

Lecture 3 - Carrier Statistics in Equilibrium

(*cont.*)

February 9, 2007

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1. Equilibrium electron concentration
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4. Location of Fermi level

Reading assignment:

del Alamo, Ch. 2, §§2.4-2.6

Key questions

- How many electrons and holes are there in thermal equilibrium in a given semiconductor?
- How does the equilibrium electron (hole) distribution in the conduction (valence) band look like?
- How can one compute n_i ?
- Where is the Fermi level in a given semiconductor? How does its location depend on doping level?

Carrier statistics in equilibrium

Question: *how many electrons and holes are there in TE in a given semiconductor?*

Answer: rigorous model exploiting energy view of semiconductors and concept of Fermi level.

Strategy to answer question:

1. derive relationship between n_o and E_F
2. derive relationship between p_o and E_F
3. derive expressions for $n_o p_o$ and n_i
4. figure out location of E_F from additional arguments (such as charge neutrality)

1. Equilibrium electron concentration

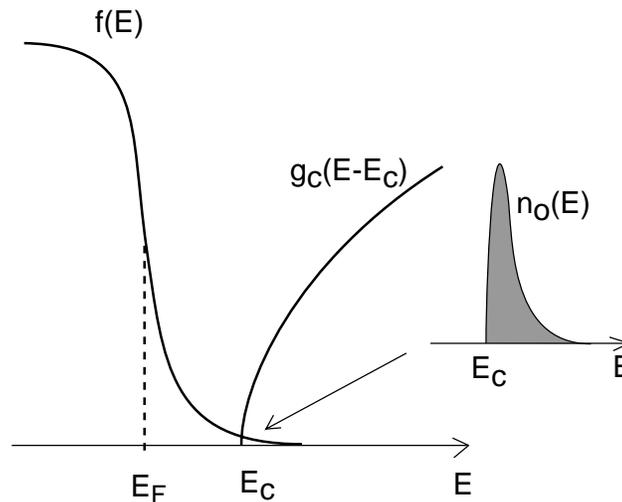
- *Q: How many electrons are there in a semiconductor in TE?*

A: It depends on location of E_F .

- Why?

Because E_F defines probability that states are occupied by electrons.

- The closer E_F is to the conduction band edge, the more electrons there are in the conduction band.



- If E_F is not too close to the conduction band edge, what is the relationship between the location of E_F and n_o ?

$$n_o \propto \exp \frac{E_F - E_c}{kT}$$

Because high energy tail of Fermi-Dirac distribution function is exponential with kT as characteristic energy.

Formally, n_o obtained by integrating electron concentration in entire conduction band:

$$n_o = \int_{E_c}^{\infty} n_o(E) dE$$

At a certain energy, $n_o(E)$ is the product of CB density of states and occupation probability:

$$n_o(E) = g_c(E) f(E)$$

Then:

$$n_o = 4\pi \left(\frac{2m_{de}^*}{h^2} \right)^{3/2} \int_{E_c}^{\infty} \frac{\sqrt{E - E_c}}{1 + \exp \frac{E - E_F}{kT}} dE$$

Refer energy scale to E_c and normalize by kT . That is, define:

$$\eta = \frac{E - E_c}{kT} \quad \eta_c = \frac{E_F - E_c}{kT}$$

Then:

$$n_o = 4\pi \left(\frac{2m_{de}^* kT}{h^2} \right)^{3/2} \int_0^{\infty} \frac{\sqrt{\eta}}{1 + e^{\eta - \eta_c}} d\eta$$

Define also:

$$N_c = 2 \left(\frac{2\pi m_{de}^* kT}{h^2} \right)^{3/2}$$

$N_c \equiv$ effective density of states of the conduction band (cm^{-3})

For Si at 300 K, $N_c \simeq 2.9 \times 10^{19} cm^{-3}$.

