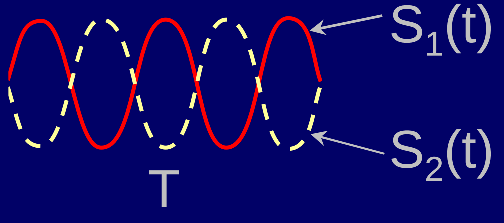
Receivers, Antennas, and Signals

Modulation and Coding

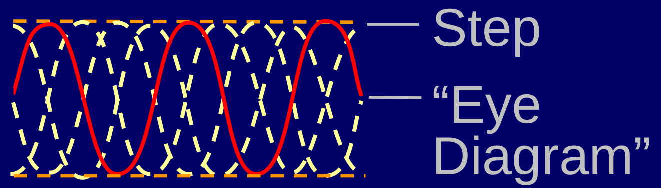
Professor David H. Staelin

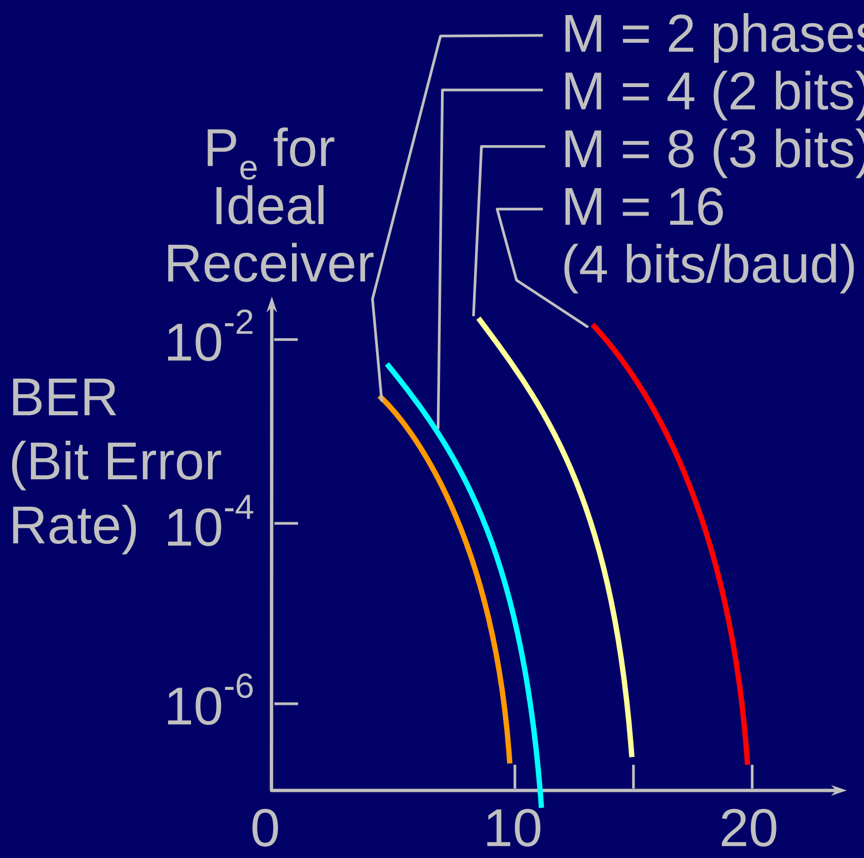
Multi-Phase-Shift Keying, “MPSK”

