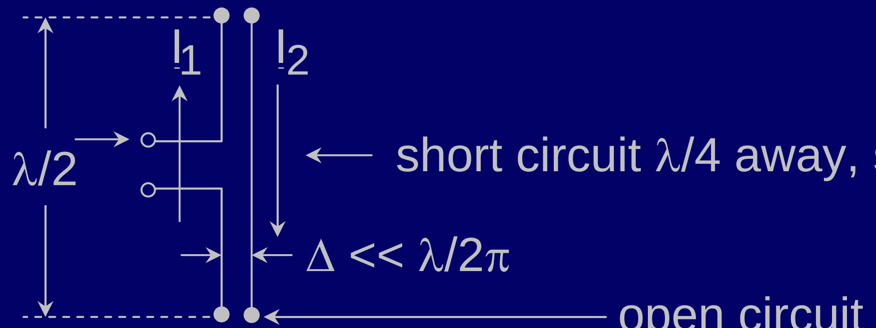
Control phase ϕ in wire by:

reducing $\omega_o \rightarrow 0° < \phi < 90°$ (lengthen wire)

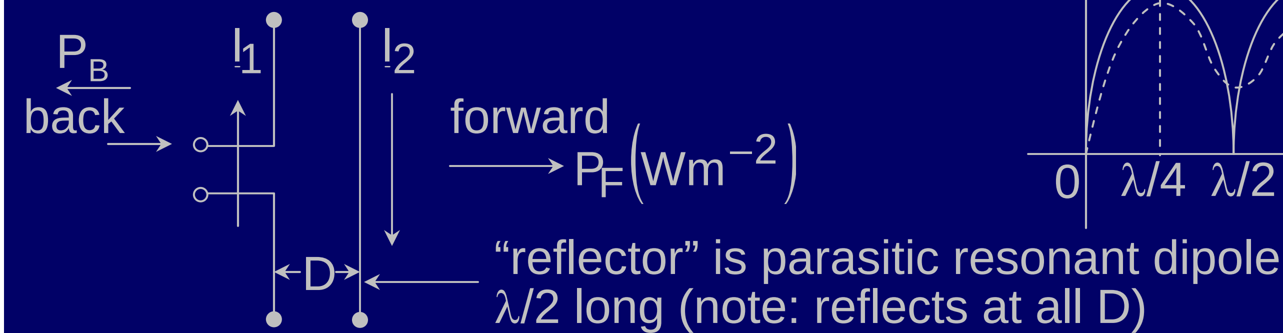
increasing $\omega_o \rightarrow -90° < \phi < 0°$ (shortening wire)

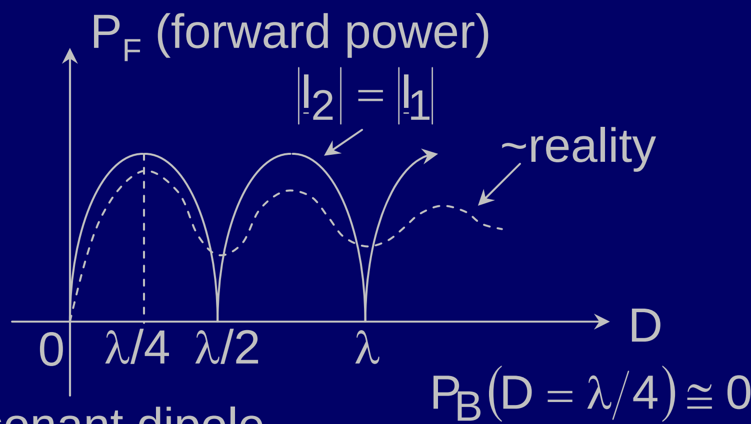
Increase Q and $\partial\phi/\partial\omega$ by increasing W_T (thinning the wire)

Directivity of parasitic wire antennas

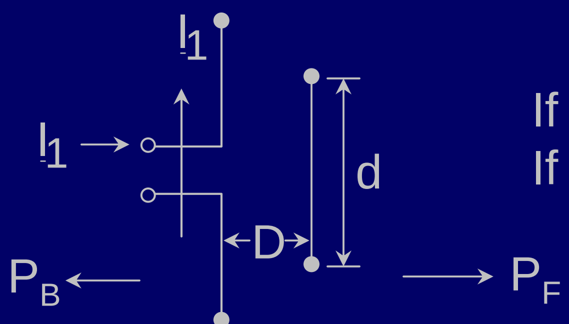


Reflectors:





Directors:

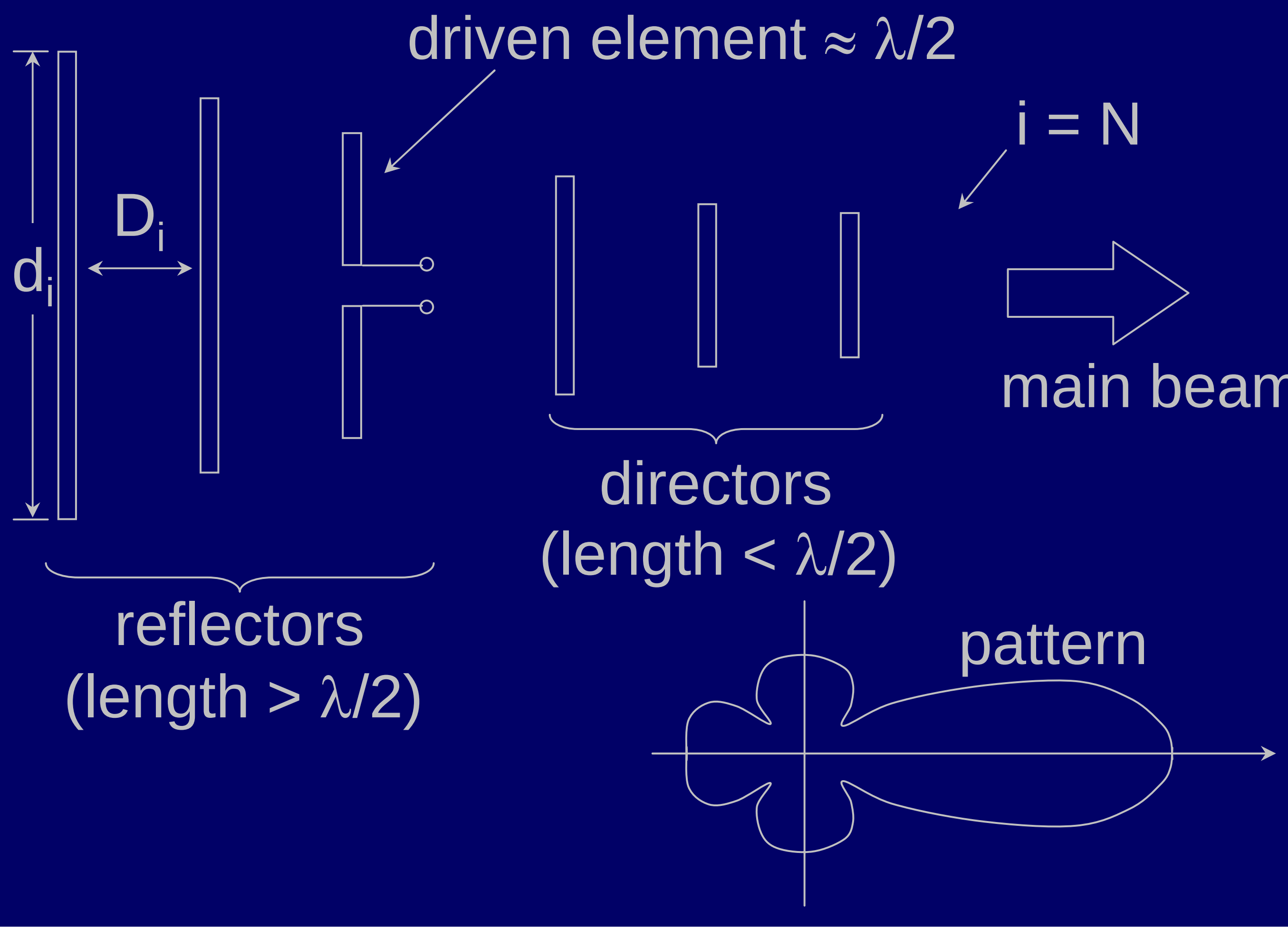


If $d \neq \lambda/2$, then $\phi \neq 0$

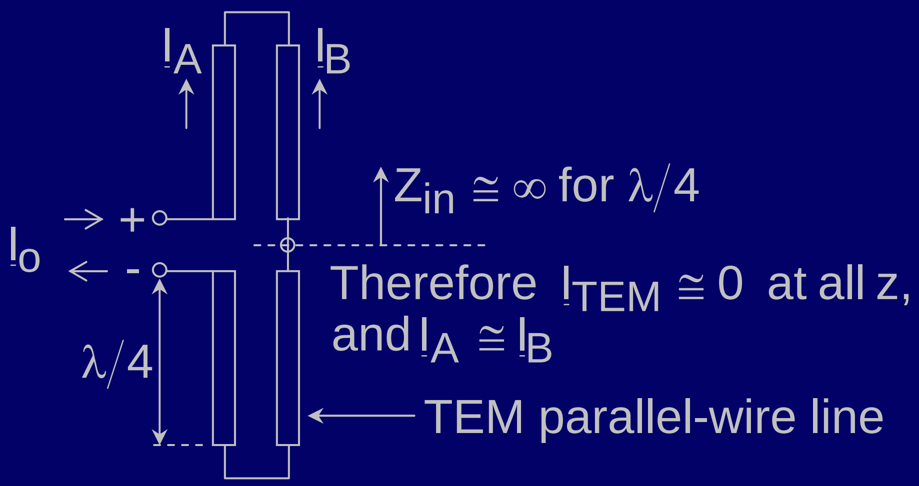
If $D,\phi \ni P_B \cong 0$ and $P_F \neq 0$, then parasitic element is "director"

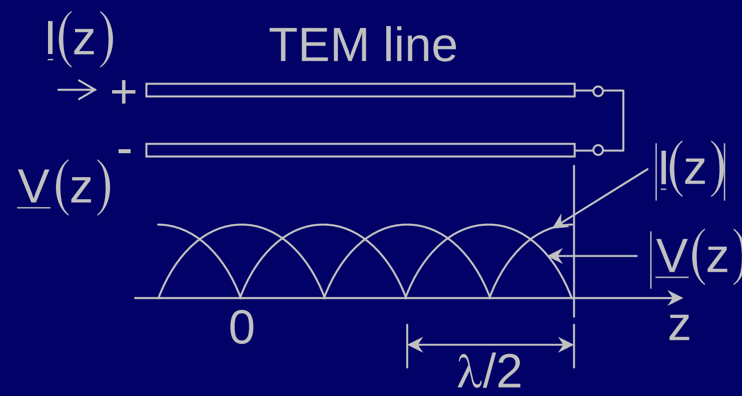
Multiple parasitic wires, Yagi antenna

Choice of d_i, D_i, (i = 1, …N) originally was an art. Now computers can optimize chosen specifications (e.g. bandwidth, reactance, directivity)

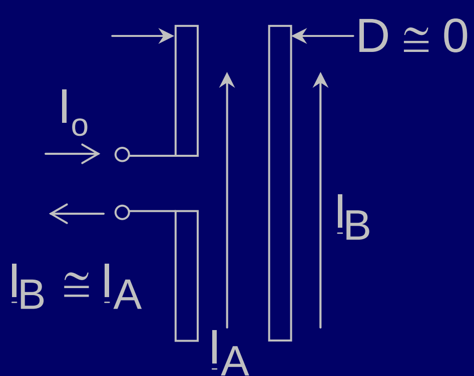


Half-wave folded dipole antenna





Equivalent to:



"Half-wave folded dipole"

$$P_t = I_o^2 R_{r_o} / 2 \quad \text{(single dipole)}$$

$$P_t = (2I_o)^2 R_{r_o} / 2 = 2I_o^2 R_r \quad \text{(folded dipole)}$$

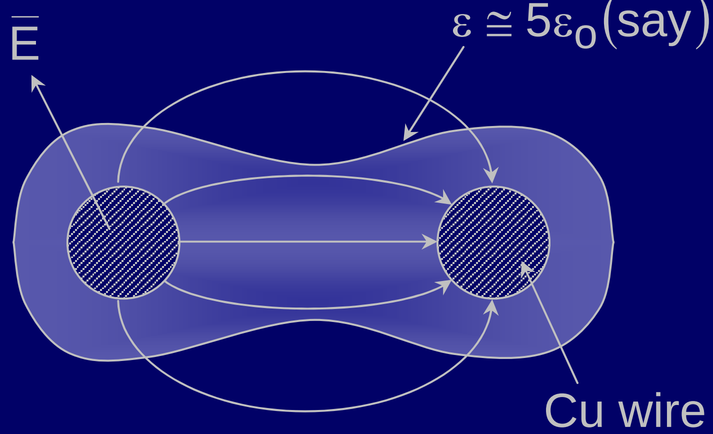
Therefore $R_r = 4R_{r_o} \cong 300\Omega$

Half - wave dipole $R_{r_o} \cong 73\Omega$

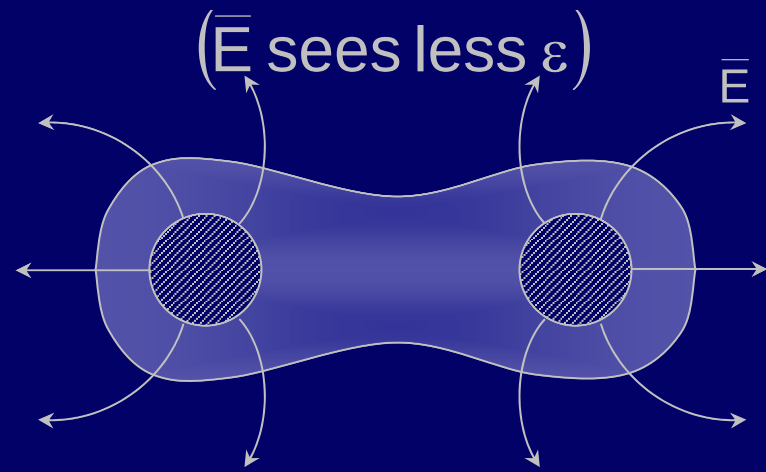
Half-wave folded dipole antenna

Cross-section of TEM “twin lead” line:

“TEM” mode



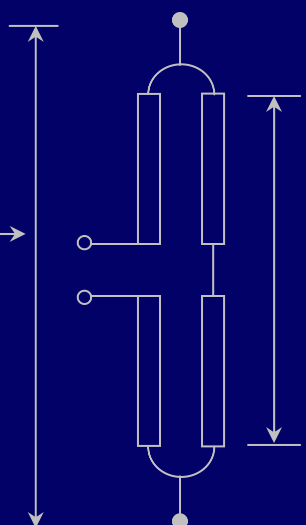
Common mode



$c_{TEM} < c_{common\ mode}$ and $v \cong 1/\sqrt{\mu\varepsilon}$, so $\lambda_{common} > \lambda_{TEM}$

Therefore

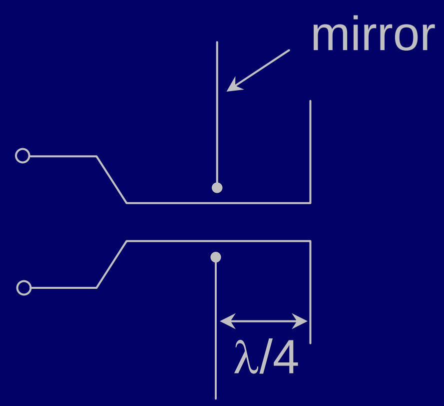
$\lambda/2$ for common mode to radiate →



