Lecture 6 - Carrier drift and diffusion

February 16, 2007

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1. Thermal motion and scattering
2. Drift
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Reading assignment:

del Alamo, Ch. 4, §§4.1-4.3.

Key questions

- Are carriers sitting still in thermal equilibrium?
- How do carriers move in an electric field? What are the key dependencies of the drift velocity?
- How do the energy band diagrams represent the presence of an electric field?
- How does a concentration gradient affect carriers?

1. Thermal motion and scattering

We can think of carriers as particles in an ideal gas.

At finite T, carriers have finite thermal energy. All this energy resides in the kinetic energy of the particles.

Carriers move in random directions: no net velocity, but average carrier velocity is *thermal velocity*:

$$v_{th} = \sqrt{\frac{8}{\pi}\frac{kT}{m_c^*}}$$

Where:

$$m_c^* \equiv \text{conductivity effective mass } [eV \cdot s^2/cm^2]$$

m_c^* accounts for all interactions between the carriers and the perfect periodic potential of the lattice.

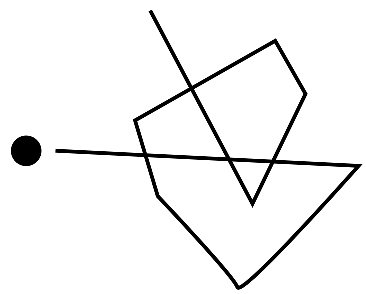
For electrons in Si at 300 K ($m_{ce}^* = 0.28 m_o$) and

$$v_{the} \simeq 2 \times 10^7 \ cm/s$$

But... semiconductor crystal is not perfect:

- the Si atoms themselves are vibrating around their equilibrium position in the lattice
- there are impurities and crystal imperfections

As carriers move around, they suffer frequent collisions:



Define:

- *Mean free path*, l_c: average distance travelled between collisions $[cm]$.
- *Scattering time*, τ_c: average time between collisions $[s]$.

Then:

$$l_c = v_{th}\tau_c$$

□ Scattering mechanisms:

1. *lattice or phonon scattering*: carriers collide with vibrating lattice atoms (phonon absorption and emission)

   ⇒ some energy exchanged ($\sim$ tens of meV)

2. *ionized impurity scattering*: Coulombic interaction between charged impurities and carriers

   ⇒ no energy exchanged

3. *surface scattering* in inversion layer
4. *neutral impurity scattering* with neutral dopants, interstitials, vacancies, etc
5. *carrier-carrier scattering*

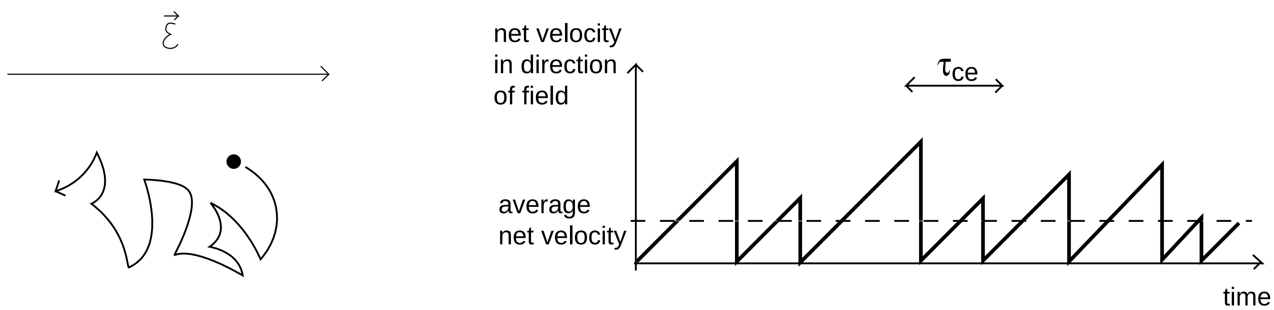
No need for detailed models.

Order of magnitude of $\tau_c < 1\ ps$ (see how to estimate in notes).