Then:

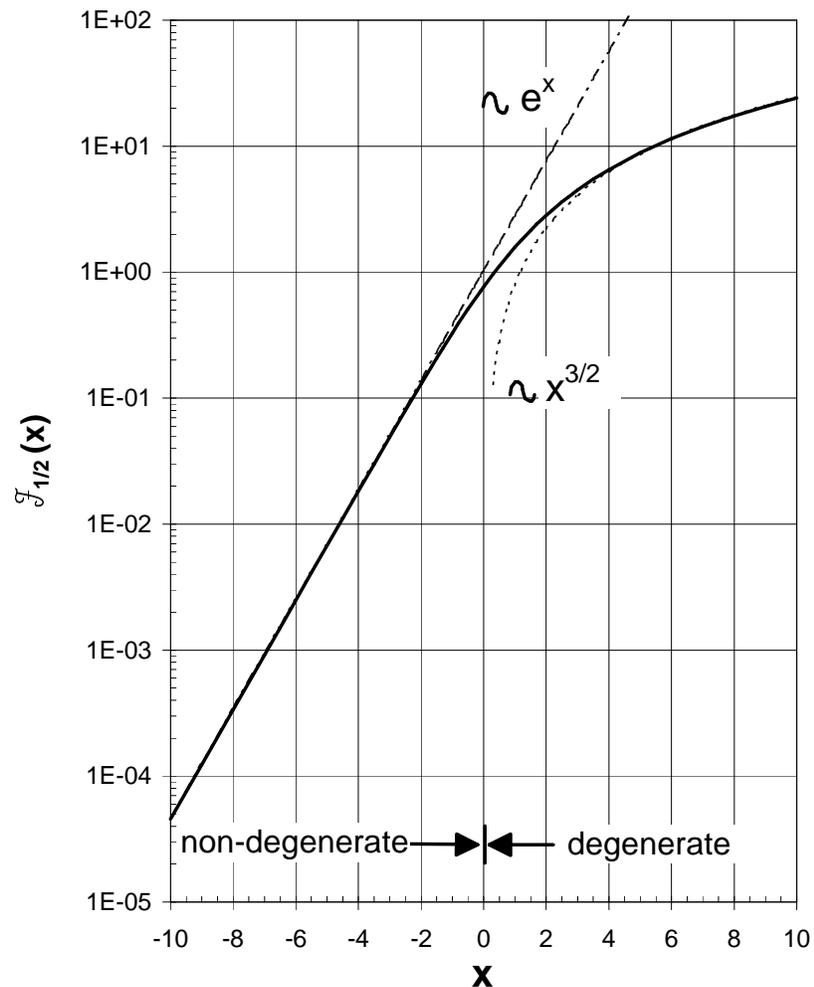
$$n_o = N_c \mathcal{F}_{1/2}(\eta_c)$$

with:

$$\mathcal{F}_{1/2}(\eta_c) = \frac{2}{\sqrt{\pi}} \int_0^\infty \frac{\sqrt{\eta}}{1 + e^{\eta - \eta_c}} d\eta$$

$\mathcal{F}_{1/2}(x)$ is **Fermi integral of order 1/2**.

Fermi integral of order 1/2:



Key result again:

$$n_o = N_c \mathcal{F}_{1/2}(\eta_c) \quad \text{with} \quad \eta_c = \frac{E_F - E_c}{kT}$$

$\eta_c \uparrow \Rightarrow$ the higher E_F is with respect to $E_c \Rightarrow n_o \uparrow$

Two regimes in n_o :