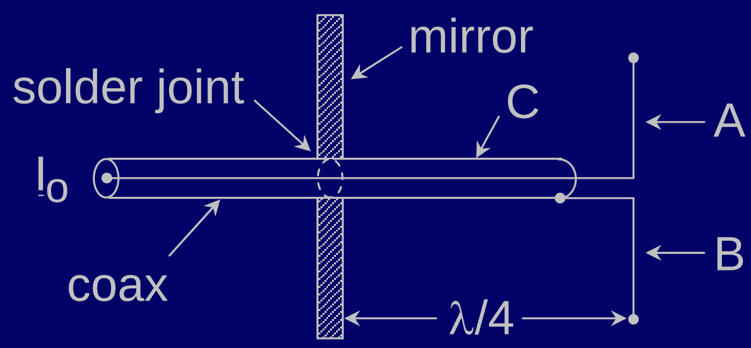
$\lambda/2$ for TEM mode to force $I_A = I_B$

A Balun couples balanced to unbalanced systems

e.g., this is okay

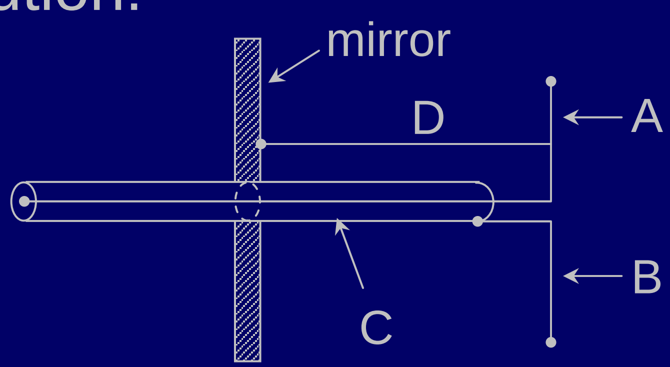


Suppose we want:



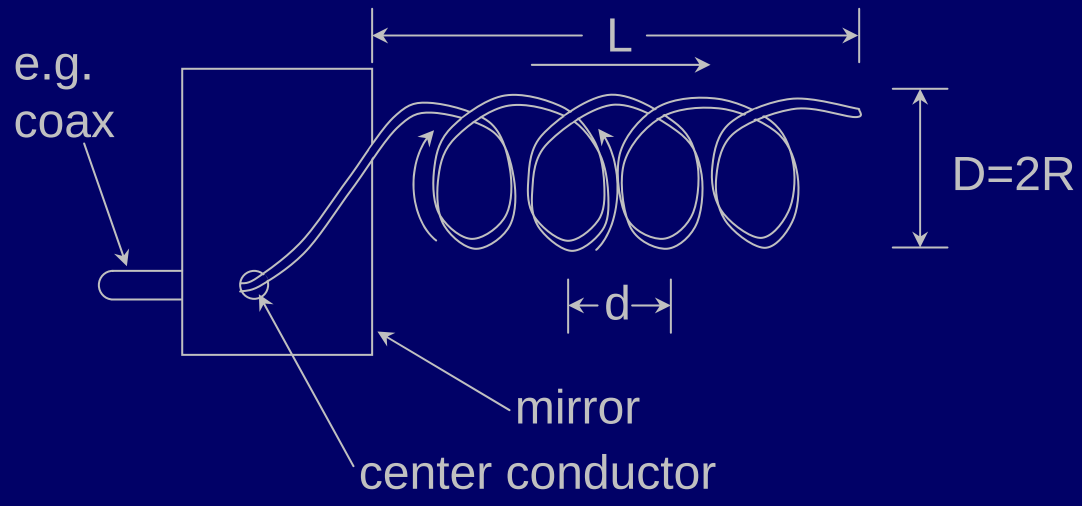
But current will flow down the outside of C instead of into B

Solution:



Conductors C and D form λ/4 TEM line shorted at the mirror, yielding an open circuit at coax end, forcing current into Ⓑ