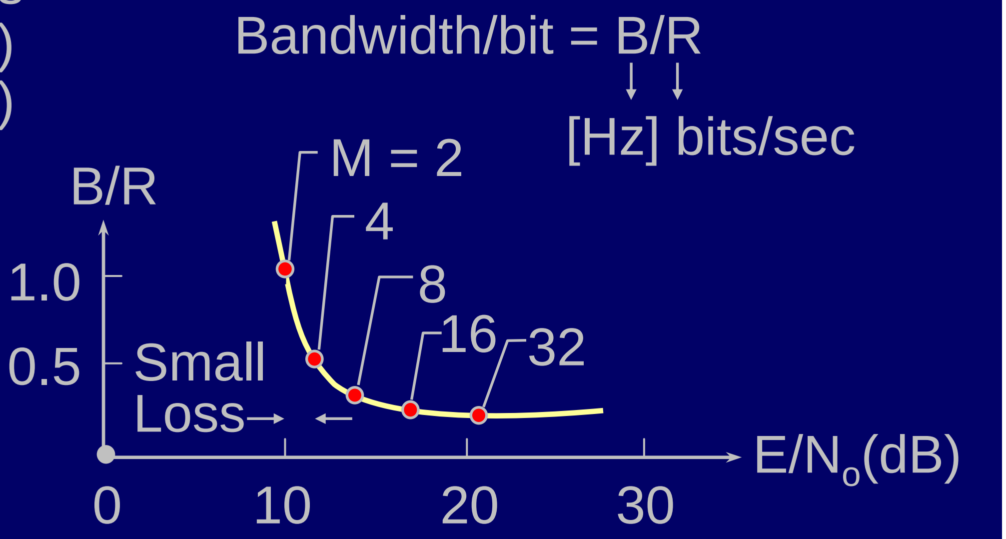
BPSK: Binary Phase-Shift Keying



QPSK: Quadrature Phase-Shift Keying

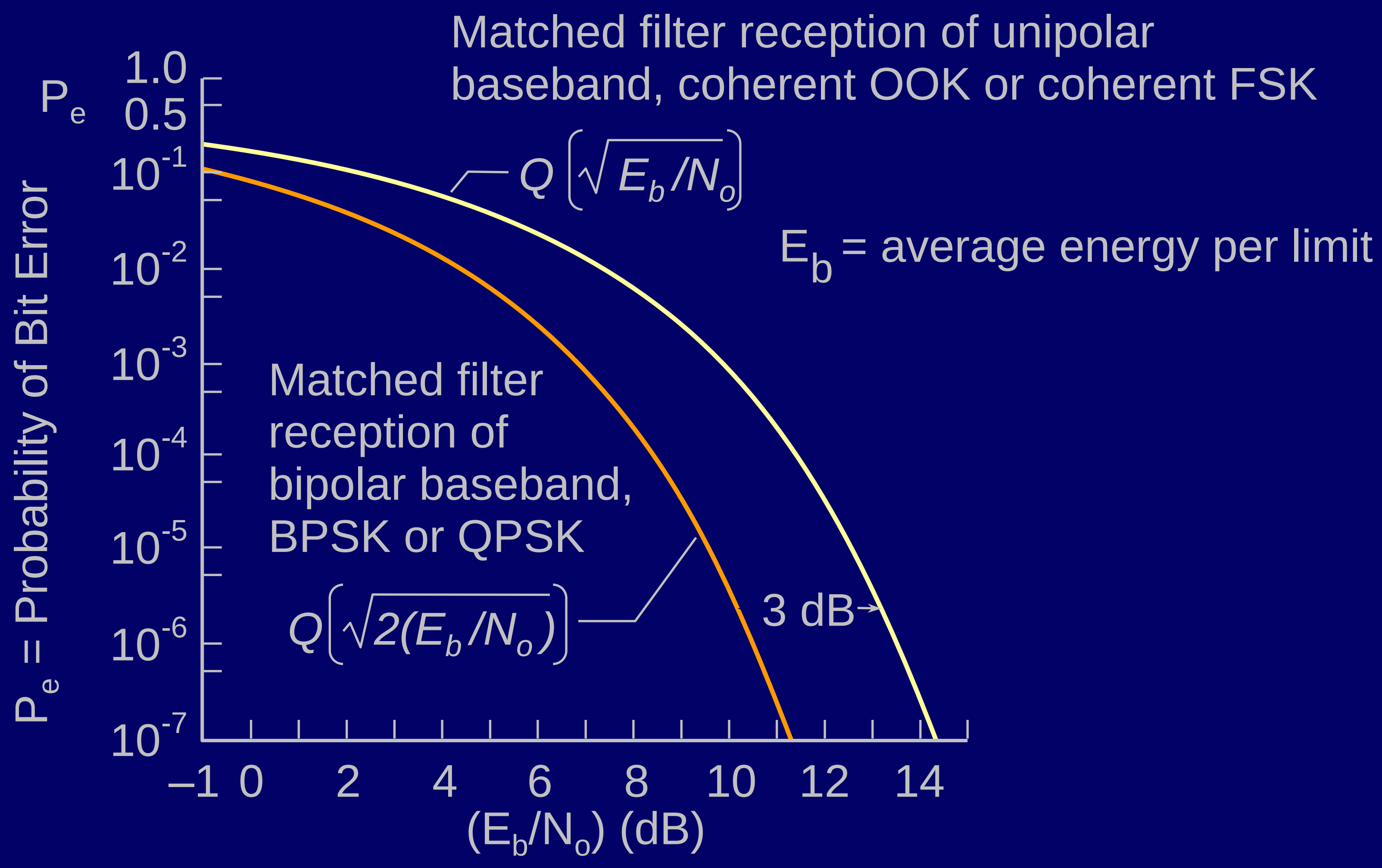






QPSK is bandwidth efficient, but has little E/N_o penalty.

Error Probabilities for Binary Signaling



Matched filter reception of unipolar baseband, coherent OOK or coherent FSK

$Q\left(\sqrt{E_b/N_o}\right)$

E_b = average energy per limit

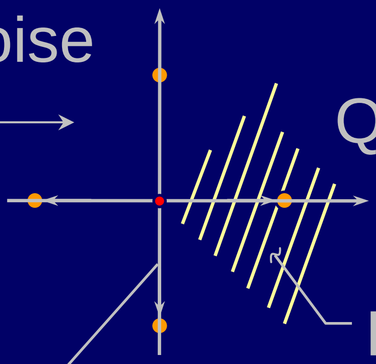
Matched filter reception of bipolar baseband, BPSK or QPSK

$Q\left(\sqrt{2(E_b/N_o)}\right)$

Source: Digital and Analog Communication Systems, L.W. Couch II, (4th Edition), Page 351