□ *Non-degenerate* regime

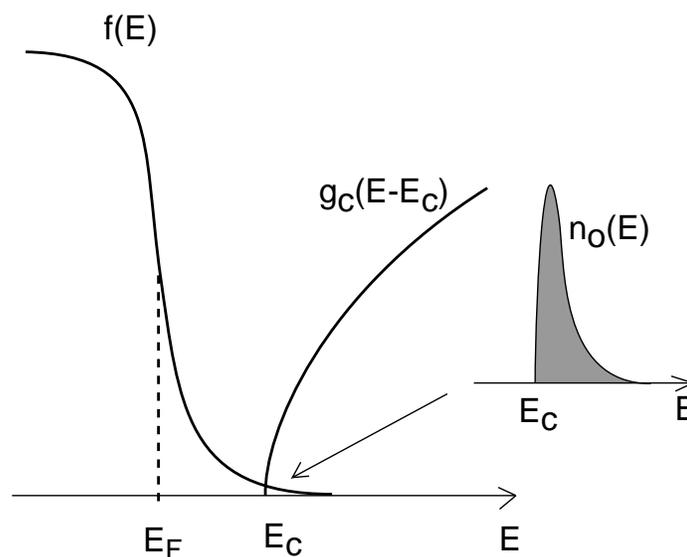
Approximation to $\mathcal{F}_{1/2}(x)$ for low values of x :

$$\mathcal{F}_{1/2}(x \ll -1) \simeq e^x$$

Then, if $\eta_c \ll -1$, or $E_C - E_F \gg kT$, or $n_o \ll N_c$:

$$n_o \simeq N_c \exp \frac{E_F - E_C}{kT}$$

Simple exponential relationship when Fermi level is well below conduction band edge.



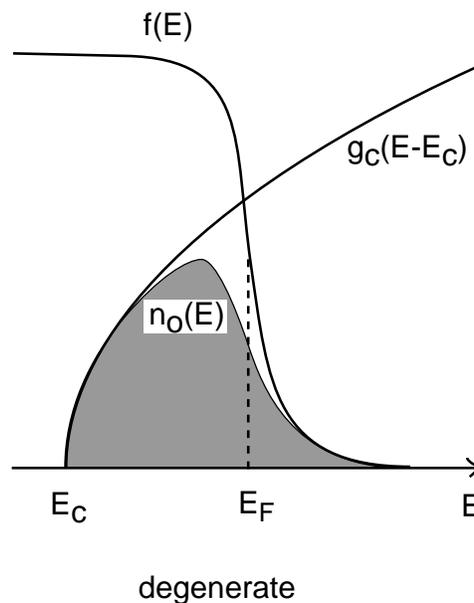
Can obtain same result with *Maxwell-Boltzmann statistics* for $f(E)$.

□ *Degenerate* regime

More complicated behavior of $\mathcal{F}_{1/2}(x)$ for high values of x (see Advanced Topic AT2.3).

Degenerate semiconductor if $\eta_c \gg -1$, or $E_F - E_C \gg kT$, or $n_o \gg N_c$.

Electron distribution inside conduction band very different from non-degenerate regime:



Will not deal with degenerate regime in 6.720 because it's even more complicated [see AT2.6 in notes]

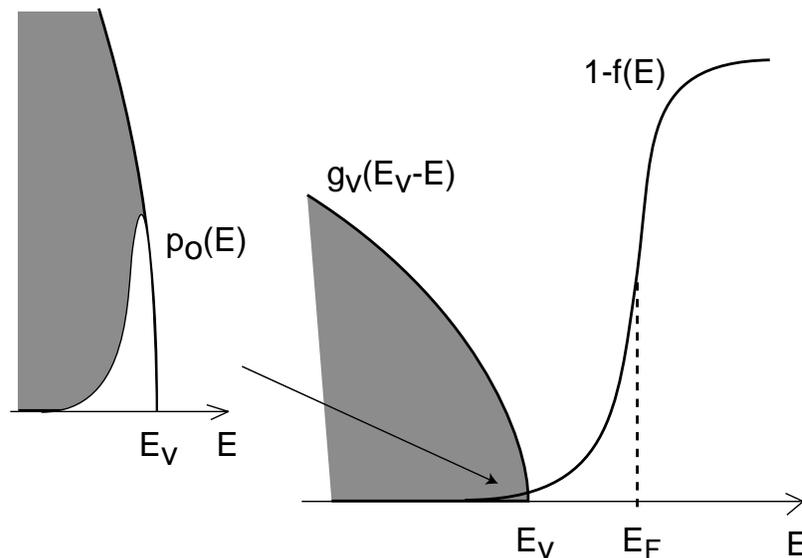
2. Equilibrium hole concentration

- *Q: How many holes are there in a semiconductor in TE?*

A: It depends on location of E_F .

