Massachusetts Institute of Technology

Department of Electrical Engineering and Computer Science

6.691 Seminar in Advanced Electric Power Systems

Problem Set 4 Solutions April 15, 2006

The Phasor Diagram for operation of this machine at the specified condition is shown in Figure 1. Note that this figure is only approximately correct: the resistive part of the diagram is quite exagerated so that the resistive part of impedance can be seen.

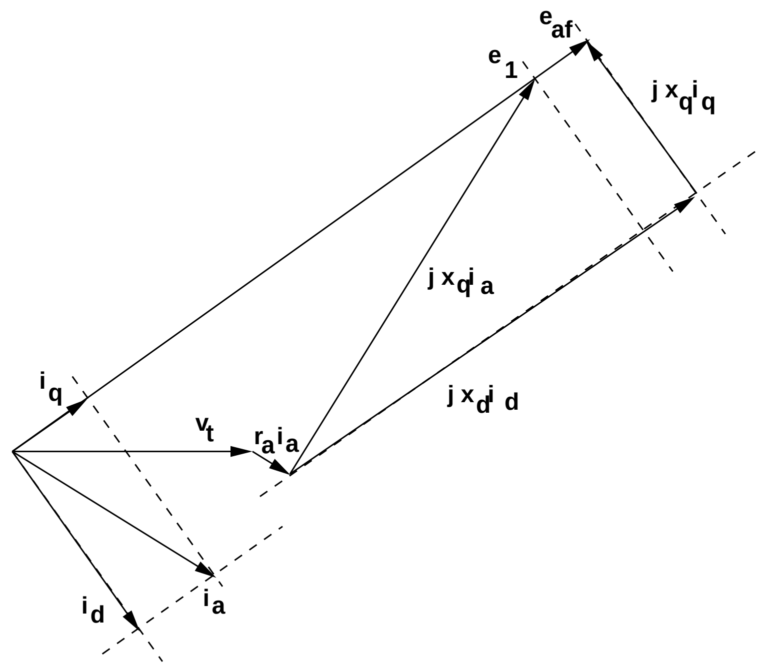

Figure 1: Cartoon of Operational Phasor Diagram

The machine is operated at rated kVA and at a power factor of 85% *lagging* (over-excited). This means that the *real* component of current is $i_r = .85$ per-unit and the *imaginary* component of current is $i_i = -\sqrt{1 - .85^2}$. If we note the actual current to be $i_a = i_r + ji_i$, we can find a point on the quadrature axis:

$$e_1 = v_t + (r_a + jx_q)i_a$$

This allows us to find the torque angle (just the angle of e_1), and then to construct the two components of current:

$$\begin{aligned} i_d &= i_a \sin \delta - \psi \\ i_q &= i_a \cos \delta - \psi \end{aligned}$$

The voltage seen by the reactive parts of the machine is inside of the stator resistance:

$$v_a = v_t + r_a i_a$$

and we may express this as:

$$v_a = |v_a| \angle v_a = |v_a| e^{j\theta_a}$$

Voltage components are:

$$\begin{aligned} v_q &= |v_a| \cos(\delta + \theta_a) \\ v_d &= |v_a| \sin(\delta + \theta_a) \end{aligned}$$

Now, the d- and q- axis fluxes are simply:

$$\begin{aligned} \psi_d &= v_q \\ \psi_q &= -v_d \end{aligned}$$

Air-gap fluxes are simply terminal fluxes plus flux produced by the stator leakeage:

$$\begin{aligned} \psi_{ad} &= \psi_{kd} = & \psi_d + x_{al} i_d \\ \psi_{aq} &= \psi_{kq} = & \psi_q + x_{al} i_q \end{aligned}$$