Phasor Diagrams

$S(t) = R_e\{\underline{S}e^{j\omega t}\}$



Im{$\underline{S}$}

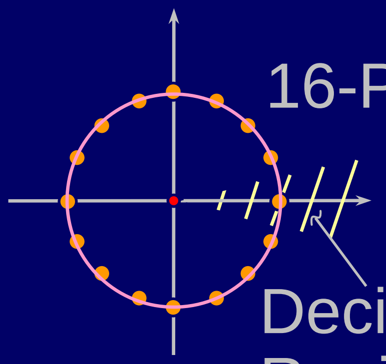
Equal-Noise Contour

QPSK

Re{$\underline{S}$}

Phasor

Decision Region for H_1

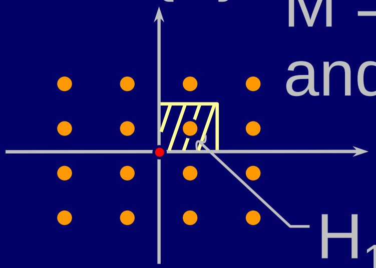


Im{$\underline{S}$}

16-PSK

Re{$\underline{S}$}

Decision Region for H_1



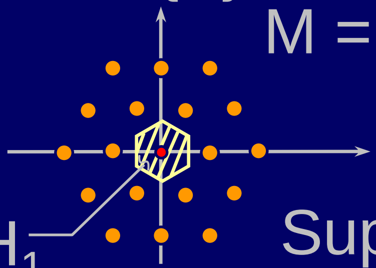
Im{$\underline{S}$}

M = 16, Amplitude and Phase

Re{$\underline{S}$}

H_1

Rectangular



Im{$\underline{S}$}

M = 19

Re{$\underline{S}$}

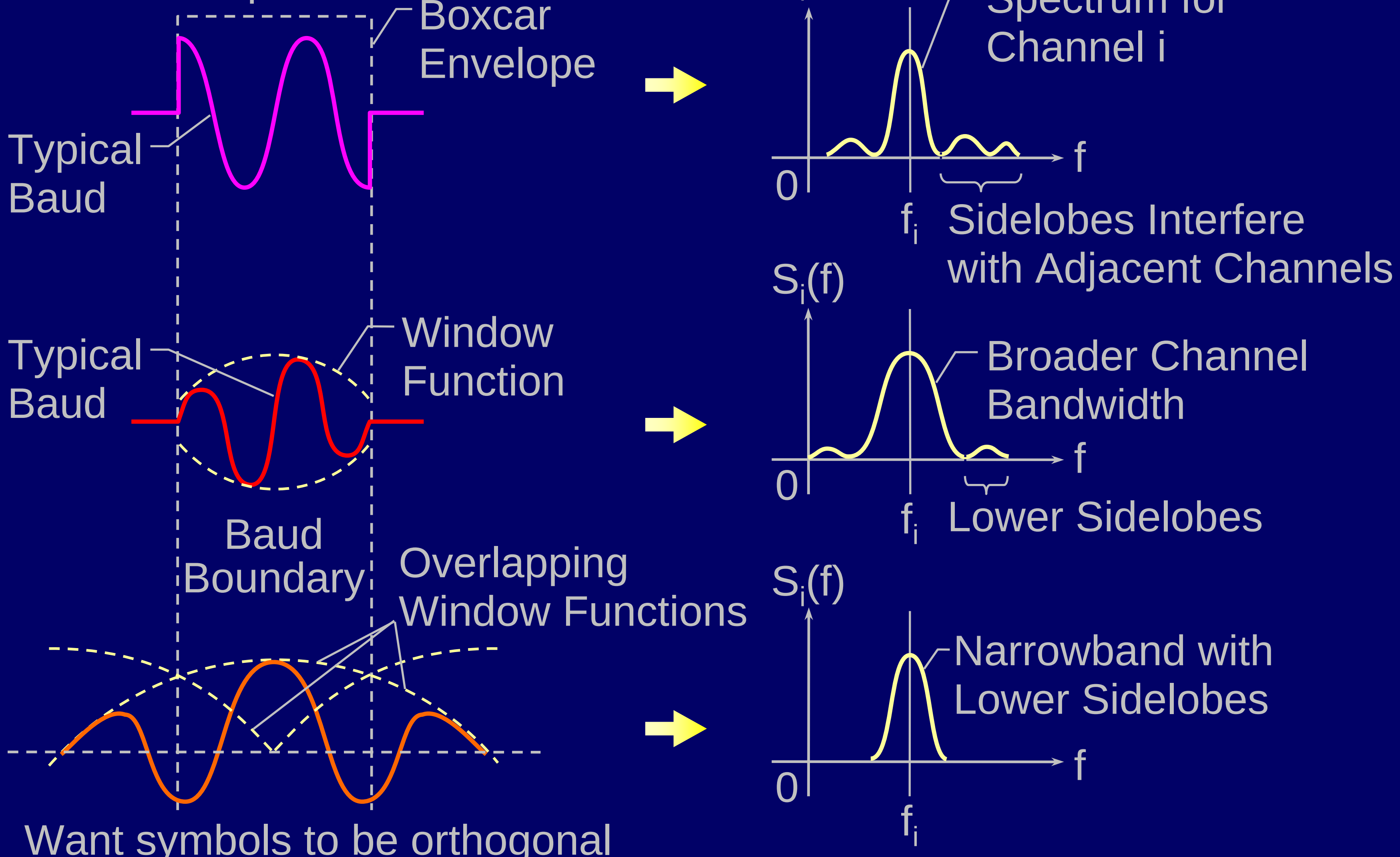
H_1

Superior to Rectangular

Hexagonal

Intersymbol Interference

MPSK Examples:



Boxcar Envelope

Typical Baud

Spectrum for Channel i

Sidelobes Interfere with Adjacent Channels

Window Function

Typical Baud

Broader Channel Bandwidth

Lower Sidelobes

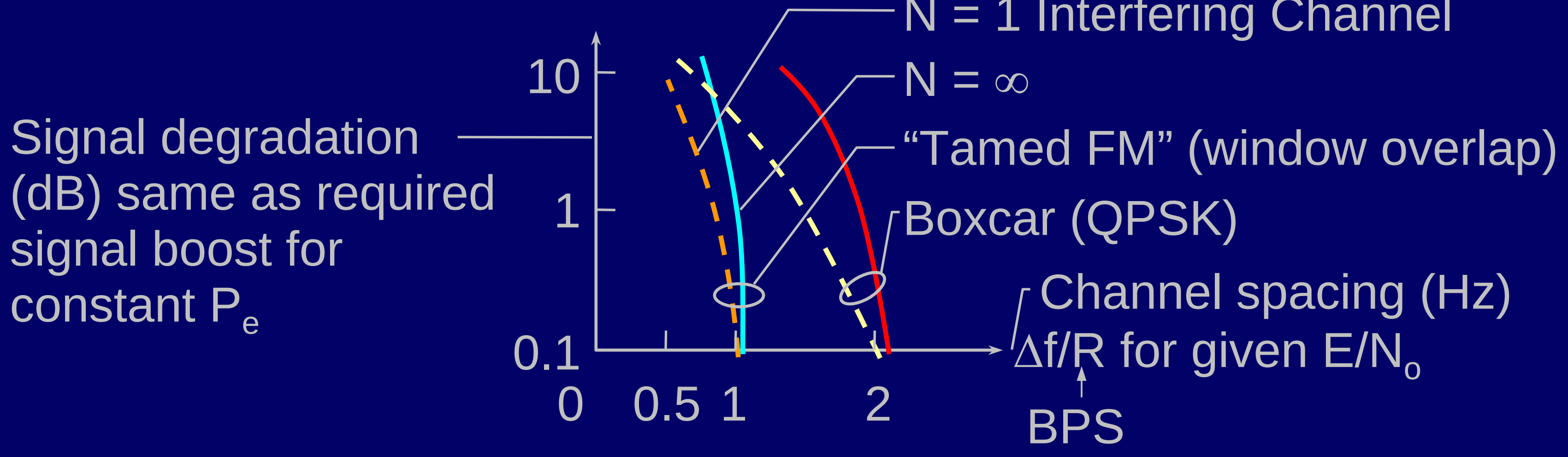
Baud Boundary

Overlapping Window Functions

Narrowband with Lower Sidelobes

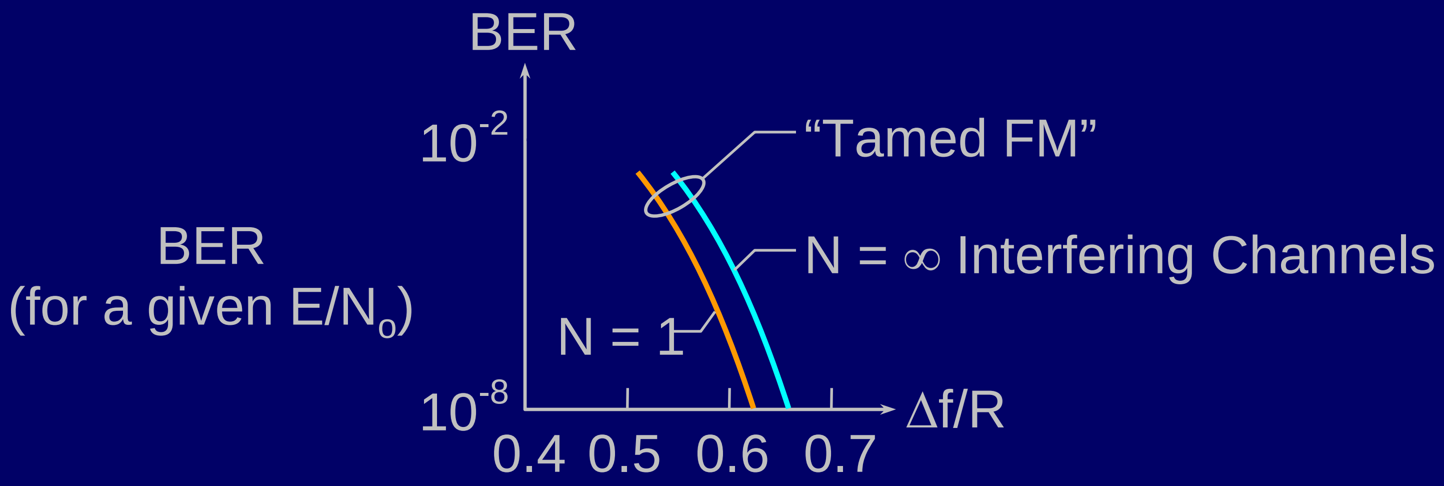
Want symbols to be orthogonal within and between windows