Helical antenna

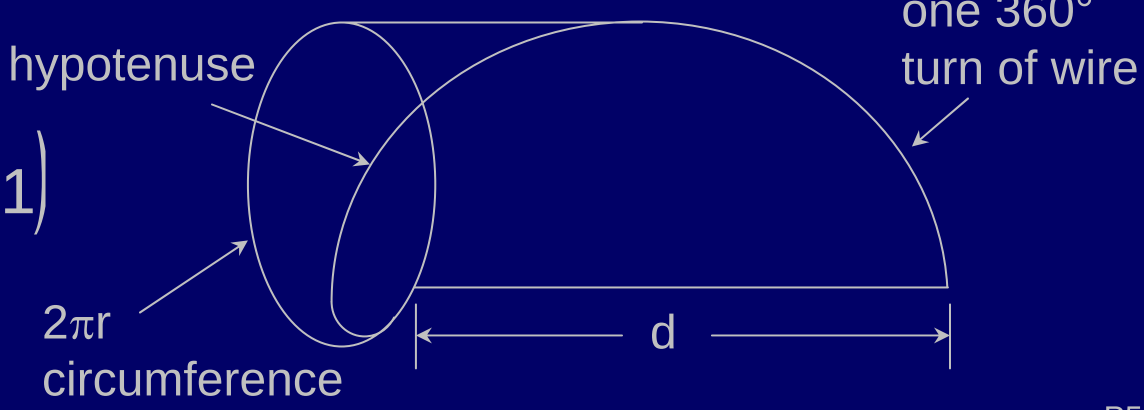


Waves add in phase in the forward direction if

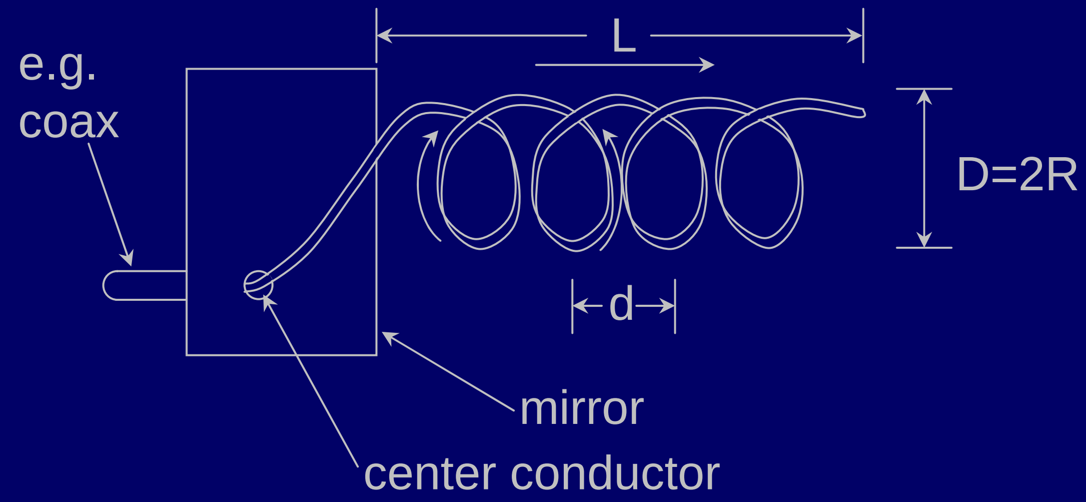
$$\sqrt{(2\pi r)^2 + d^2} - d = n\lambda$$

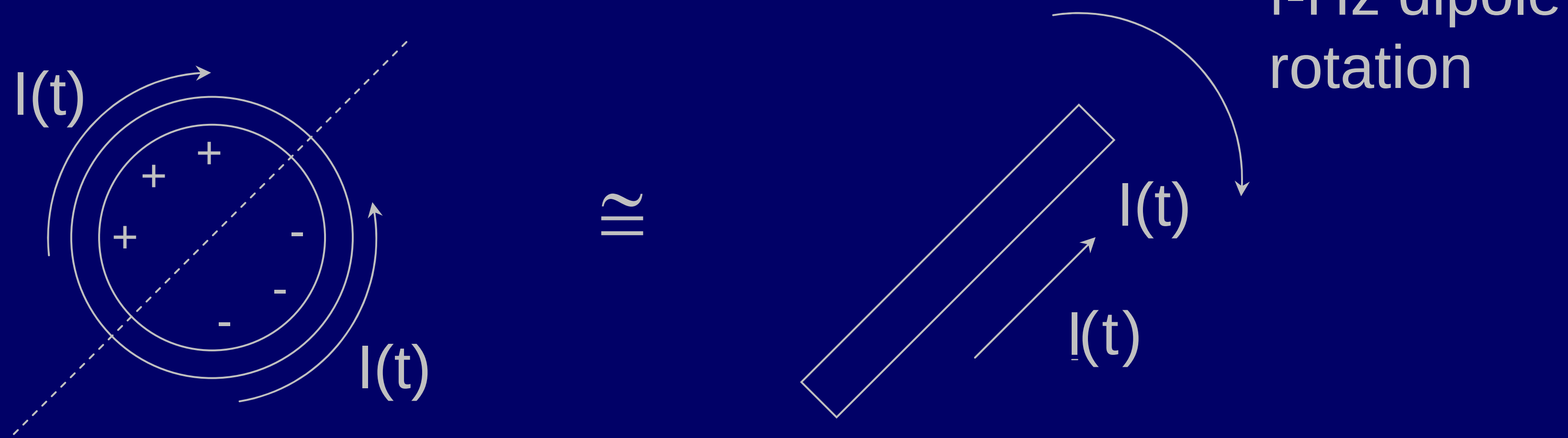
$$\left(\text{If } r = d,\ d = n\lambda / \sqrt{4\pi^2 + 1} - 1\right)$$

