Lecture 10 - Carrier Flow *(cont.)*

February 28, 2007

Contents:

1. Minority-carrier type situations

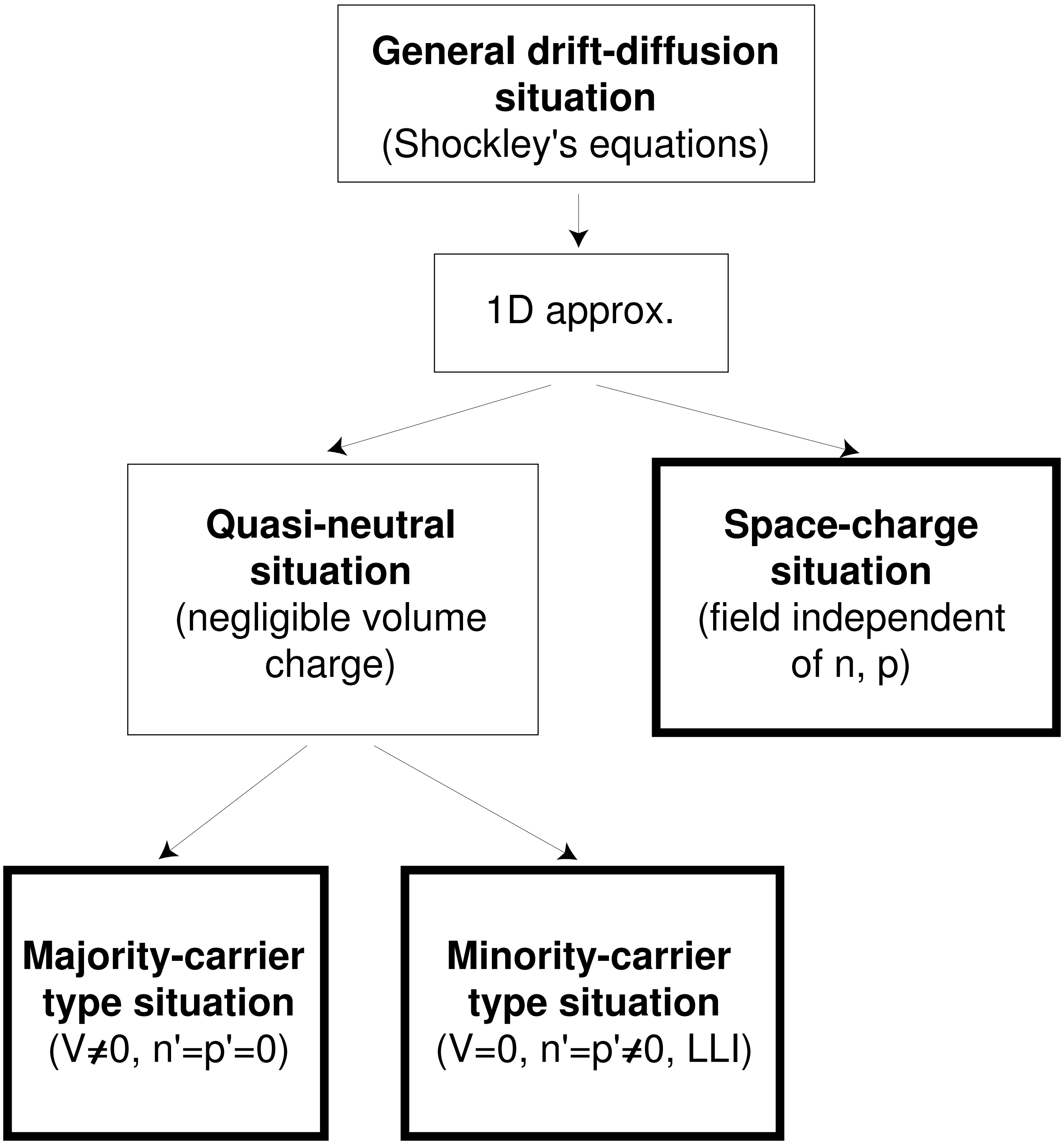
Reading assignment:

del Alamo, Ch. 5, §5.6

Key questions

- What characterizes *minority*-carrier type situations?
- What is the length scale for minority-carrier type situations?
- What do majority carriers do in minority-carrier type situations?