Performance Degradation Due to Interchannel Interfence



Closer channel spacing requires more signal power to maintain P_e

Recover by boosting signal power (works until N_o becomes negligible)

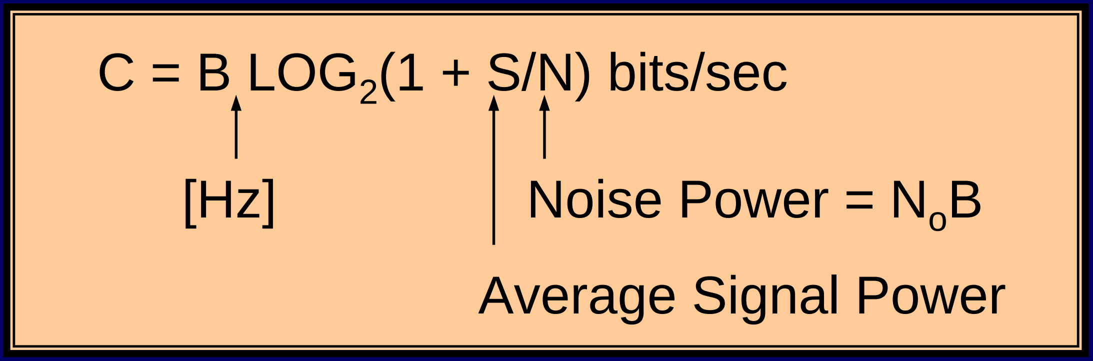


Error Reduction via Channel Coding

Shannon’s Channel Capacity Theorem

We want $P_e \rightarrow 0$ (banking, etc.) $\Rightarrow E/N_o \rightarrow \infty$ using prior methods

Theorem: $P_e \rightarrow 0$ if channel capacity “C” not exceeded, in bits/sec



$C = B\, LOG_2(1 + S/N)$ bits/sec

[Hz]

Noise Power = N_oB

Average Signal Power

(Shannon showed “can,” not “how”)

Examples: S/N = 10 yields C = 3B (3 bits/Hz), $S/N = \underbrace{127}_{\sim 21\text{ dB}}$ yields C = 7B (7 bits/Hz)

e.g. 3-kHz phone at 9600 bps requires $S/N \gtrsim 10$ dB

Channel Codes

Definitions:

1. "Channel codes" reduce P_e
2. "Source codes" reduce redundancy
3. "Cryptographic" codes conceal

Solomon Golomb: A message with content and clarity
has gotten to be quite a rarity;
to combat the terror of serious error,
use bits of appropriate parity.

Channel codes are our principal approach to letting $R \rightarrow C$ with acceptable P_e

Coding Delays Message and Increases Bandwidth

Can show: $P_e \leq 2^{-Tk(C,R)}$, T = time delay in coding process

e.g. use $M = 2^{RT}$ possible messages in T sec.
(RT = #bits in T sec; “block coding”)

use $M = 2^{RT}$ frequencies spaced at ~1/T Hz

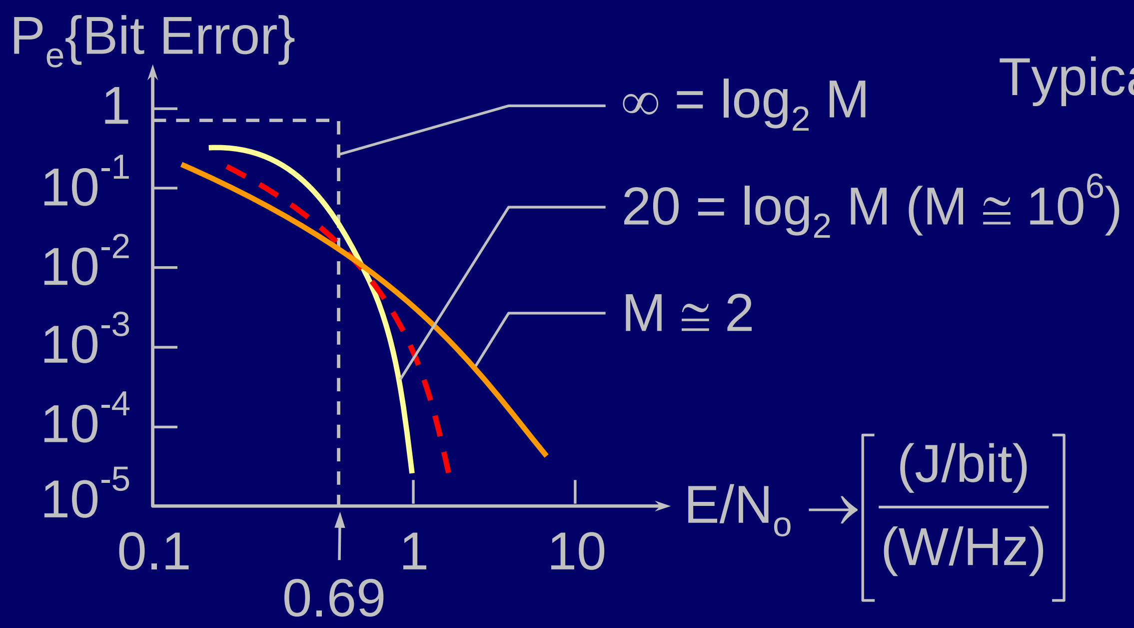
then $B = 2^{RT}/T$ (can → ∞!)

Minimum S/N_o for $P_e \rightarrow 0$

$$\text{Can show}: \; C_\infty \overset{\Delta}{=} \lim_{B\rightarrow\infty} C = S/\left(N_o \ln 2\right) \geq R_{\left(P_e \rightarrow 0\right)} \; \text{bits/sec}$$

Therefore

$$\boxed{S/N_o \geq 0.69 \; R \text{ for } P_e \rightarrow 0 \text{ as } B \rightarrow \infty}$$



Typically $2 \leq M \leq 64$

M = number of frequencies

Error Detection K + R Code

Blocks: $\underbrace{\text{message}}_{\text{K bits}} \qquad \underbrace{\text{check bits}}_{\text{R bits}}$

Simple parity check – $\underbrace{\text{xxx…x}}_{\text{K bits}}\ \underbrace{\text{P}}_{\text{R = 1 bit}}$ where P ∍ Σ 1’s = $\left\{\begin{array}{l}\text{even}\\ \text{or}\\ \text{odd}\end{array}\right.$ (2 standards)

i.e. = A single bit error transforms its block to “illegal” message set (half are illegal here).