Then, order of magnitude of $l_c < 50\ nm$.

2. Drift

In the presence of an electric field, electrons drift:



□ Drift velocity

-electric field: $\mathcal{E}$
-electrostatic force on electron: $-q\mathcal{E}$
-acceleration between collisions: $\frac{-q\mathcal{E}}{m_{ce}^*}$
-velocity acquired during time τ_{ce}:

$$v_e^{drift} = -\frac{q\mathcal{E}\tau_{ce}}{m_{ce}^*}$$

or

$$v_e^{drift} = -\mu_e\mathcal{E}$$

$$\mu_e \equiv \text{electron mobility } [cm^2/V \cdot s]$$

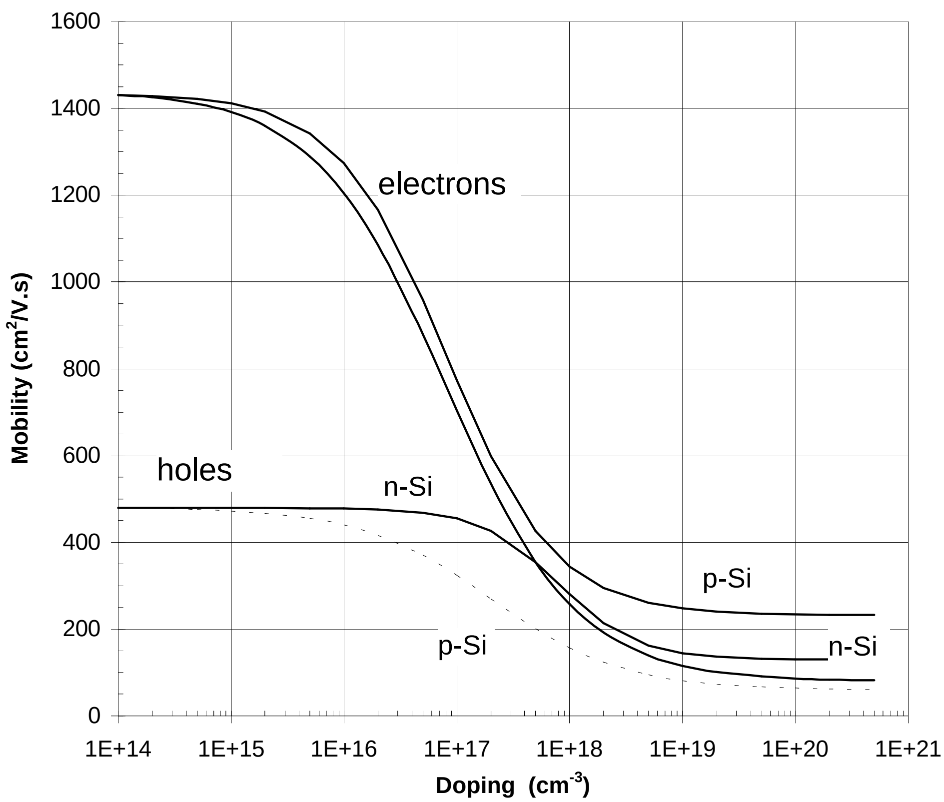
Mobility suggests ease of carrier motion in response to $\mathcal{E}$.

$$v_e^{drift} = -\mu_e \mathcal{E}$$

$$v_h^{drift} = \mu_h \mathcal{E}$$

Mobility depends on doping level and whether carrier is majority or minority-type.

