Lecture 9 - Carrier Flow *(cont.)*

February 23, 2007

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1. Shockley's Equations
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3. Majority-carrier type situations

Reading assignment:

del Alamo, Ch. 5, §§5.3-5.5

Quote of the day:

"If in discussing a semiconductor problem, you cannot draw an energy band diagram, then you don't know what you are talking about."

-H. Kroemer, IEEE Spectrum, June 2002.

Key questions

- How can the equation set that describes carrier flow in semiconductors be simplified?
- In regions where carrier concentrations are high enough, quasi-neutrality holds in equilibrium. How about out of equilibrium?
- What characterizes *majority*-carrier type situations?

1. Shockley's Equations

Gauss' law:	$\vec{\nabla} \cdot \vec{\mathcal{E}} = \frac{q}{\epsilon}(p - n + N_D^+ - N_A^-)$
Electron current equation:	$\vec{J}_e = -qn\vec{v}_e^{\,drift} + qD_e\vec{\nabla}n$
Hole current equation:	$\vec{J}_h = qp\vec{v}_h^{\,drift} - qD_h\vec{\nabla}p$
Electron continuity equation:	$\frac{\partial n}{\partial t} = G_{ext} - U(n,p) + \frac{1}{q}\vec{\nabla} \cdot \vec{J}_e$
Hole continuity equation:	$\frac{\partial p}{\partial t} = G_{ext} - U(n,p) - \frac{1}{q}\vec{\nabla} \cdot \vec{J}_h$
Total current equation:	$\vec{J}_t = \vec{J}_e + \vec{J}_h$

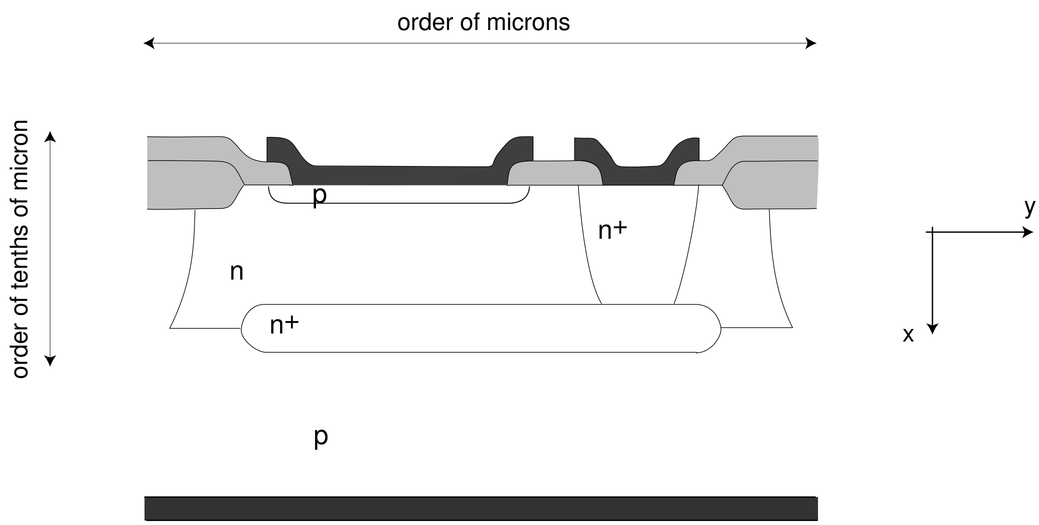
System of non-linear, coupled partial differential equations.

2. Simplifications of Shockley equations to 1D quasi-neutral situations

□ One-dimensional approximation

In many cases, complex problems can be broken into several 1D subproblems.