Error Correction Code

Message = $m_1\ m_2 \ldots m_K$　　　Checks = $m_{K+1} \ldots . m_{K+R}$

Any of these K + R bits can be erroneous

Receive:　$\hat{m}_1$　$\hat{m}_2 \ldots\ldots\ldots\ldots . \hat{m}_{K+R}$

Correct:　m_1　$m_2 \ldots\ldots\ldots\ldots . m_{K+R}$

Sum (modulo 2) = 0's if no error → 0　0　0

Consider locations of "1"s in K + R slots of Sum

If we wish to detect and correct 0 or 1 bit error in the block of K + R bits, we need

$$\underbrace{K + R}_{\text{Original Information}} \geq K + \text{LOG}_2\ (\underbrace{K + R}_{\text{Number of Slots for 1-bit error}} \underbrace{+ 1}_{\text{“No Error” Message}})\ \text{bits/block}$$

Single-Bit Error Correction

If we wish to detect and correct 0 or 1 bit error in the block of K + R bits, we need

$$\underbrace{K + R}_{\text{Original Information}} \geq K + LOG_2\,(\underbrace{K + R}_{\text{Number of Slots for 1-bit error}} + \underbrace{1}_{\text{“No Error” Message}})\text{ bits/block}$$

K =	R ≥	R/(R +K)
1	2	0.67
2	3	0.6
3	3	0.5
4	3	0.4
5	4	0.4
100	7	0.07
10^3	~10	0.01
10^6	~20	0.002

Not too efficient

R = Check bits needed to detect and fix ≤ 1 error in a block of K + R

Two-Bit Error Correction

If we wish to correct two errors:

We need

$$K + R \geq K + LOG_2 \left(\underbrace{1}_{\text{“No Error” Message}} + \underbrace{K + R}_{\text{1 Error}} + \underbrace{\frac{(K + R)(K + R - 1)}{2}}_{\text{2 Errors}} \right)$$

K =	$R \geq$	R/(R +K)
5	7	0.6
10^3	~20	0.02
10^6	~40	0.004

R = Check bits needed to detect and fix $\leq$ 2 errors in a block of K + R bits

Implementation: Single-Error Correction

Block $\overset{\Delta}{=}$ $\begin{bmatrix} m_1 & m_2 & m_3 & m_4 & C_1 & C_2 & C_3 \end{bmatrix}$

(K = 4, R = 3) (4 message bits, 3 check bits)

Let $C_1 \overset{\Delta}{=} m_1 \oplus m_2 \oplus m_3$

$C_2 \overset{\Delta}{=} m_1 \oplus m_2 \oplus m_4 \quad C_3 \overset{\Delta}{=} m_1 \oplus m_3 \oplus m_4$

(Note: $C_1 \oplus C_1 \overset{\Delta}{=} m_1 \oplus m_2 \oplus m_3 \oplus C_1 \equiv 0$)

⊕	0	1
0	0	1
1	1	0

"Sum, modulo-2"
Truth Table

Modulo-2
matrix
multiply

$$\underbrace{\begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}}_{\overset{\Delta}{=} \bar{\bar{H}}} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ C_1 \\ C_2 \\ C_3 \end{bmatrix} \overset{\Delta}{=} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\bar{Q} \overset{\Delta}{=} \longrightarrow$

$\uparrow$ If no errors

(Note: $m_1 \oplus m_2 \oplus m_3 = C_1$)

$\bar{\bar{H}}\bar{Q} = \bar{0}$ defines legal codewords $\bar{Q}$

Implementation: Single-Error Correction

$\bar{\bar{H}}\bar{Q} = \bar{0}$ defines legal codewords $\bar{Q}$

Only 1/8 of all 7-bit words are legal because C_1, C_2, and C_3 are each correct only half the time and $(0.5)^3 = 1/8$

Suppose transmitted $\bar{Q}$ is legal and received $\bar{R} = \bar{Q} + \bar{E}$ then

$$\bar{\bar{H}}\bar{R} = \underbrace{\bar{\bar{H}}\bar{Q}}_{\equiv 0} + \underbrace{\bar{\bar{H}}\bar{E}}_{\underset{\uparrow}{\neq 0}}$$

Interpret to yield error-free $\bar{Q}$ from $\bar{R}$

Say $\bar{\bar{H}}\bar{R} = \begin{bmatrix}1\\0\\1\end{bmatrix} \Rightarrow$ Error in m_3 $\left(\text{Note that } H_{i3} = \begin{bmatrix}1\\0\\1\end{bmatrix}\right)$

Can even rearrange transmitted word so:

$$\begin{array}{rl} & \left[\begin{array}{ccccccc} 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{array}\right] \\ L = & \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \end{array} \begin{bmatrix} C_3 \\ C_2 \\ m_4 \\ C_1 \\ m_3 \\ m_2 \\ m_1 \end{bmatrix} = \bar{E} = \text{Binary representation of error location “L”}$$

P_e Benefits of Channel Coding

Suppose $P_e = 10^{-5}$, then $P\{\text{error in 4-bit word}\} = 1 - \underbrace{(1 - 10^{-5})^4}_{P\{\text{no errors}\}} \cong 4 \times 10^{-5}$ (no-coding case)

If we add 3 bits to block (4 + 3 = 7) for single-error correction,
and send it in the same time $\Rightarrow \frac{4}{7}$ less E/N_o (2.4 dB loss)

$P_e \to 6 \times 10^{-4}$ (per bit; depends on modulation)

$$p\{\text{2 errors in 7 bits @ } 6 \times 10^{-4}\} = p\{\text{no error}\}^5 \bullet \underbrace{p\{\text{error}\}^2}_{(6 \times 10^{-4})^2} \underbrace{\binom{7}{2}}_{7 \bullet 6/2!} \cong 8 \times 10^{-6}$$