Field flux is air-gap flux on the d- axis plus flux across the field leakage inductance:

$$\psi_f = \psi_d + x_{fl} i_f$$

where, for the steady state condition:

$$i_f = \frac{e_a}{x_{ad}}$$

This has all been built into a MATLAB script which is appended and which yields the following rendition of numbers:

```
Problem Set 4, Part 1
Terminal Voltage = 1
Terminal Current = 0.85 + j -0.526783
Terminal Current Magnitude = 1
Terminal Current Angle = -0.554811 radians = -31.7883 deg
Torque Angle = 0.68942 radians = 39.5009 deg
Torque Angle = 0.68942 PF angle = -0.554811
Voltage behind sync reactance eq = 2.85721
Voltage Behind Resistance = 1.00493 + j -0.00305534
D-axis voltage = 0.641583 Q-Axis voltage = 0.773476
Terminal Flux psid = 0.773476 psiq = -0.641583
Air-Gap Flux psiad = 0.868191 psiaq = -0.609504
Field Current if = 1.36057 Field Flux psif = 1.43963
Torque by generation plus loss = 0.8558
Torque by flux times current = 0.8558
Audit Difference = 1.11022e-16
```

Note the last few lines relate to the last sub-question which was asked: torque from power balance would be, in per-unit:

$$t_e = P_{out} + r_a \left(i_d^2 + i_q^2 \right)$$

and torque from the crossed-current times flux:

$$t_e = \psi_d i_q - \psi_q i_d$$

which do appear to be pretty close to the same.