If L >> D, standing wave at end is small because of radiation losses. Assume ~ TEM propagation



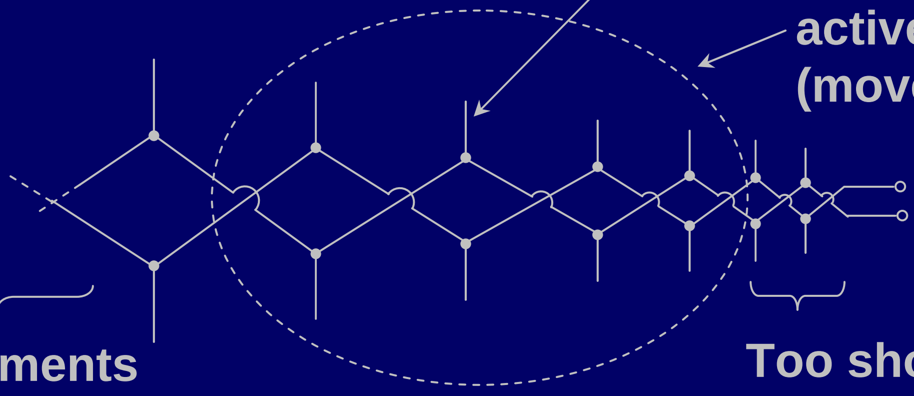
Helical antenna



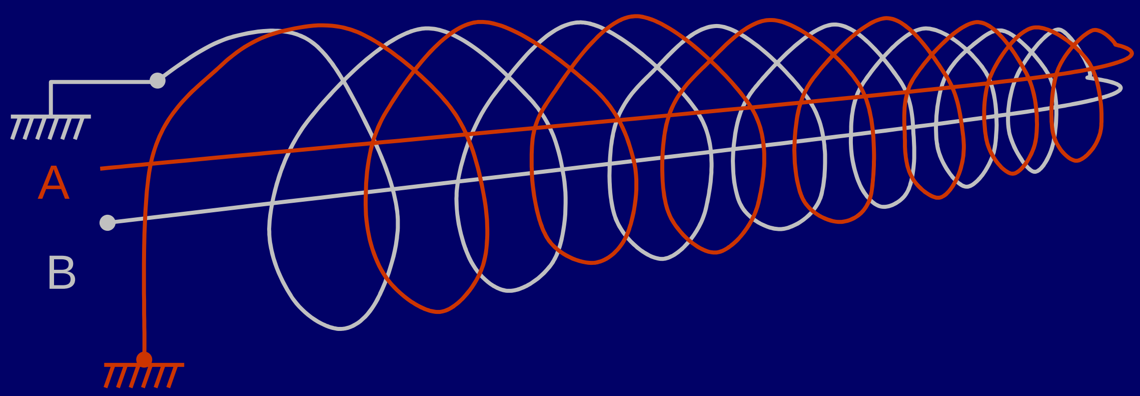


Long helices have weaker standing waves (less current at end)

Log-periodic antennas



Pattern, impedance $\neq f(f)$ (approximately)



Log spiral, radiates circular polarization