Example: integrated p-n diode



1D approximation: $\vec{\nabla} \Rightarrow \frac{\partial}{\partial x}$

Shockley's equations in 1D:

Gauss' law:	$\frac{\partial \mathcal{E}}{\partial x} = \frac{q}{\epsilon}(p - n + N_D - N_A)$
Electron current equation:	$J_e = -qnv_e^{drift}(\mathcal{E}) + qD_e\frac{\partial n}{\partial x}$
Hole current equation:	$J_h = qpv_h^{drift}(\mathcal{E}) - qD_h\frac{\partial p}{\partial x}$
Electron continuity equation:	$\frac{\partial n}{\partial t} = G_{ext} - U(n, p) + \frac{1}{q}\frac{\partial J_e}{\partial x}$
Hole continuity equation:	$\frac{\partial p}{\partial t} = G_{ext} - U(n, p) - \frac{1}{q}\frac{\partial J_h}{\partial x}$
Total current equation:	$J_t = J_e + J_h$

Equation set difficult because of coupling through Gauss' law.

Two broad classes of important situations break Gauss'law coupling:

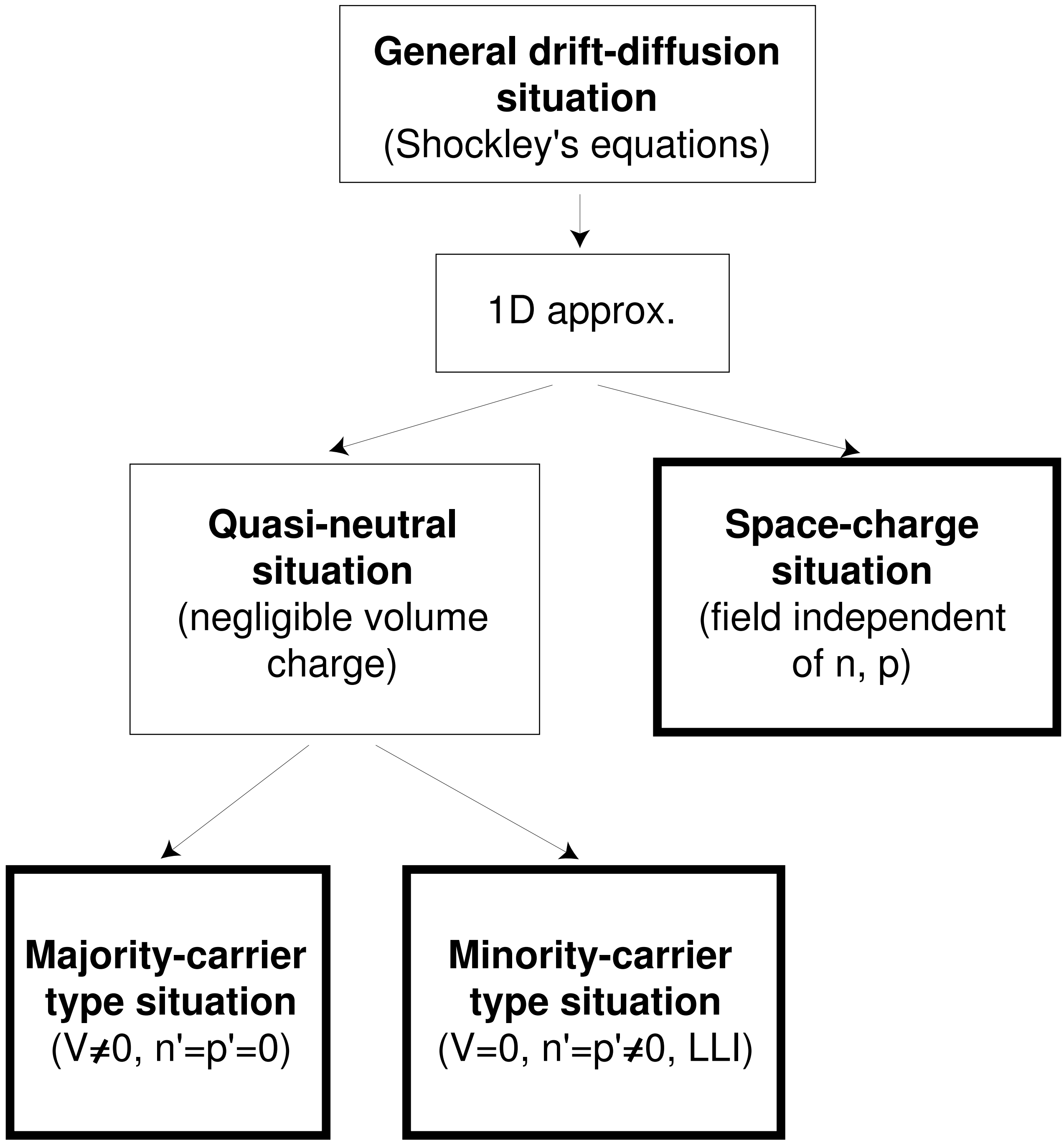
1. Carrier concentrations are high: *quasi-neutral* situation:

$$\rho \simeq 0 \ \Rightarrow \ \frac{\partial \mathcal{E}}{\partial x} \simeq 0$$

2. Carrier concentrations are very low: *space-charge* and *high-resistivity* situations:

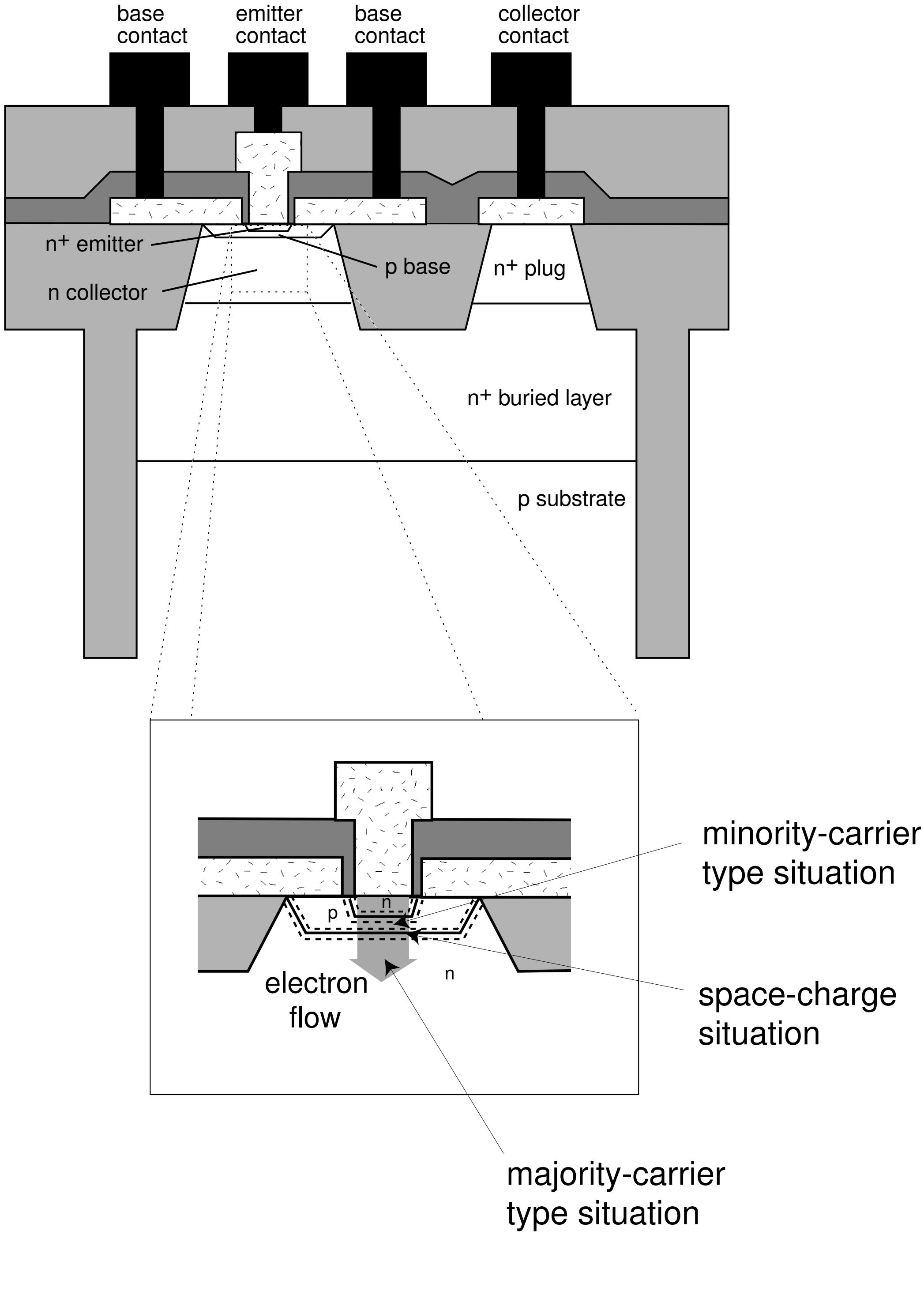
$$\mathcal{E} \text{ independent of } n, p$$