Overview of simplified carrier flow formulations



Simplified set of Shockley equations for 1D quasi-neutral situations

$$p - n + N_D - N_A \simeq 0$$

$$J_e = -qnv_e^{drift} + qD_e \frac{\partial n}{\partial x}$$

$$J_h = qpv_h^{drift} - qD_h \frac{\partial p}{\partial x}$$

$$\frac{\partial n}{\partial t} = G_{ext} - U + \frac{1}{q}\frac{\partial J_e}{\partial x} \quad \text{or} \quad \frac{\partial p}{\partial t} = G_{ext} - U - \frac{1}{q}\frac{\partial J_h}{\partial x}$$

$$\frac{\partial J_t}{\partial x} \simeq 0$$

$$J_t = J_e + J_h$$

1. Minority-carrier type situations

Situations characterized by:

- excess carriers over TE
- no external electric field applied (but small internal field generated by carrier injection: $\mathcal{E} = \mathcal{E}_o + \mathcal{E}'$)

Example: electron transport through base of npn BJT.

Two approximations:

1. $\mathcal{E}$ small $\Rightarrow |v^{drift}| \propto |\mathcal{E}|$
2. Low-level injection

   $\Rightarrow$ for n-type:

   - $n \simeq n_o$
   - $p \simeq p'$
   - $U \simeq \frac{p'}{\tau}$
   - negligible minority carrier drift due to $\mathcal{E}'$
     (but can't say the same about majority carriers)

Shockley equations for 1D quasi-neutral situations

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$$\frac{\partial J_t}{\partial x} \simeq 0$$

$$J_t = J_e + J_h$$

□ Further simplifications for n-type minority-carrier-type situations

- Majority-carrier current equation:

$$J_e \simeq q(n_o + n')\mu_e(\mathcal{E}_o + \mathcal{E}') + qD_e(\frac{\partial n_o}{\partial x} + \frac{\partial n'}{\partial x})$$

but in TE:

$$J_{eo} = qn_o\mu_e\mathcal{E}_o + qD_e\frac{\partial n_o}{\partial x} = 0$$

Then:

$$J_e \simeq qn_o\mu_e\mathcal{E}' + qn'\mu_e\mathcal{E}_o + qD_e\frac{\partial n'}{\partial x}$$

- Minority-carrier current equation:

$$J_h \simeq q(p_o + p')\mu_h(\mathcal{E}_o + \mathcal{E}') - qD_h(\frac{\partial p_o}{\partial x} + \frac{\partial p'}{\partial x})$$

In TE, $J_{ho} = 0$, and:

$$J_h \simeq qp'\mu_h\mathcal{E}_o + qp'\mu_h\mathcal{E}' - qD_h\frac{\partial p'}{\partial x} \simeq qp'\mu_h\mathcal{E}_o - qD_h\frac{\partial p'}{\partial x}$$

- Minority-carrier continuity equation:

$$\frac{\partial p'}{\partial t} = G_{ext} - \frac{p'}{\tau} - \frac{1}{q}\frac{\partial J_h}{\partial x}$$

Now plug in J_h from above:

$$D_h\frac{\partial^2 p'}{\partial x^2} - \mu_h\mathcal{E}_o\frac{\partial p'}{\partial x} - \frac{p'}{\tau} + G_{ext} = \frac{\partial p'}{\partial t}$$

One differential equation with one unknown: p'.

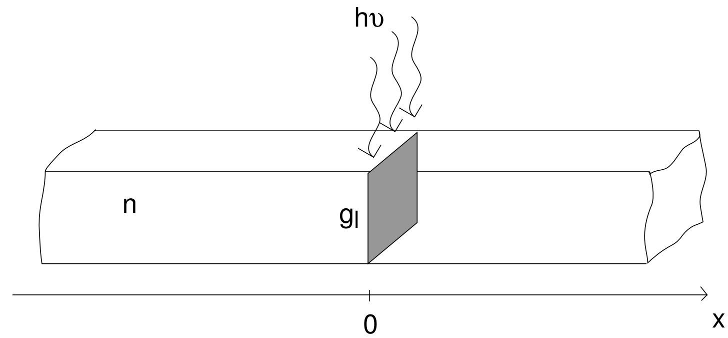
If G_{ext} and BC's are specified, problem can be solved.

