Lecture 11 - Carrier Flow *(cont.)*

March 1, 2007

Contents:

1. Dynamics of majority-carrier-type situations
2. Dynamics of minority-carrier-type situations

Reading assignment:

del Alamo, Ch. 5, §§5.4

Key questions

- What is the characteristic time constant of majority-carrier-type stuations?
- What is the characteristic time constant of minority-carrier-type situations? Always?

1. Dynamics of majority-carrier-type situations

Continuity equation for net volume charge:

$$\frac{\partial J_t}{\partial x} = -\frac{\partial \rho}{\partial t}$$

in 3D: $\int_S \vec{J_t} \cdot \vec{ds} = -\frac{\partial}{\partial t} \int_V \rho \, dV$

in 3D: $\int_S \vec{J_t} \cdot \vec{ds} = 0$ ⇒ no sources or sinks of charge

Under static conditions: $\frac{\partial \rho}{\partial t} = 0 \Rightarrow \frac{\partial J_t}{\partial x} = 0 \Rightarrow J_t$ uniform in space

Under dynamic conditions: $\frac{\partial \rho}{\partial t} \neq 0 \Rightarrow$ charge redistribution

$\Rightarrow \frac{\partial J_t}{\partial x} \neq 0 \Rightarrow$ charge flow

eventually: $\Rightarrow \rho \simeq 0$, sample quasi-neutral again

How long does it take for quasi-neutrality to be restablished?

Answer: *Dielectric relaxation time* [see AT5.1 in notes]:

$$\tau_d = \frac{\epsilon}{\sigma}$$

Hence, for $t \gg \tau_d$:

$$\frac{\partial J_t}{\partial x} \simeq 0$$

in 3D:

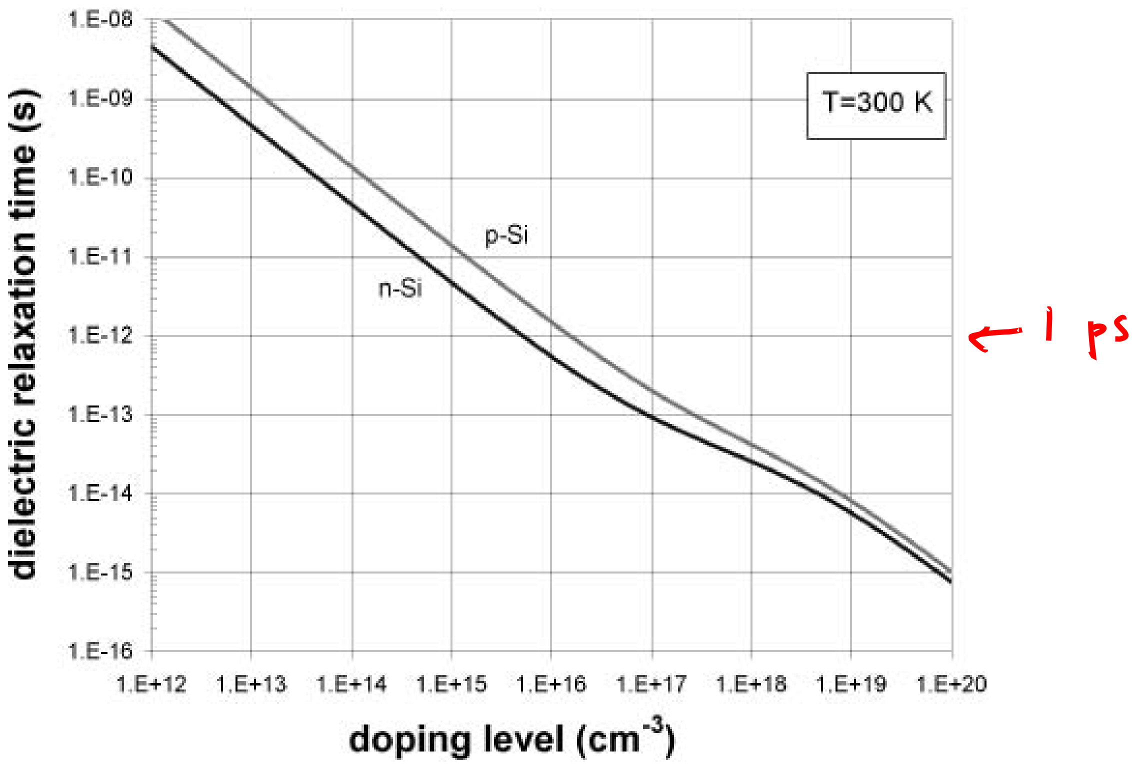
$\int_S \vec{J_t} \cdot \vec{ds} \simeq 0$

no sources or sinks of charge

□ Dielectric relaxation time

$$\tau_d = \frac{\epsilon}{\sigma}$$

Depends on doping level:



The higher the doping level, the faster quasi-neutrality is established after a perturbation.