Si at 300 K:



- at low N: limited by phonon scattering
- at high N: limited by ionized impurity scattering

□ Velocity saturation

Implicit assumption: *quasi-equilibrium*, that is, scattering rates not much affected from equilibrium.

$$v^{drift} \sim \mathcal{E} \quad \text{only if} \quad v^{drift} \ll v_{th}$$

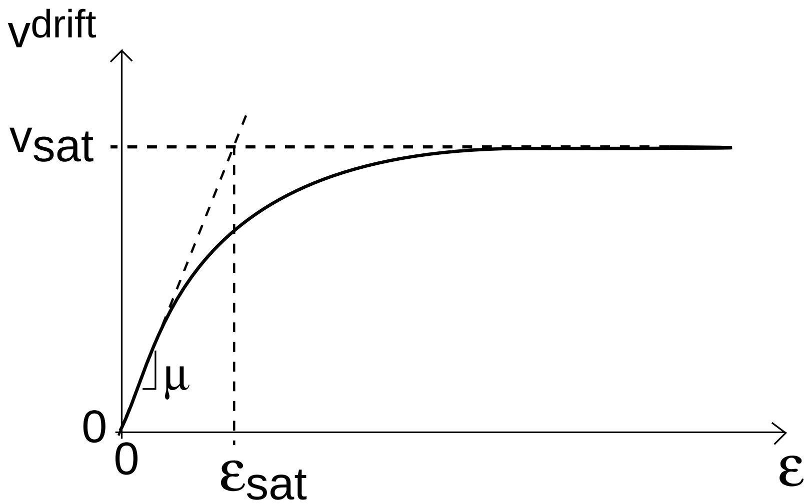
For high $\mathcal{E}$: carriers acquire substantial energy from $\mathcal{E}$

→ optical phonon emission strongly enhanced
→ scattering time $\sim 1/\mathcal{E}$
→ **drift velocity saturates**

$$v_{sat} \simeq \sqrt{\frac{8}{3\pi}\frac{E_{opt}}{m_c^*}}$$

For Si at 300 K:

- $v_{sat} \simeq 10^7\ cm/s$ for electrons
- $v_{sat} \simeq 6 \times 10^6\ cm/s$ for holes
- independent of T



Drift velocity vs. electric field fairly well described by:

$$v^{drift} = \mp \frac{\mu \mathcal{E}}{1 + |\frac{\mu \mathcal{E}}{v_{sat}}|}$$

Field required to saturate velocity:

$$\mathcal{E}_{sat} = \frac{v_{sat}}{\mu}$$

Velocity saturation crucial in modern devices:

if $\mu = 500\ cm^2/V.s$, $\mathcal{E}_{sat} = 2 \times 10^4\ V/cm$ (2 V across 1 μm)

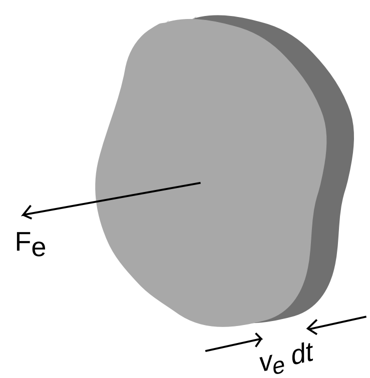
Since μ depends on doping, $\mathcal{E}_{sat}$ depends on doping too.

□ Particle flux and current density

particle flux ≡ # particles crossing unity surface (normal to flow) per unit time $[cm^{-2} \cdot s^{-1}]$

current density ≡ electrical charge crossing unity surface (normal to flow) per unit time $[C \cdot cm^{-2} \cdot s^{-1}]$

$$J_e = -qF_e$$



$$F_e = \frac{nv_e dt}{dt} = nv_e$$

Then

$$J_e = -qnv_e$$

$$J_h = qpv_h$$

• Drift current (low fields):

$$J_e = q\mu_e n\mathcal{E}$$

$$J_h = q\mu_h p\mathcal{E}$$

total:

$$J = q(\mu_e n + \mu_h p)\mathcal{E}$$

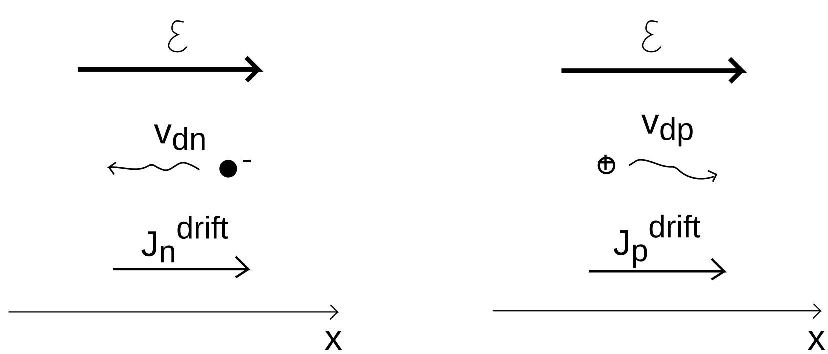
Electrical conductivity $[(\Omega \cdot cm)^{-1}]$:

$$\sigma = q(\mu_e n + \mu_h p)$$

Electrical resistivity $[\Omega \cdot cm]$:

$$\rho = \frac{1}{q(\mu_e n + \mu_h p)}$$

Check signs:



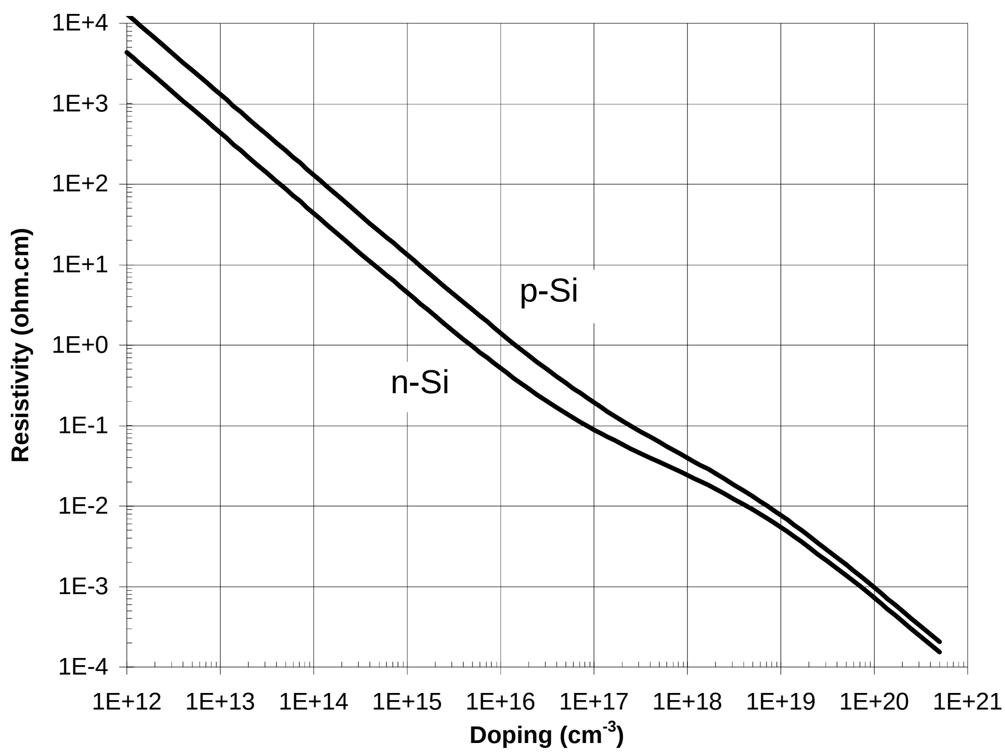
$$\rho = \frac{1}{q(\mu_e n + \mu_h p)}$$

ρ strong function of doping $\Rightarrow$ frequently used by wafer vendors to specify doping level of substrates

-for n-type: $\rho_n \simeq \frac{1}{q\mu_e N_D}$

-for p-type: $\rho_p \simeq \frac{1}{q\mu_h N_A}$

Si at 300K:



• Drift current (high fields):

$$J_{esat} = qnv_{esat}$$

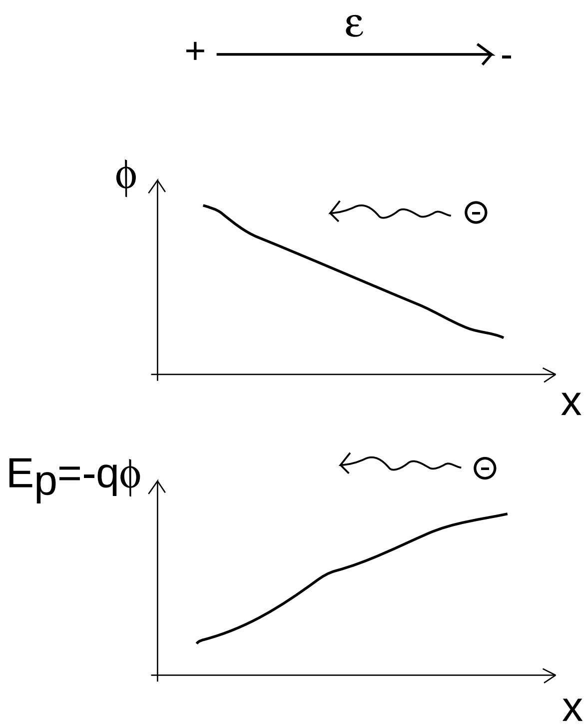
$$J_{hsat} = qpv_{hsat}$$

The only way to get more current is to increase carrier concentration.

□ Energy band diagram under electric field

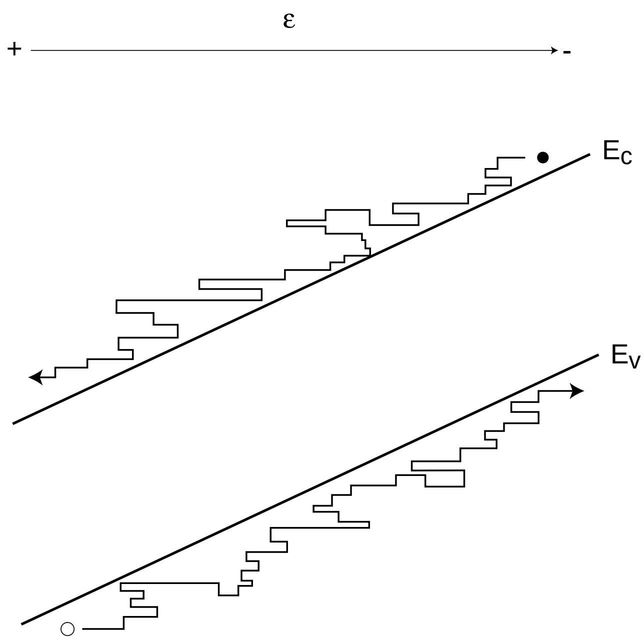
Energy band diagram needs to account for potential energy of electric field

• Vacuum:



Electron trades potential energy by kinetic energy as it moves to the left → *total electron energy unchanged*

• Energy band diagram is picture of electron energy ⇒ must add E_p to semiconductor energy band diagram ⇒ bands tilt



Meauring from an arbitrary energy reference, E_{ref}:

$$E_c + E_{ref} = E_p = -q\phi$$

Then:

$$\mathcal{E} = -\frac{d\phi}{dx} = \frac{1}{q}\frac{dE_c}{dx} = \frac{1}{q}\frac{dE_v}{dx}$$

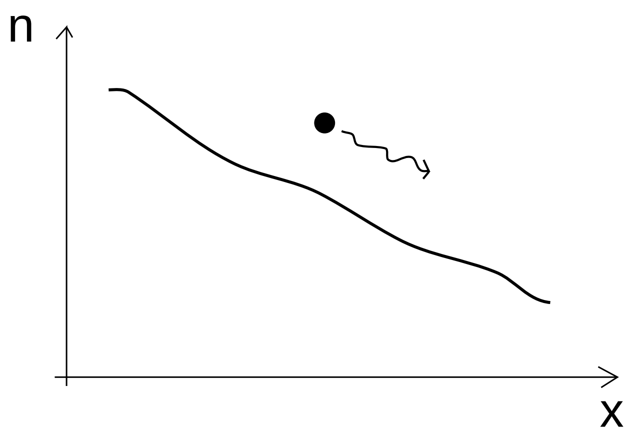
Shape of energy bands = shape of ϕ with a minus sign.