Shockley equations for 1D minority-carrier type situations

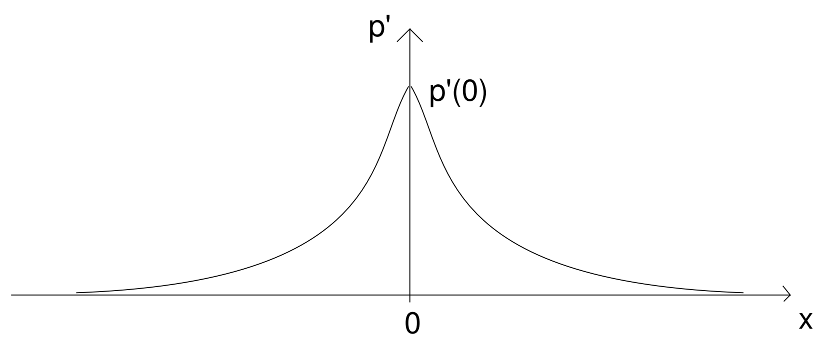
n-type	p-type
$p_o - n_o + N_D - N_A \simeq 0$	
$p' \simeq n'$	
$J_e = qn_o\mu_e\mathcal{E}' + qn'\mu_e\mathcal{E}_o + qD_e\frac{\partial n'}{\partial x}$	$J_e = qn'\mu_e\mathcal{E}_o + qD_e\frac{\partial n'}{\partial x}$
$J_h = qp'\mu_h\mathcal{E}_o - qD_h\frac{\partial p'}{\partial x}$	$J_h = qp_o\mu_h\mathcal{E}' + qp'\mu_h\mathcal{E}_o - qD_h\frac{\partial p'}{\partial x}$
$D_h\frac{\partial^2 p'}{\partial x^2} - \mu_h\mathcal{E}_o\frac{\partial p'}{\partial x} - \frac{p'}{\tau} + G_{ext} = \frac{\partial p'}{\partial t}$	$D_e\frac{\partial^2 n'}{\partial x^2} + \mu_e\mathcal{E}_o\frac{\partial n'}{\partial x} - \frac{n'}{\tau} + G_{ext} = \frac{\partial n'}{\partial t}$
$\frac{\partial J_t}{\partial x} \simeq 0$	
$J_t = J_e + J_h$	

Example 1:
Diffusion and bulk recombination in a ”long” bar

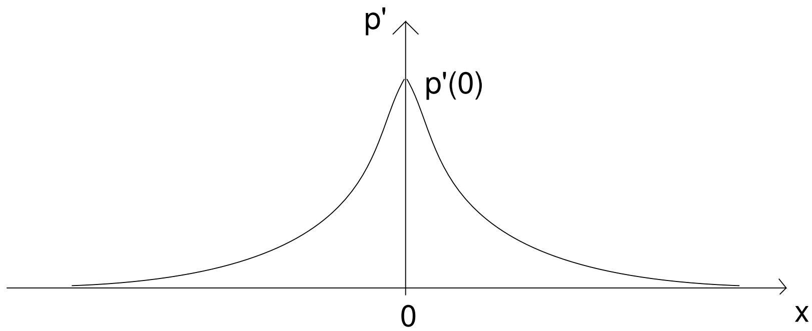
Uniform doping: $\mathcal{E}_o = 0$; static conditions: $\frac{\partial}{\partial t} = 0$



Minority carrier profile:



Majority carrier profile? $n' = p'$ exactly?