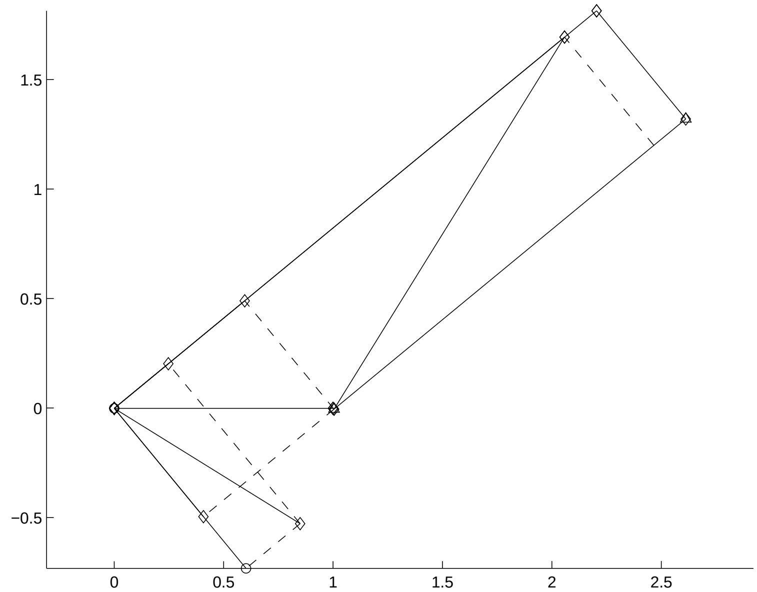

Figure 2: Operational Phasor Diagram

Shown in Figure 2 is the vector diagram drawn by MATLAB. It is not annotated but by comparison with the cartoon of Figure 1 you can see what vectors are which.

```
% 6.691 Problem Set 4 Solution
% Part 1: Steady operation of the machine
% data
xad = 2.1;
xaq = 1.9;
xal = 0.1;
xkdl = 0.183;
xfl = 0.42;
xkql = 0.128;
ra = .0058;
vt = 1.0;
it = 1.0;
pf = 0.85;
% a couple of little things
xd = xad+xal;
xq = xaq+xal;

% take advantage of MATLAB's ability to do complex numbers
ia = it*(pf - j * sqrt(1 - pf^2)); %this is the complex current
e1 = vt + ia*ra+j*xq*ia;       % this is on the q- axis
va = vt + ia*ra;               % this is voltage 'inside' resistance
tha = angle(va);               % pretty small angle
delta = angle(e1);             % this is the torque angle
psi = angle(ia);               % and this is power factor angle
idg = it*sin(delta - psi);         % since the psi we calculate is negative
iqg = it*cos(delta - psi);
i_d = idg*exp(j*(delta - pi/2));
i_q = iqg*exp(j*delta);
ea = abs(e1) + (xd - xq) * idg;  % voltage behind synchronous reactance
e_a = ea*exp(j*delta);
vq = abs(va)*cos(delta - tha);  % voltage on q axis is d axis flux
vd = abs(va)*sin(delta - tha);
v_q = vq*exp(j*delta);
v_d = vd*exp(j*(delta-pi/2));
psid = vq;
psiq = - vd;
psiad = psid + xal*idg;        % air-gap flux
psiaq = psiq + xal*iqg;        % these will be damper fluxes too
i_f = ea/xad;                  % field current
psif = psiad + xfl*i_f;        % flux at field terminal

% now do an audit
te1 = .85+ra*(idg^2+iqg^2);     % input is output plus losses
te2 = psid*iqg - psiq*idg;        % done by flux times current
```

```
% now some snazzy looking output
fprintf('Problem Set 4, Part 1\n');
fprintf('Terminal Voltage = %g\n', vt);
fprintf('Terminal Current = %g + j %g\n',real(ia), imag(ia));
fprintf('Terminal Current Magnitude = %g \n', abs(ia));
fprintf('Terminal Current Angle = %g radians = %g deg \n', psi, 180*psi/pi);
fprintf('Torque Angle = %g radians = %g deg\n', delta, 180*delta/pi);
fprintf('Torque Angle = %g  PF angle = %g\n',delta, psi);
fprintf('Voltage behind sync reactance eq = %g\n',ea);
fprintf('Voltage Behind Resistance = %g + j %g\n',real(va), imag(va));
fprintf('D-axis voltage = %g  Q-Axis voltage = %g\n',vd, vq);
fprintf('Terminal Flux psid = %g  psiq = %g\n',psid, psiq);
fprintf('Air-Gap Flux psiad = %g  psiaq = %g\n',psiad, psiaq);
fprintf('Field Current if = %g  Field Flux psif = %g\n',i_f, psif);
fprintf('Torque by generation plus loss = %g\n', te1);
fprintf('Torque by flux times current = %g\n', te2);
fprintf('Audit Difference = %g\n',te1-te2);

% and now we can draw a scaled version of the vector diagram: here are the
% vectors
v_1 = [0 vt];                                % terminal voltage
v_1a = [vt va];                              % inside of resistances
v_2 = [0 ia];                                % terminal current
v_3 = [0 e1];                                % on the q- axis
v_4 = [0 ea*exp(j*delta)];                   % eaf
v_5 = [0 i_d];                               % id
v_6 = [0 i_q];                               % iq
v_7 = [va e1];                               % vector to q axis
v_8 = [va va+j*xd*i_d];                      % voltage from d- axis current
v8a = [va va+j*xq*i_d];
v_9 = [va+j*xd*i_d e_a];                      % from there to e1
d_9a = [va+j*xq*i_d e1];
v_10 = [0 v_q];                              % q-axis voltage
v_11 = [0 v_d];                              % d-axis voltage
d_q = [va v_q];                              % dotted line from va to vq
d_d = [va v_d];                              % another dotted line from va to vd
d_qi = [ia i_q];
d_di = [ia i_d];


figure(1)
clf
hold on
plot(real(v_1), imag(v_1), '-d')
plot(real(v_1a), imag(v_1a), '-d')
```

```
plot(real(v_2), imag(v_2), '-d')
plot(real(v_3), imag(v_3), '-d')
plot(real(v_4), imag(v_4), '-d')
plot(real(v_5), imag(v_5), '-o')
plot(real(v_6), imag(v_6), '-d')
plot(real(v_7), imag(v_7), '-d')
plot(real(v_8), imag(v_8), '-^')
plot(real(v_9), imag(v_9), '-d')
plot(real(v_10), imag(v_10), '-d')
plot(real(v_11), imag(v_11), '-d')
plot(real(d_q), imag(d_q), '--');
plot(real(d_d), imag(d_d), '--');
plot(real(d_qi), imag(d_qi), '--');
plot(real(d_di), imag(d_di), '--');
plot(real(d_9a), imag(d_9a), '--');
axis square
axis equal
```