Can easily compute $\mathcal{E}$ from energy band diagram.

3. Diffusion

Movement of particles from regions of high concentration to regions of low concentration.

Diffusion produced by collisions with background medium (*i.e.*, vibrating Si lattice).



- Diffusion flux $\propto$ concentration *gradient* [Fick's first law]

$$F_e = -D_e \frac{dn}{dx}$$

$$F_h = -D_h \frac{dp}{dx}$$

$$D \equiv \text{diffusion coefficient } [cm^2/s]$$

$$F_e = -D_e \frac{dn}{dx}$$

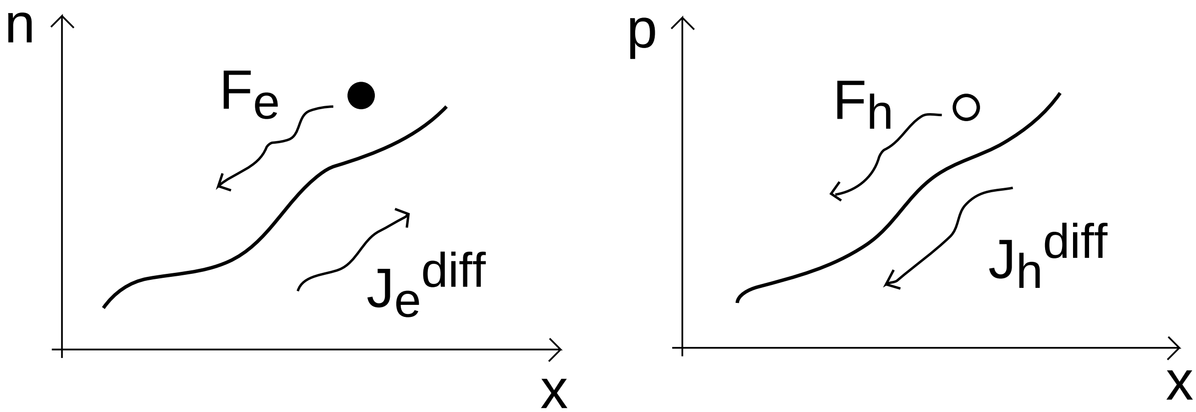
$$F_h = -D_h \frac{dp}{dx}$$

- Diffusion current:

$$J_e = qD_e \frac{dn}{dx}$$

$$J_h = -qD_h \frac{dp}{dx}$$

Check signs:



Key conclusions

- At finite temperatures, carriers move around in a random way suffering many collisions *(thermal motion)*.
- Dominant scattering mechanisms in bulk Si at 300K: phonon scattering and ionized impurity scattering.
- Two processes for carrier flow in semiconductors: drift and diffusion.
- General relationship between carrier net velocity (by drift or diffusion) and current density:

$$J_e = -qnv_e \qquad J_h = qpv_h$$

- For low fields, $v^{drift} \sim \mathcal{E}$.
- For high fields, $v^{drift} \sim v_{sat}$.
- Driving force for diffusion: concentration gradient.
- Order of magnitude of key parameters for Si at 300K:
  - $v_{th} \sim 2 \times 10^7\ cm/s$
  - $\tau_c < 1\ ps$
  - $l_c < 50\ nm$
  - electron mobility: $\mu_e \sim 100 - 1400\ cm^2/V \cdot s$
  - hole mobility: $\mu_h \sim 50 - 500\ cm^2/V \cdot s$
  - saturation velocity: $v_{sat} \sim 10^7\ cm/s$

Self study

- Study estimation of τ_c and l_c.
- Study doping dependence of $\mathcal{E}_{sat}$.
- Study phenomenological diffusion model in §4.3.