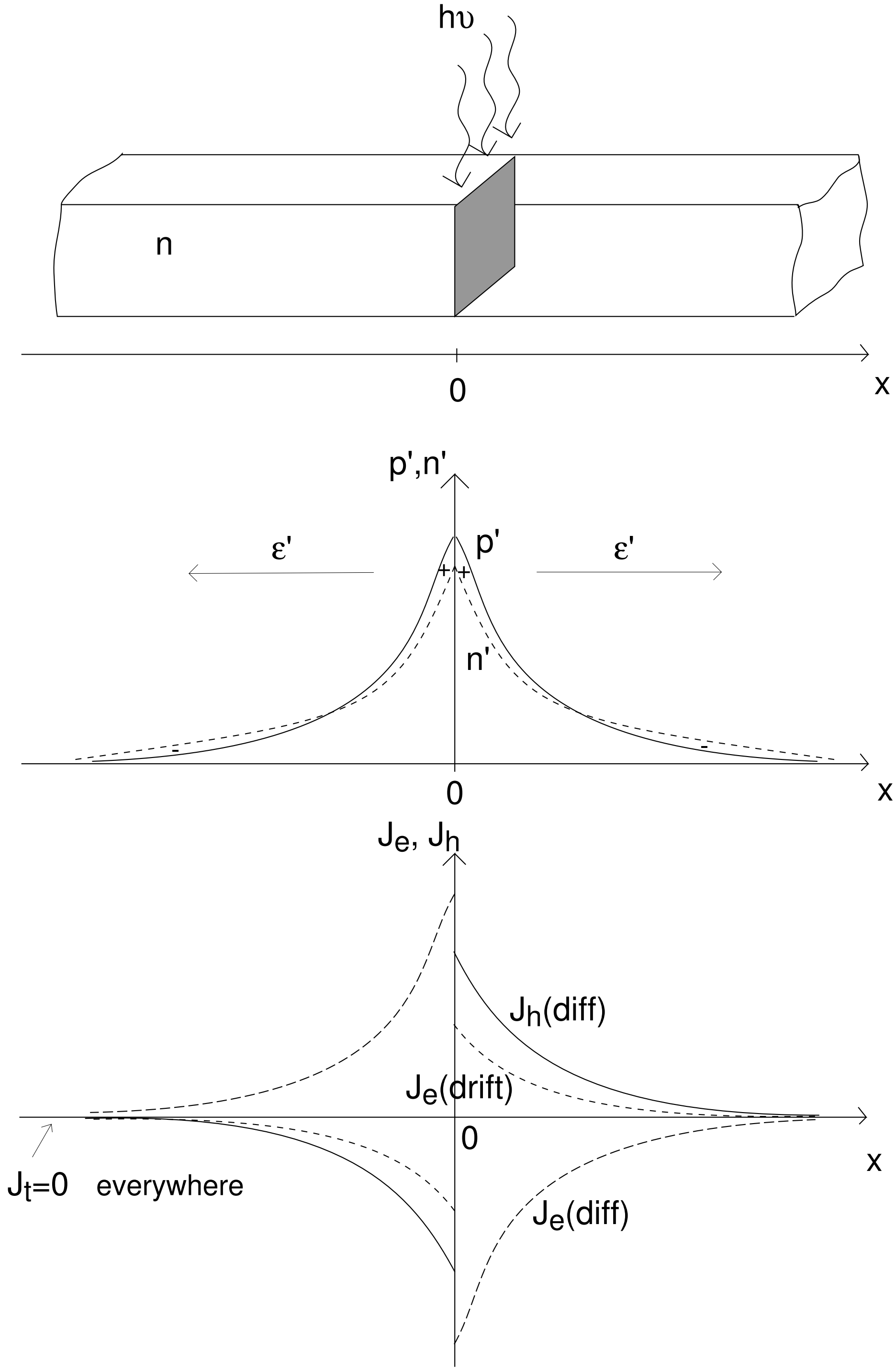


Far away $J_t = 0 \Rightarrow J_t = 0$ everywhere.

$$J_t = J_e + J_h \simeq q n_o \mu_e \mathcal{E}' + q(D_e - D_h)\frac{dp'}{dx} = 0$$

If $D_e = D_h \Rightarrow$ diffusion term $= 0$
$\Rightarrow$ drift term $= 0$
$\Rightarrow \mathcal{E}' = 0$
$\Rightarrow n' = p'$