Compare new p{block error} 8×10^{-6} to 4×10^{-5} without coding

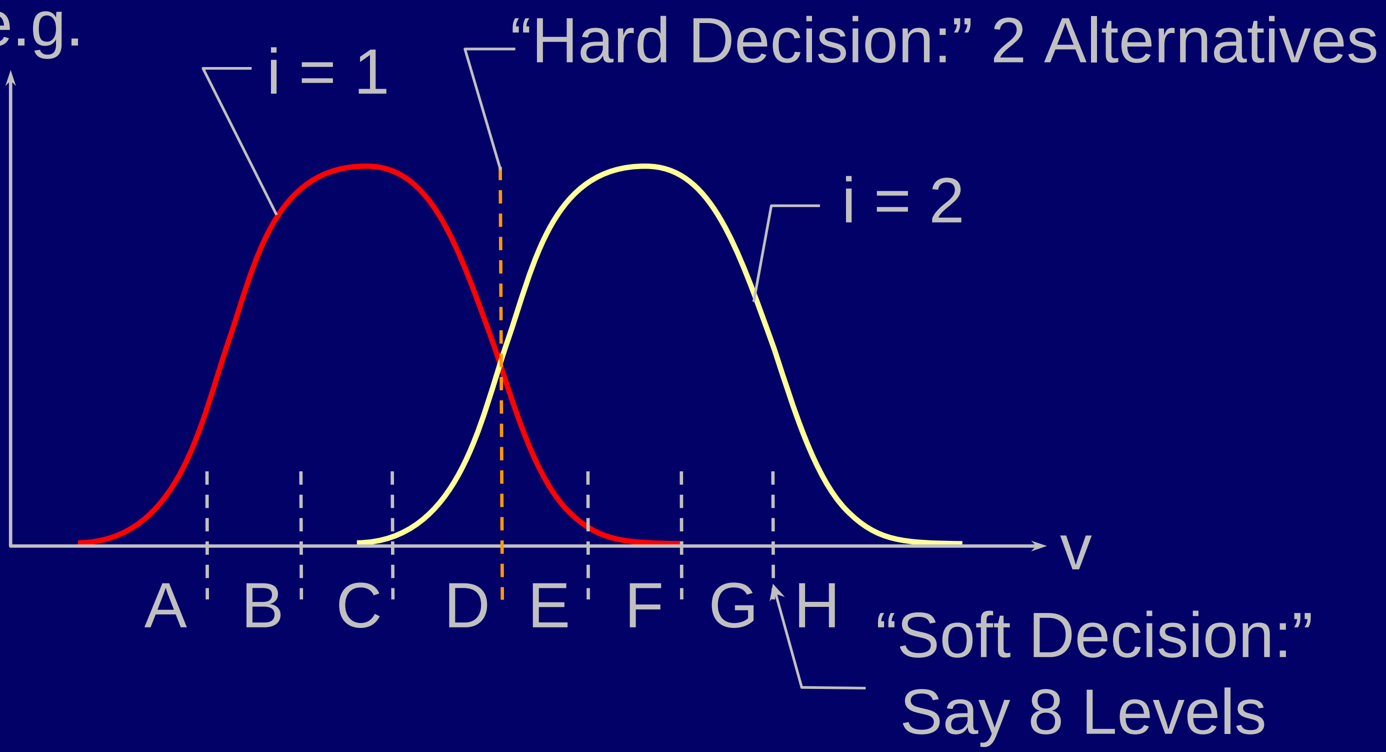
Coding reduced block errors by a factor of 5 with same transmitter power

Alternatively, reduce power and maintain P_e
Benefits depend on $P_e(E/N_o)$ relation

Benefits of Soft Decisions

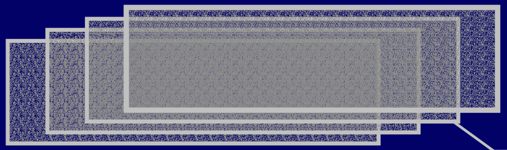
Soft decisions can yield ~2 dB SNR improvement for same P_e

e.g.



Example: Parity bit implies one of n bits was received incorrectly. Choose the one bit for which the decision was least clear.

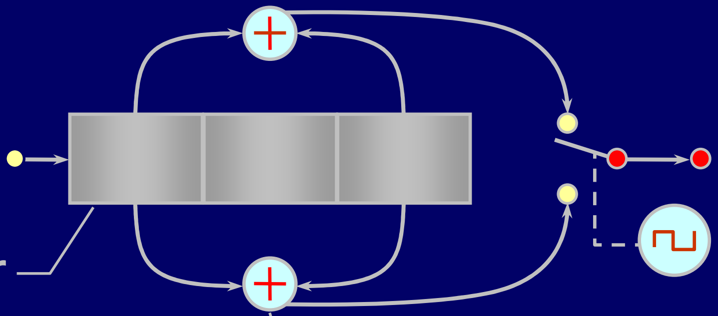
Convolutional Codes



Constraint Length

Convolutional codes employ overlapping blocks (sliding window)

Example:



R (bits/sec)

In

2R bits/sec output

3-bit shift register

Sum modulo 2

This is a “rate 1/2, constraint-length-3 convolutional coder”

One advantage: accommodates soft decisions

Here each message bit impacts 3 output bits and therefore impacts decoder decisions impacting 3 or more reconstructed bits, so soft decisions help identify erroneous bits.

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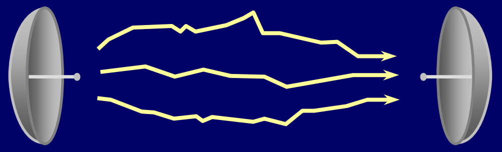
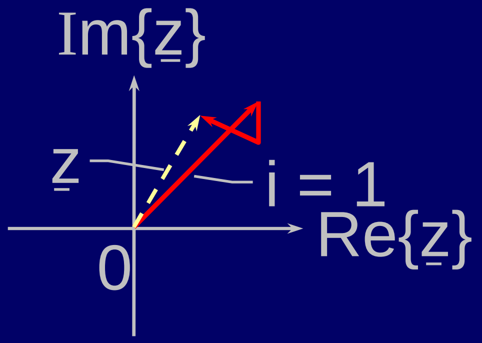
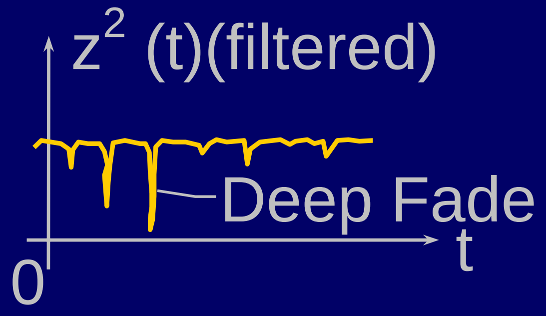
Rayleigh Fading Channels