Overview of simplified carrier flow formulations



Each formulation uniquely applies to a different region in a device.

Example: npn BJT in forward-active regime



□ Quasi-neutral approximation

At every location, the net volume charge that arises from a discrepancy of the concentration of positive and negative species is negligible in the scale of the charge density that is present.

QN approximation eliminates Gauss' law from the set:

$$\rho = q(p - n + N_D^+ - N_A^-) = q(p_o - n_o + N_D^+ - N_A^-) + q(p' - n')$$

- Quasi-neutrality in equilibrium:

$$|\frac{p_o - n_o + N_D^+ - N_A^-}{N_D^+ - N_A^-}| \ll 1$$

which implies

$$n_o - p_o \simeq N_D^+ - N_A^-$$

- Additionally, quasi-neutrality outside equilibrium:

$$|\frac{p' - n'}{n'}| \simeq |\frac{p' - n'}{p'}| \ll 1$$

which implies:

$$p' \simeq n'$$

- QN approximation good if $n,\ p$ high $\Rightarrow$ carriers move to erase ρ.

- QN holds if *length scale of problem* $\gg$ *Debye length*

□ Consequence of quasi-neutrality

1. Uncouple Gauss' law from rest of system:

$$\rho \simeq 0 \;\Rightarrow\; \frac{\partial \mathcal{E}}{\partial x} \simeq 0$$

If, in general define:

$$\mathcal{E} = \mathcal{E}_o + \mathcal{E}'$$

Then, in equilibrium:

$$\frac{\partial \mathcal{E}_o}{\partial x} = \frac{q}{\epsilon}(p_o - n_o + N_D^+ - N_A^-)$$

and out of equilibrium:

$$\frac{\partial \mathcal{E}'}{\partial x} = \frac{q}{\epsilon}(p' - n')$$

$\mathcal{E}_o$ computed as in Ch. 4. Here will learn to compute $\mathcal{E}'$.

2. Subtract one continuity equation from the other:

$$\frac{\partial J_t}{\partial x} = q\frac{\partial(n-p)}{\partial t} = -\frac{\partial \rho}{\partial t}$$

continuity equation for net volume charge: if J_t changes with position, ρ changes with time.

Easier to see in integral form:

$$\int_S \vec{J}_t.d\vec{S} = -\frac{\partial}{\partial t}\int_V \rho\, dV$$

- In *Static case*:

  $\frac{\partial \rho}{\partial t} = 0 \Rightarrow \frac{\partial J_t}{\partial x} = 0,\ J_t$ independent of x

- In *Dynamic case*, we also have in most useful situations:

  $\frac{\partial \rho}{\partial t} \simeq 0$ in times scale of interest

  [will discuss soon]

Simplified set of Shockley equations for 1D quasi-neutral situations

$$p - n + N_D - N_A \simeq 0$$

$$J_e = -qnv_e^{drift} + qD_e\frac{\partial n}{\partial x}$$

$$J_h = qpv_h^{drift} - qD_h\frac{\partial p}{\partial x}$$

$$\frac{\partial n}{\partial t} = G_{ext} - U + \frac{1}{q}\frac{\partial J_e}{\partial x} \quad \text{or} \quad \frac{\partial p}{\partial t} = G_{ext} - U - \frac{1}{q}\frac{\partial J_h}{\partial x}$$