But, typically $D_e > D_h \Rightarrow$ diffusion term < 0 (for $x > 0$)
$\Rightarrow$ drift term > 0
$\Rightarrow \mathcal{E}' > 0$ (for $x > 0$)
$\Rightarrow$ *and* $\mathcal{E}' \propto D_e - D_h$
$\Rightarrow n' \neq p'$ (but still $n' \simeq p'$)
$\Rightarrow$ *and* $|n' - p'| \propto D_e - D_h$


hυ
n
0
x
p',n'
ε'
p'
ε'
n'
0
x
Je, Jh
Jh(diff)
Je(drift)
0
x
Jt=0 everywhere
Je(diff)

Solution

STEP 1. Minority carrier flow problem (for $x > 0$):

$$\frac{d^2p'}{dx^2} - \frac{p'}{L_h^2} = 0$$

with

$$L_h = \sqrt{D_h\tau}$$

solution of the form:

$$p' = A\exp\frac{x}{L_h} + B\exp\frac{-x}{L_h}$$

B.C. at $x = 0$:

$$\frac{g_l}{2} = \frac{1}{q}J_h(0) = -D_h\frac{dp'}{dx}|_{x=0}$$

Then:

$$p' = \frac{g_l L_h}{2D_h}\exp\frac{-x}{L_h}$$

This works for $x \geq 0$ because p' has to be continuous.

STEP 2. Hole current:

Assuming $J_h(drift) \ll J_h(diff)$

$$J_h \simeq -qD_h \frac{dp'}{dx} = \frac{qg_l}{2} \exp \frac{-x}{L_h}$$

STEP 3. Total current:

$$J_t = 0 \quad everywhere$$

STEP 4. Electron current:

$$J_e = -J_h = -\frac{qg_l}{2} \exp \frac{-x}{L_h}$$

STEP 5. Electron profile:

$$n' \simeq p' = \frac{g_l L_h}{2D_h} \exp \frac{-x}{L_h}$$

STEP 6. Electron diffusion current:

$$J_e(diff) = qD_e\frac{dn'}{dx} = -\frac{qg_l}{2}\frac{D_e}{D_h}\exp\frac{-x}{L_h}$$

STEP 7. Electron drift current:

$$J_e(drift) = J_e - J_e(diff)$$
$$= \frac{qg_l}{2}\frac{D_e - D_h}{D_h}\exp\frac{-x}{L_h}$$

Note: if $D_e = D_h \Rightarrow J_e(drift) = 0$

STEP 8. Average velocity of hole diffusion:

$$v_h^{diff} = \frac{J_h^{diff}(x)}{qp(x)} \simeq \frac{J_h(x)}{qp'(x)} = \frac{D_h}{L_h}$$

independent of x.

[will use when deriving I-V characteristics of pn junction diode]

Now verify assumptions

STEP 9. Verify *quasi-neutrality*: $|\frac{p'-n'}{p'}| \ll 1$

Compute $\mathcal{E}'$ from $J_e(drift)$:

$$\mathcal{E}' = \frac{J_e(drift)}{q\mu_e n_o} = \frac{kT}{q}\frac{g_l}{2n_o}\frac{D_e - D_h}{D_e D_h}\exp\frac{-x}{L_h}$$

From Gauss' law, get difference between n' and p':

$$p' - n' = -\frac{\epsilon kT}{q^2 n_o}\frac{g_l}{2L_h}\frac{D_e - D_h}{D_e D_h}\exp\frac{-x}{L_h}$$

Then

$$|\frac{p' - n'}{p'}| = (\frac{L_D}{L_h})^2\frac{D_e - D_h}{D_e}$$

If characteristic length of problem is much longer than L_D (Debye length), quasi-neutrality applies in minority-carrier-type situations.

Put numbers: for $N_D = 10^{16}\ cm^{-3}$, $L_D \sim 0.04\ \mu m$, $L_h \sim 400\ \mu m$, and $(L_D/L_h)^2 \sim 10^{-8}$.