- Why? Because E_F defines probability that states are occupied by electrons.

- The closer E_F is to the valence band edge, the more holes there are in the valence band.



- If E_F is not too close to the valence band edge, what is the relationship between the location of E_F and p_o ?

$$p_o \propto \exp \frac{E_v - E_F}{kT}$$

Because high energy tail of Fermi-Dirac distribution function is exponential with kT as characteristics energy.

Formally, p_o obtained by integrating hole concentration in entire valence band:

$$p_o = \int_{-\infty}^{E_v} p_o(E) dE$$

At a certain energy, $p_o(E)$ is product of VB DOS times probability that state is *empty*:

$$p_o(E) = g_v(E) [1 - f(E)]$$

Proceed as with electrons. Define:

$$\eta_v = \frac{E_v - E_F}{kT}$$

$$N_v = 2 \left(\frac{2\pi m_{dh}^* kT}{h^2} \right)^{3/2}$$

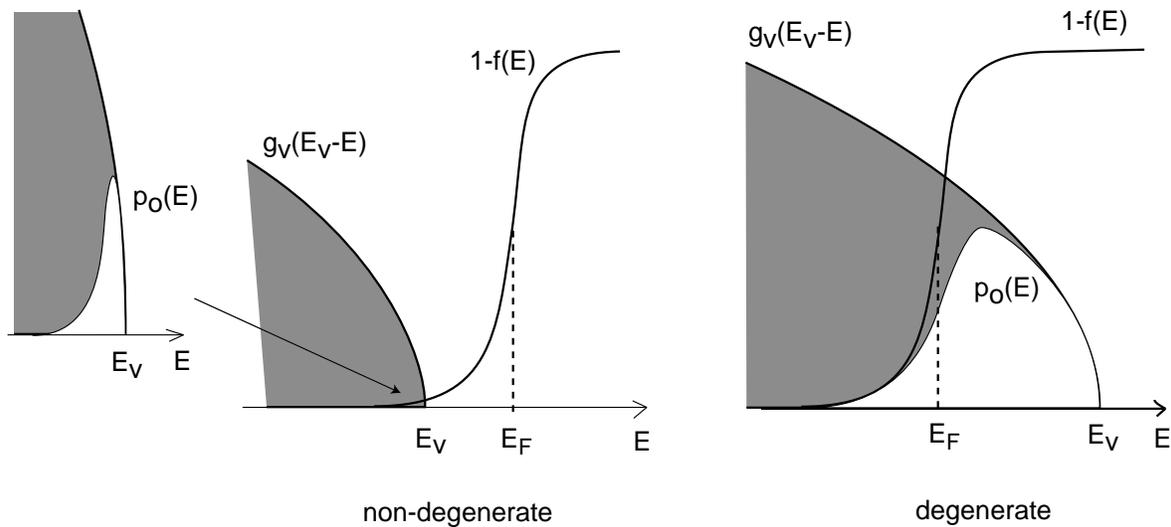
$N_v \equiv$ effective density of states of valence band (cm^{-3})

For Si at 300 K, $N_v \simeq 3.1 \times 10^{19} cm^{-3}$

Then:

$$p_o = N_v \mathcal{F}_{1/2}(\eta_v)$$

Two regimes again:



□ *Non-degenerate* regime:

If $\eta_v \ll -1$, or $E_F - E_v \gg kT$, or $p_o \ll N_v$:

$$p_o \simeq N_v \exp \frac{E_v - E_F}{kT}$$

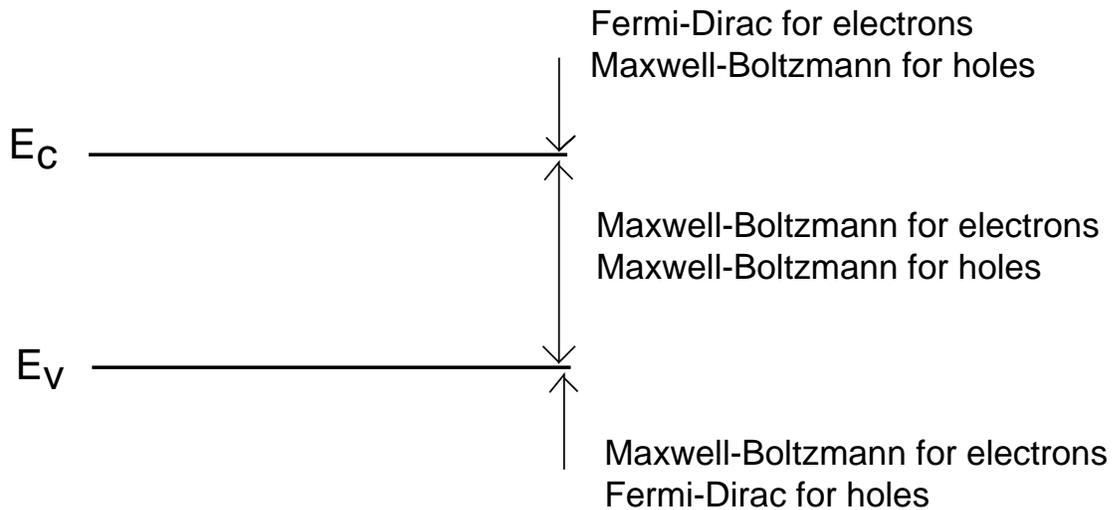
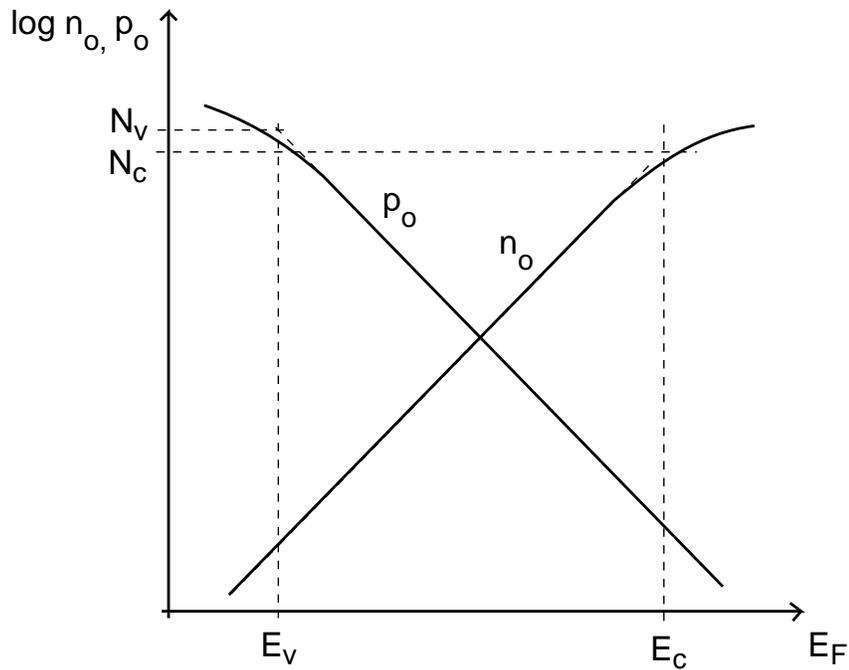
Fermi level well above valence band edge.

□ *Degenerate* regime:

If $\eta_v \gg 1$, or $E_v - E_F \gg kT$, or $p_o \gg N_v$, more complicated dependence of p_o on E_F .

Fermi level inside valence band.