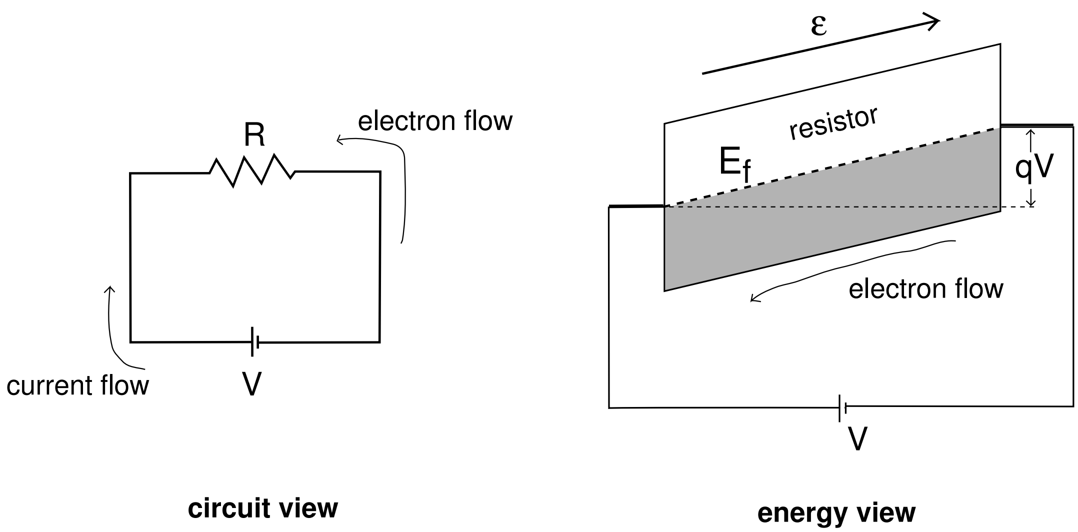
$$\frac{\partial J_t}{\partial x} \simeq 0$$

$$J_t = J_e + J_h$$

3. Majority-carrier type situations

Voltage applied to extrinsic quasi-neutral semiconductor without upsetting the equilibrium carrier concentrations.

□ Remember what a battery does:



• Battery picks up electrons from positive terminal, increases their potential energy and puts them at the negative terminal.

• If provided with a path (resistor), electrons flow.

□ Characteristics of majority carrier-type situations:

- electric field imposed from outside
- electrons and holes drift
- electron and hole concentrations unperturbed from TE

Simplifications:

- neglect contribution of minority carriers
- neglect time derivatives of carrier concentrations

⇒ problem becomes completely *quasi-static*

□ Simplification of majority carrier current (n-type):

Must distinguish between internal field in TE ($\mathcal{E}_o$) and total field outside equilibrium ($\mathcal{E}$).

For simplicity, do in low-field limit (exact case done in notes).

In equilibrium:

$$J_{eo} = q\mu_e n_o \mathcal{E}_o + qD_e \frac{dn_o}{dx} = 0$$

Out of equilibrium:

$$J_e \simeq q\mu_e n_o \mathcal{E} + qD_e \frac{dn_o}{dx}$$

Hence:

$$J_e = q\mu_e n_o(\mathcal{E} - \mathcal{E}_o) = q\mu_e n_o \mathcal{E}'$$

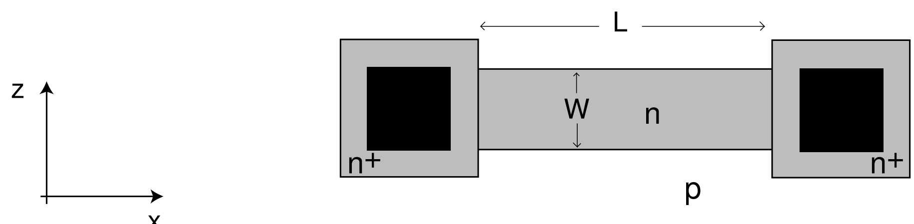
In the more general case (see notes):