STEP 10. Verify $J_h(drift) \ll J_h(diff)$

$$\left|\frac{J_h(drift)}{J_h(diff)}\right| = \left|\frac{q\mu_h p'\mathcal{E}'}{-qD_h\frac{dp'}{dx}}\right| = \frac{1}{2}\frac{p'}{n_o}\frac{D_e - D_h}{D_e}$$

as good as low-level injection

STEP 11. *Limit to injection* to maintain LLI: $p'(0) \ll n_o$

$$g_l \ll \frac{2D_h n_o}{L_h}$$

STEP 12. Verify linearity between v^{drift} and $\mathcal{E}'$

At $x = 0$ (worst point):

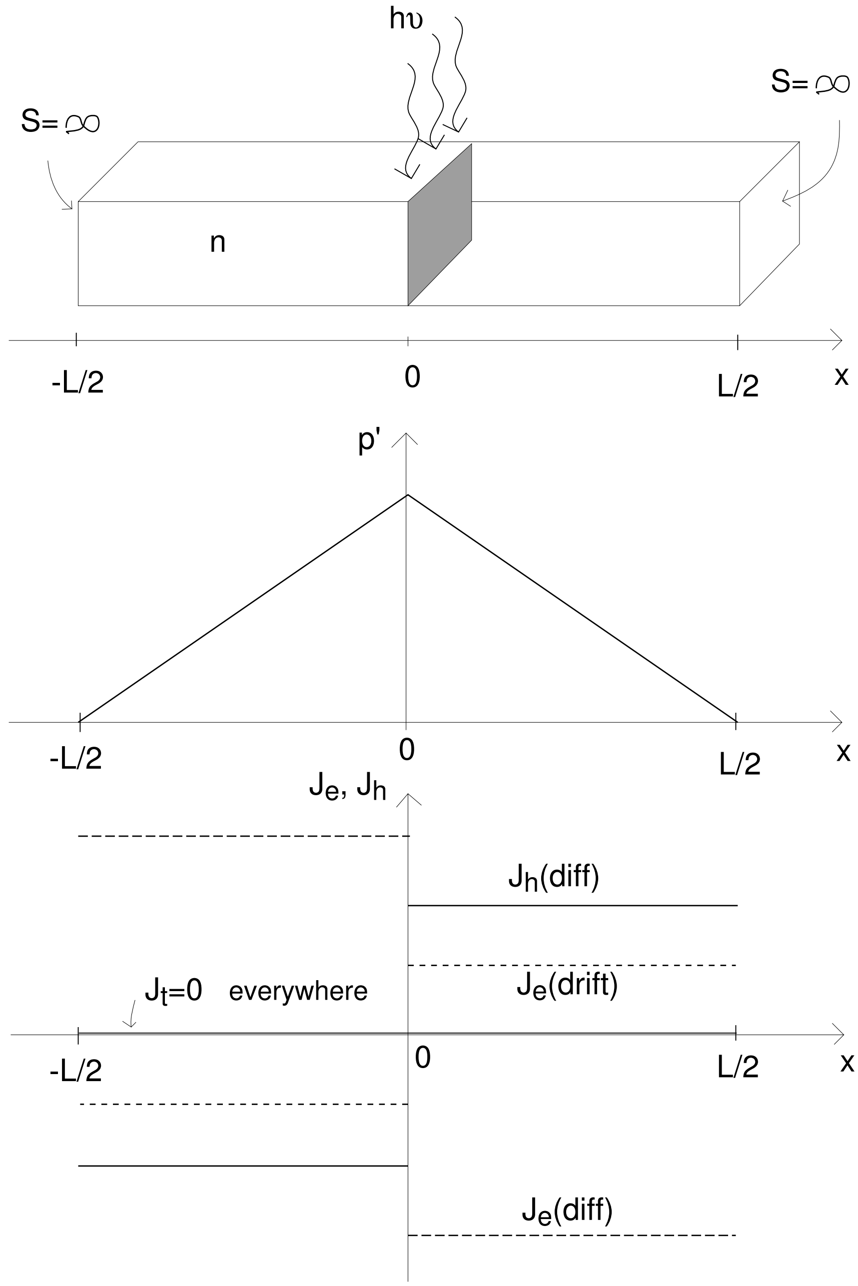
$$\mu_e \mathcal{E}' = \frac{g_l}{2n_o}\frac{D_e - D_h}{D_h} \ll \frac{D_e - D_h}{L_h}$$

$\sim 1000\ cm/s \ \ll\ v_{sat}.$

□ Example 2: Diffusion and surface recombination in a "short" or "transparent" bar

Uniform doping: $\mathcal{E}_o = 0$; static conditions: $\frac{\partial}{\partial t} = 0$

Bar length: $L \ll L_h$; $S = \infty$ at bar ends.