e.g. Fading from deep vigorous multipath

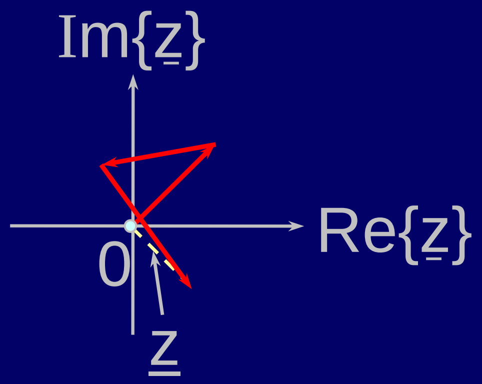


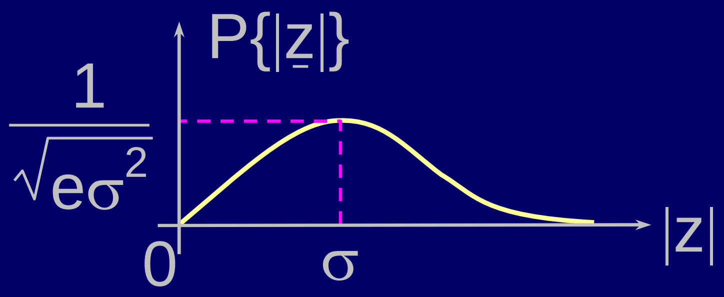
Consider multipath with output signal $z(t) = \sum_{i=1}^{N} x_i \cos \omega t + y_i \sin \omega t$
(sum of N phasors, one per path)





Rayleigh fading: x_i and y_i are independent g.r.v.z.m.





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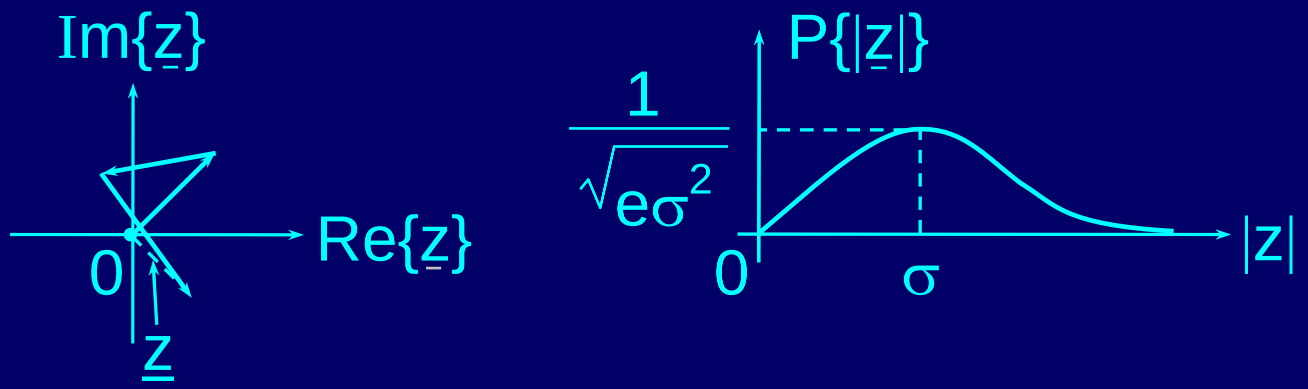
Rayleigh fading: x_i and y_i are independent g.r.v.z.m.



Variance of $\mathrm{Re}\{\underline{z}\}$, $\mathrm{Im}\{\underline{z}\} \equiv \sigma^2$

$$\langle|\underline{z}|\rangle = \sigma\sqrt{\pi/2}$$

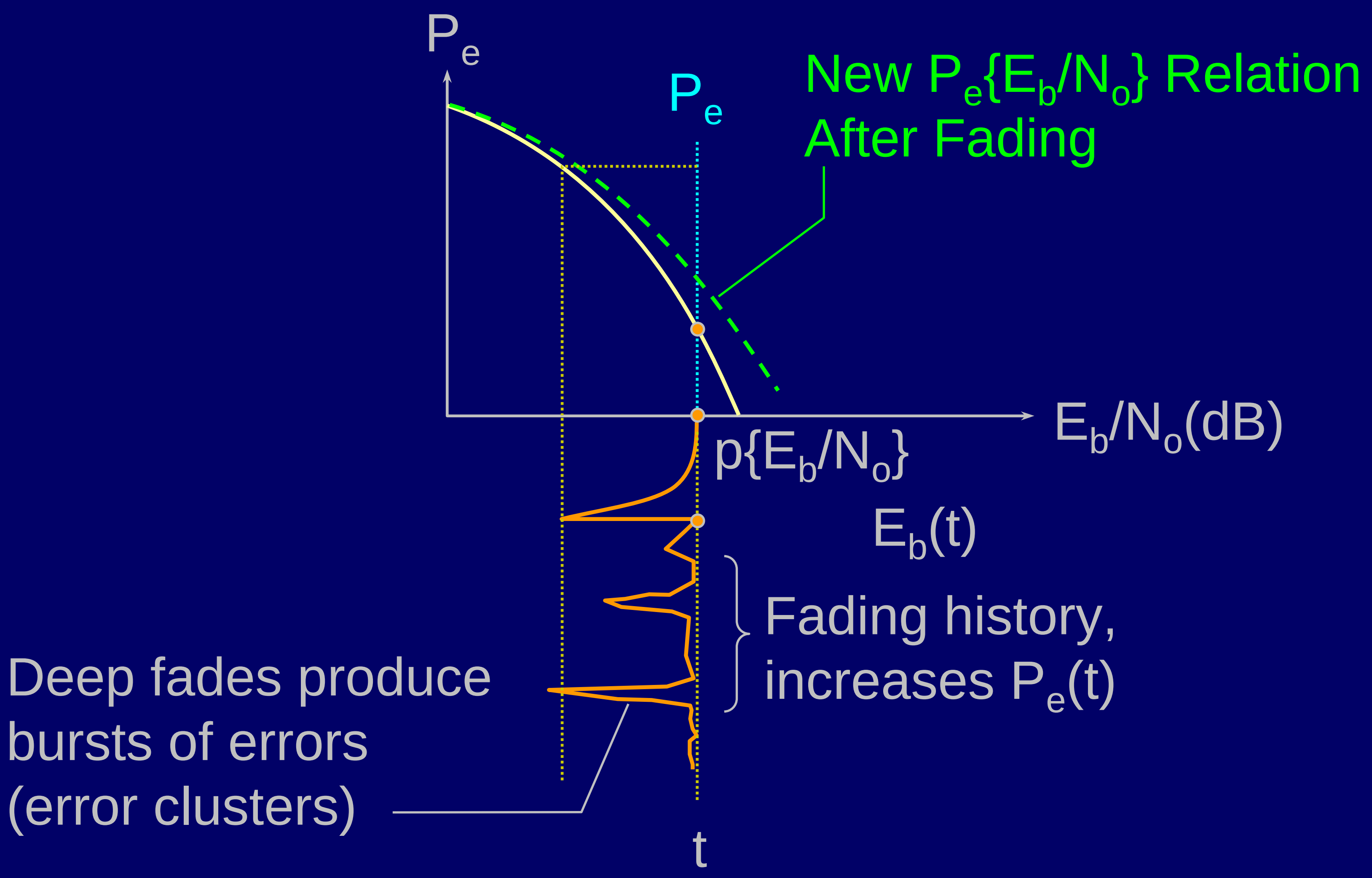
$$\langle|\underline{z}|^2\rangle = \sigma^2(2 - \pi/2)$$