$$J_e = -qn_o[v_{de}(\mathcal{E}) - v_{de}(\mathcal{E}_o)]$$

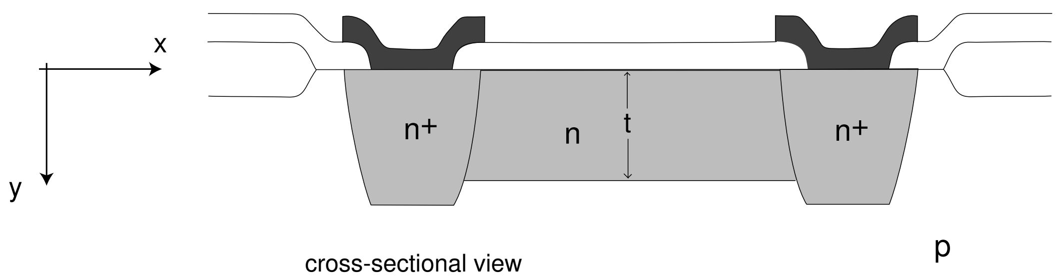
□ Equation set for 1D majority-carrier type situations:

n-type	p-type
$n \simeq n_o \simeq N_D$	$p \simeq p_o \simeq N_A$
$J_e = -qn_o[v_{de}(\mathcal{E}) - v_{de}(\mathcal{E}_o)]$	$J_h = qp_o[v_{dh}(\mathcal{E}) - v_{dh}(\mathcal{E}_o)]$
$\frac{dJ_e}{dx} \simeq 0,\ \frac{dJ_h}{dx} \simeq 0,\ \frac{dJ_t}{dx} \simeq 0$	
$J_t \simeq J_e$	$J_t \simeq J_h$

□ Example 1: *Integrated Resistor* with uniform doping (n-type)



top view



cross-sectional view

Uniform doping ⇒ $\mathcal{E}_o = 0$, then:

$$J_t = -qN_D v_e^{drift}(\mathcal{E})$$

• If $\mathcal{E}$ not too high,

$$J_t \simeq qN_D \mu_e \mathcal{E}$$

I-V characteristics:

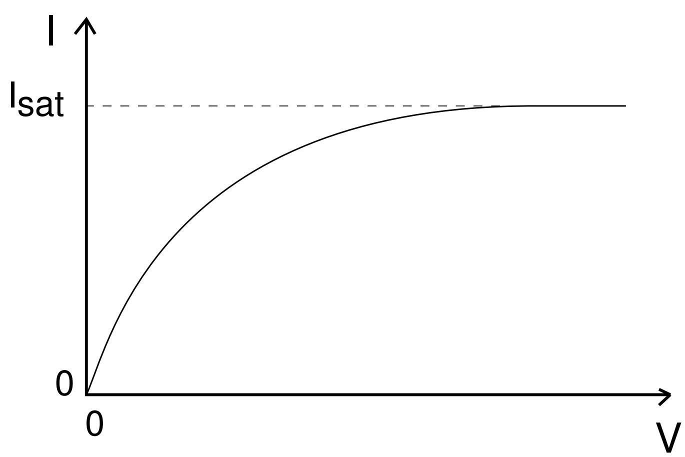
$$I = WtqN_D\mu_e \frac{V}{L}$$

• In general (low and high fields):

$$I = WtqN_D \frac{v_{sat}}{1 + \frac{v_{sat}}{\mu_e}\frac{L}{V}}$$

which for high fields saturates to:

$$I_{sat} = WtqN_D v_{sat}$$



Key conclusions

- *Shockley equations*: system of equations that describes carrier phenomena in semiconductors in the drift-diffusion regime.
- *Quasi-neutral* approximation appropriate if semiconductor is sufficiently extrinsic: $\rho \simeq 0 \Rightarrow$

$$n_o - p_o \simeq N_D - N_A \qquad n' \simeq p'$$

- Consequence of quasi-neutrality:

$$J_t \neq J_t(x)$$

- *Majority carrier-type situations* characterized by application of external voltage without perturbing carrier concentrations.
- Majority-carrier type situations dominated by drift of majority carriers.
- Integrated resistor:
  - for low voltages, current proportional to voltage across
  - for high voltages, current saturates due to v_{sat}

Self-study

- Integral form of continuity equations and consequences.
- Exercises 5.1, 5.2.
- Non-uniformly doped resistor
- Sheet resistance