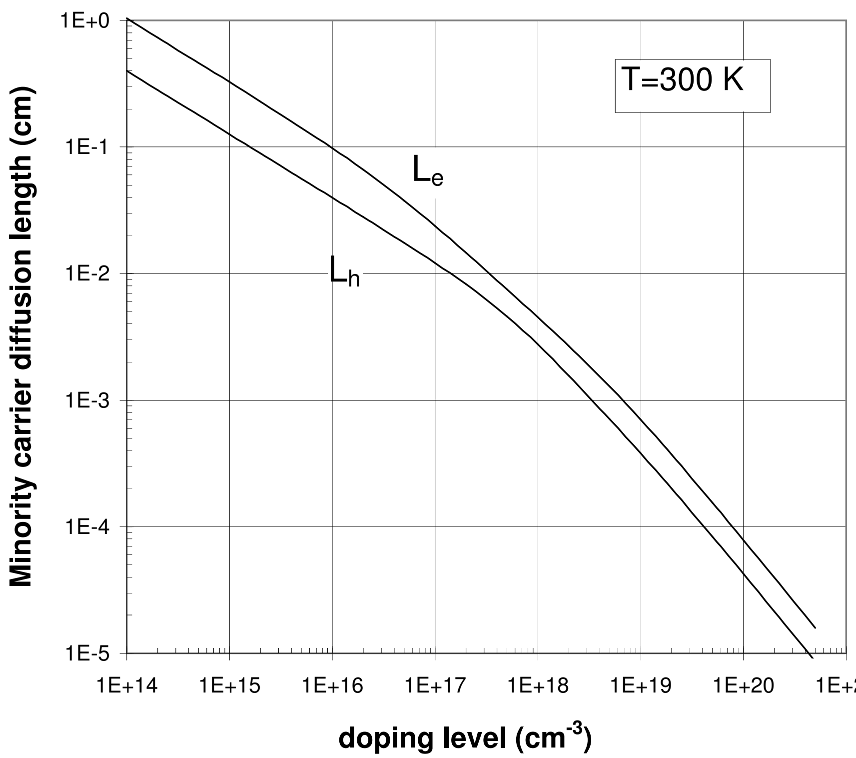


Length scales of minority-carrier situations

□ *Diffusion Length*: mean distance that a carrier diffuses in a bulk semiconductor before recombining

$$L_{diff} = \sqrt{D\tau}$$

L_{diff} strong function of doping:



□ *Sample size*, L

- If $L \gg L_{diff}$, L_{diff} is characteristic length of problem
- If $L \ll L_{diff}$, L is characteristic length of problem

Key conclusions

- Minority-carrier type situations dominated by behavior of minority carriers: diffusion, recombination and drift.
- Two characteristic lengths in minority-carrier type situations dominated by diffusion and recombination:
  - *diffusion length*, $L = \sqrt{D\tau}$, average distance that a carrier diffuses in a bulk semiconductor before recombining;
  - *sample size*, L
  - whichever one is smallest, L or L_{diff}, dominates behavior of minority carriers.
- Minority-carrier type situations called that way because:
  - length and time scales of problem dominated by minority carrier behavior (diffusion, recombination, and drift)
  - role of majority carriers is to preserve quasi-neutrality and total current continuity
- Order of magnitude of key parameters in Si at $300K$:
  - Diffusion length: $L_{diff} \sim 0.1 - 1000\ \mu m$ (depends on doping level).

Self-study

- Work out example 2: diffusion and surface recombination in a "short bar" (§5.6.2)