For $N > 10^{16}\ cm^{-3}$, $\tau_d < 1\ ps \Rightarrow$ typically can ignore dynamics of quasi-neutrality.

2. Dynamics of minority-carrier-type situations

□ MINORITY CARRIER SITUATIONS: characteristic time constant dominated by minority carrier physics

⇒ Substantial memory effects

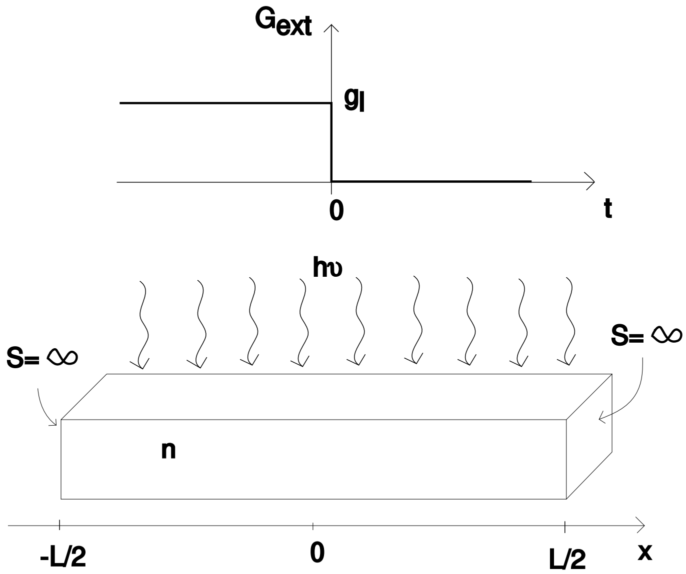
- in uniform situations characteristic time constant is *carrier lifetime*
- in non-uniform situations?

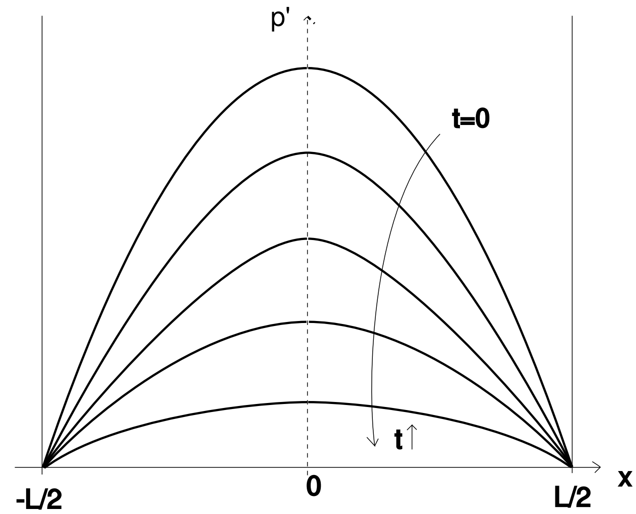
such as with surfaces

□ Example: Transient in semiconductor bar with $S = \infty$

Uniformly-doped n-type bar.

Switch-off transient after uniform illumination





Two recombination paths:

- Bulk recombination: time constant τ (carrier lifetime)
- Surface recombination: limited by carrier diffusion to surfaces; time constant: $\propto L$, $\propto 1/D$

  ↖ sample size

Combined time constant: $< \tau$

□ For $t \leq 0$ (steady-state solution under illumination):

$$D_h \frac{d^2p'}{dx^2} - \frac{p'}{\tau} + G_{ext} = 0$$

Boundary conditions:

$$\frac{dp'}{dx}|_{x=0} = 0$$

$$p'(\pm\frac{L}{2}) = 0$$

Solution:

$$p'(x, t = 0) = g_l\tau(1 - \frac{\cosh \frac{x}{L_h}}{\cosh \frac{L}{2L_h}})$$

□ For $t \geq 0$:

$$D_h \frac{\partial^2 p'}{\partial x^2} - \frac{p'}{\tau} = \frac{\partial p'}{\partial t}$$