Summary of carrier statistics depending on E_F location

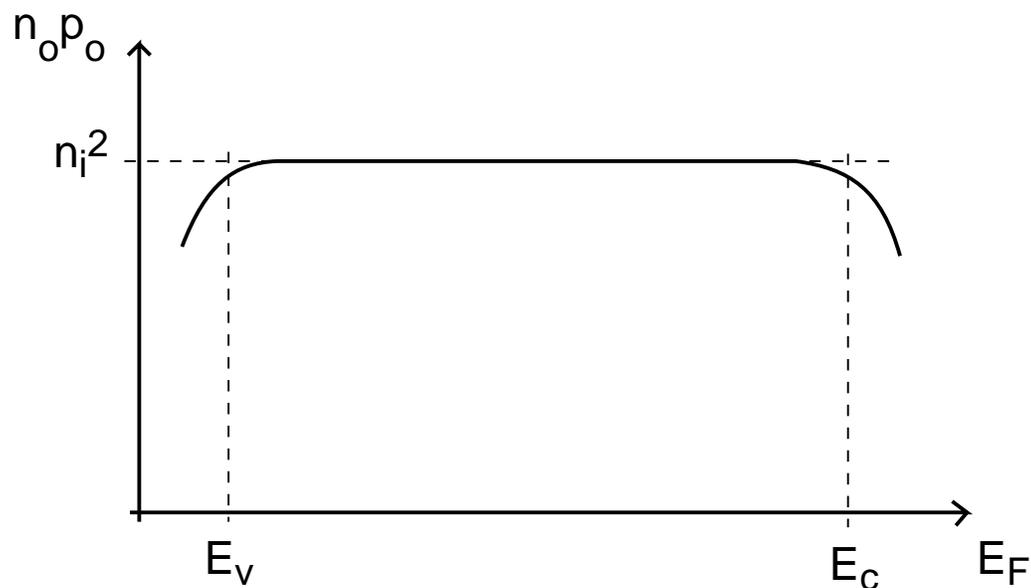


3. np product in equilibrium

Compute product:

$$n_o p_o = N_c N_v \mathcal{F}_{1/2}(\eta_c) \mathcal{F}_{1/2}(\eta_v)$$

Sketch:



If E_F is inside bandgap:

$$n_o p_o \simeq N_c N_v \exp -\frac{E_g}{kT}$$

For a given semiconductor, $n_o p_o$ depends only on T and is independent of precise location of E_F .

But only if semiconductor is non-degenerate.

In intrinsic semiconductor, $n_o = p_o$ and usually fairly small \Rightarrow semiconductor non-degenerate. Hence:

$$n_i = \sqrt{n_o p_o} = \sqrt{N_c N_v} \exp -\frac{E_g}{2kT}$$

Now we have the prefactor we could not obtain last time.

Review key dependencies of n_i :

- $T \uparrow \Rightarrow n_i$
- $E_g \uparrow \Rightarrow n_i$
- $N_c \uparrow, N_v \uparrow \Rightarrow n_i$

Remember: in Si at RT: $n_i \simeq 10^{10} \text{ cm}^{-3}$ ($\ll N_c, N_v$)

4. Location of Fermi level

Location of E_F in band structure completely defines equilibrium carrier concentrations.

To pin-point location of E_F , need $n_o(E_F)$, $p_o(E_F)$, plus one additional argument.