$$\sqrt{\left(|\underline{z}| - \langle|\underline{z}|\rangle\right)^2} \cong 2\sigma/3 \neq f(N)$$

$$P\{|\underline{z}| > z_o\} = e^{-(z_o/\sigma)^2/2}$$

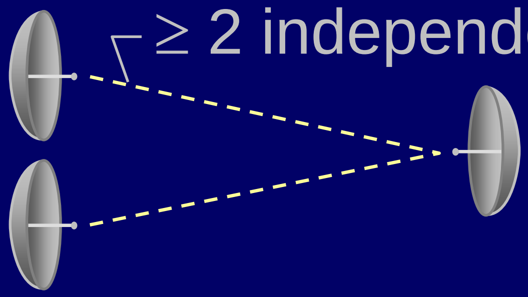
Effect of Fading on $P_e(E_b/N_o)$

P_e curve increases and flattens when there is fading



Remedies for Error Bursts

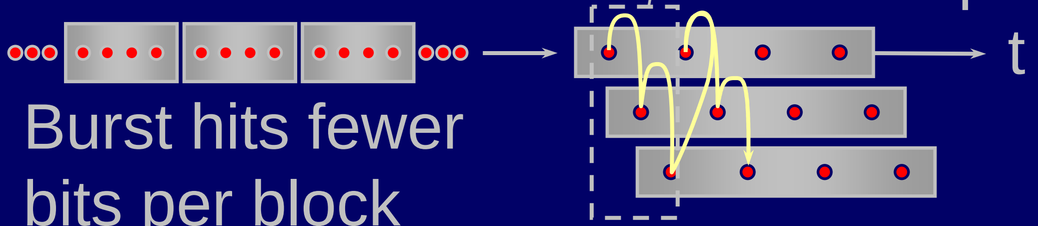
1. Diversity – Space
   – Frequency
   – Polarization



≥ 2 independent paths

2. General error-correcting codes

3. Same, plus interleaving:



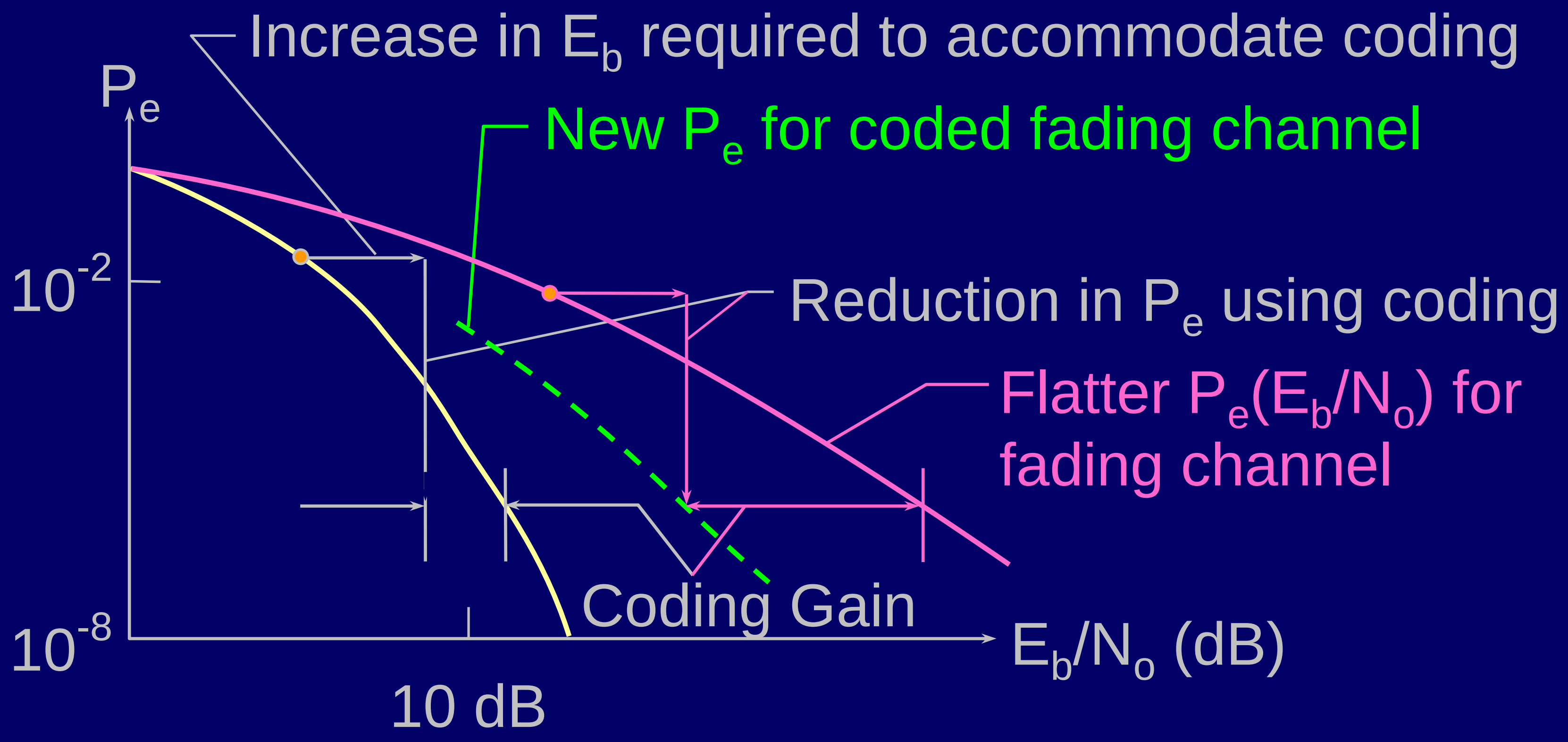
Burst hits fewer bits per block

Error burst, hits only, one bit per block

4. Reed-Solomon codes
   (Tolerate adjacent errors better than random ones)
   e.g. multivalue symbols A (say 4 bits each, 16 possibilities)
   so then block error-correct the symbols A:

Remedies for Error Bursts

Fading flattens $P_e(E_b/N_o)$ curve, so potential coding gain can exceed 10 dB sometimes



Note: Coding gain greater for flatter $P_e(E_b/N_o)$