Solve by method of separation of constants:

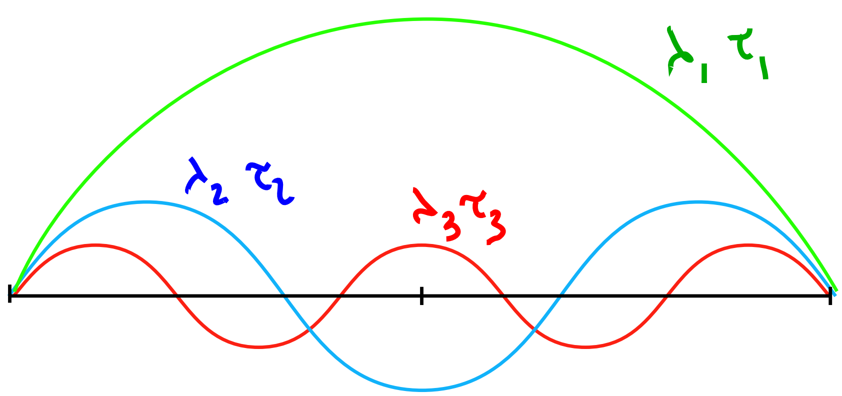
$$p'_n(x,t) = \underbrace{\exp\frac{-t}{\tau} \sum_n K_n \exp\frac{-D_h t}{\lambda_n^2}}_{\text{only } f(t)} \underbrace{\cos\frac{x}{\lambda_n}}_{\text{only } g(x)} \qquad \text{for } n = 1, 2, 3, \ldots$$

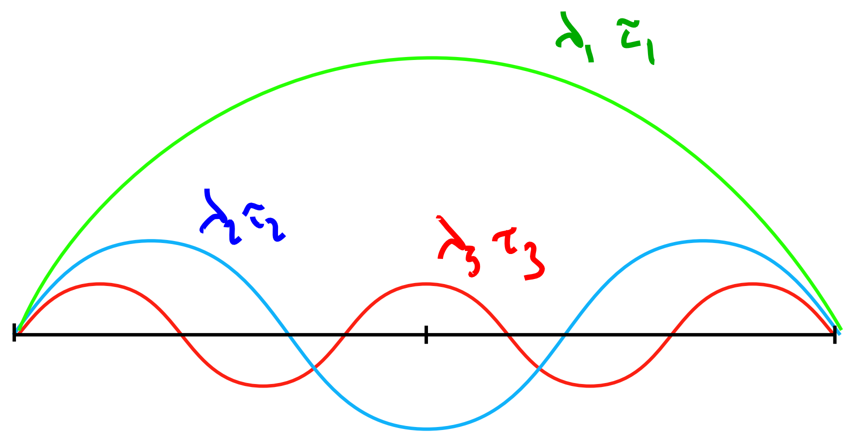
K_n are proper weighting coefficients and

$$\lambda_n = \frac{L}{(2n-1)\pi} \qquad \text{for } n = 1, 2, 3, \ldots$$

Time decay is not simple exponential but sum of individual exponentials. Time constant of nth mode:

$$\frac{1}{\tau_n} = \frac{1}{\tau} + D_h[\frac{(2n-1)\pi}{L}]^2 > \frac{1}{\tau} \qquad \text{for } n = 1, 2, 3, \ldots$$





Study this result:

$$\frac{1}{\tau_n} = \frac{1}{\tau} + D_h[\frac{(2n-1)\pi}{L}]^2 > \frac{1}{\tau} \qquad \text{for } n = 1, 2, 3, ...$$

- For all values of n, time constant of nth mode is smaller than carrier lifetime:

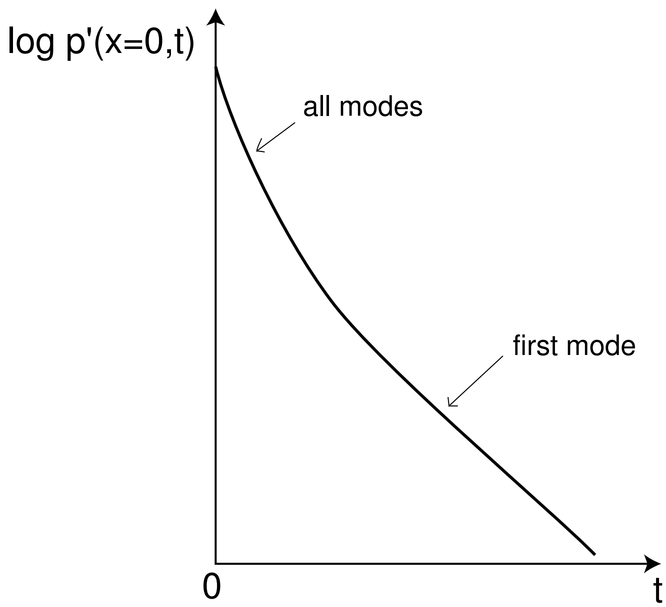
$$\tau_n < \tau$$

Always faster decay than uniform situation due to surface recombination.

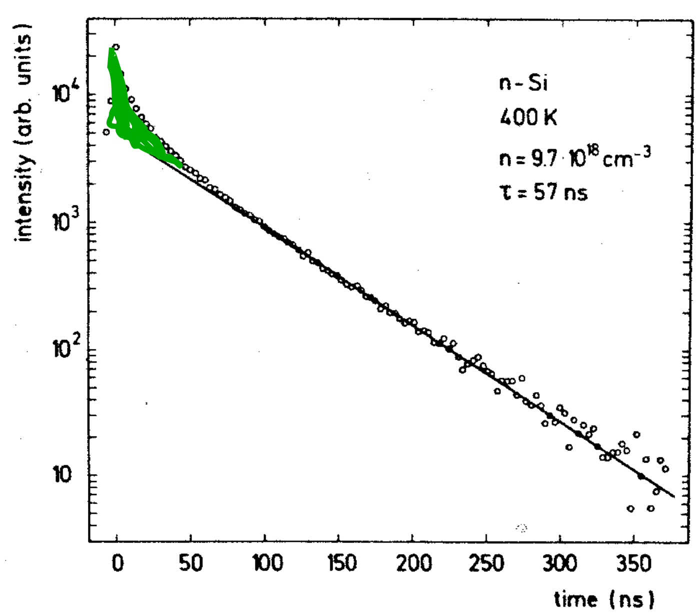
- Higher order modes decay faster:

$$n \uparrow \rightarrow \ \tau_n \downarrow$$

High-order components decay quickly ⇒ initial fast decay followed by slow decay dominated by 1st order time constant



This is seen in experiments:



Reprinted with permission from Dziewior, J., and W. Schmid. "Auger Coefficients for Highly Doped and Highly Excited Silicon." *Applied Physics Letters* 31, no. 5 (1977): 346-348. Copyright 1977, American Institute of Physics.

[Dziewior & Silber]

After short time, decay dominated by first mode with time constant:

$$\frac{1}{\tau_1} = \frac{1}{\tau} + D_h(\frac{\pi}{L})^2$$

This is the dominant time constant of the problem.

In a general way:

$$\frac{1}{\tau_1} = \frac{1}{\tau} + \frac{1}{\tau_t}$$

with $\tau_t \equiv$ *transit time* or average time for excess carrier to reach surface

$$\tau_t = \frac{L^2}{\pi^2 D_h}$$

Surface recombination speeds up excess minority carrier decay by providing additional recombination paths:

$$\tau_1 < \tau$$

In the limit of very slow bulk recombination,

$$\tau_1 \simeq \tau_t$$

Getting the excess carriers to the surface becomes the bottleneck to the recombination rate.

Key conclusions

- In a quasi-neutral, charge redistribution takes place in scale of *dielectric relaxation time.*
- Majority-carrier type situations can be considered quasi-static.
- Minority-carrier type situations show substantial memory.
- Time constants in minority-carrier type situations:
  - carrier lifetime
  - transit time $\propto L^2/D$
  - whichever one is smallest dominates
- Order of magnitude of key parameters in Si at $300K$:
  - Dielectric relaxation time: $\tau_d < 1\ ps$ (for typical doping levels).

Self study

- Simplification of Shockley's equations for space-charge and high-resistivity regions
- Comparison between SCR and QNR transport.