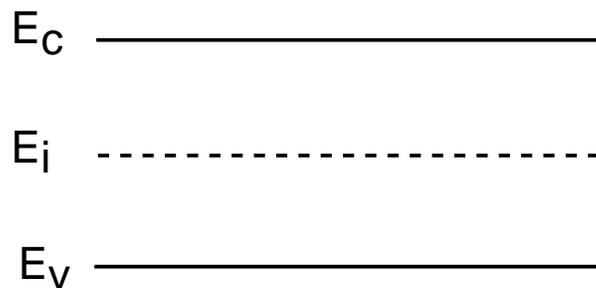
□ *Intrinsic semiconductor*

Require $n_o = p_o \Rightarrow$

$$E_i = \frac{E_c + E_v}{2} + kT \ln \sqrt{\frac{N_v}{N_c}}$$

Intrinsic Fermi level is close to the middle of the bandgap

In Si at 300 K, E_i is 1 *meV* above midgap



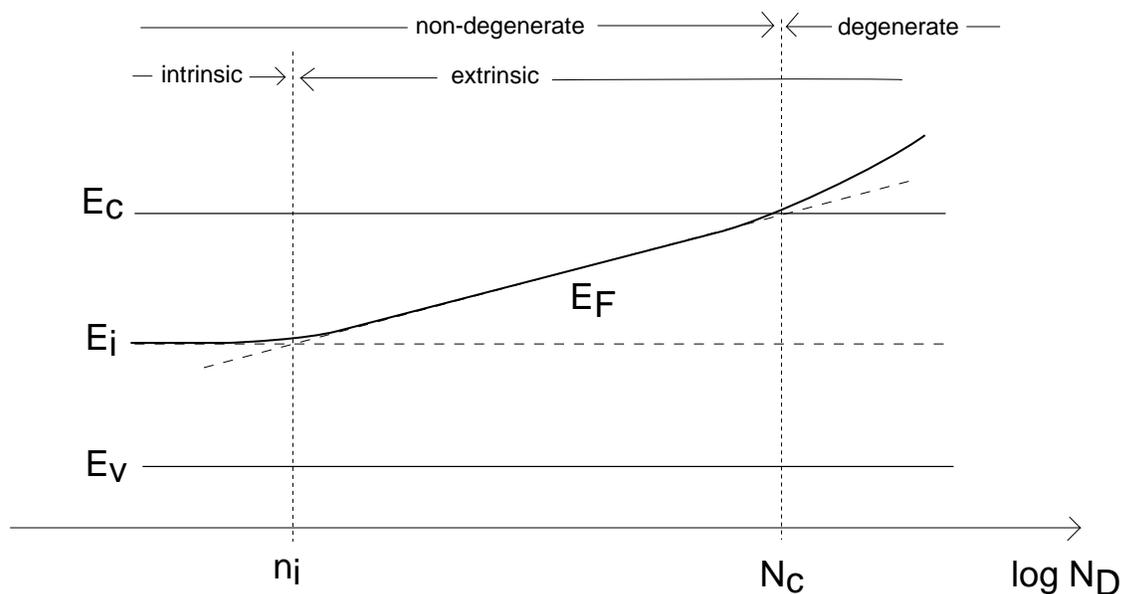
[Consistent with use of Maxwell-Boltzmann statistics in n_i expression]

□ *n-type extrinsic semiconductor*

Require $n_o \simeq N_D$. If non-degenerate ($N_D \ll N_c$):

$$E_F - E_c \simeq kT \ln \frac{N_D}{N_c}$$

Evolution of E_F with doping:

□ *p-type extrinsic semiconductor*

Require $p_o \simeq N_A$. If non-degenerate ($N_A \ll N_v$):

$$E_F - E_v \simeq kT \ln \frac{N_v}{N_A}$$

Key conclusions

- *Non-degenerate* semiconductor: $n_o \ll N_c$ and $p_o \ll N_v$: Maxwell-Boltzmann statistics apply:

$$n_o = N_c \exp \frac{E_F - E_c}{kT}, \quad p_o = N_v \exp \frac{E_v - E_F}{kT}$$

- *Intrinsic semiconductor*: ideally pure semiconductor. Under M-B statistics:

$$n_o = p_o = n_i = \sqrt{N_c N_v} \exp -\frac{E_g}{2kT}$$

- In non-degenerate semiconductor $n_o p_o$ is a constant that only depends on T :

$$n_o p_o = n_i^2$$

- In intrinsic semiconductor, E_F is close to middle of E_g .
- In extrinsic semiconductor, E_F location depends on doping level:
 - n-type non-degenerate semiconductor:

$$n_o \simeq N_D, \quad E_F - E_c \simeq kT \ln \frac{N_D}{N_c}$$

- p-type non-degenerate semiconductor:

$$p_o \simeq N_A, \quad E_F - E_v \simeq kT \ln \frac{N_v}{N_A}$$

- Order of magnitude of key parameters for Si at 300 K:
 - effective density of states of CB and VB: $N_c, N_v \sim 10^{19} \text{ cm}^{-3}$