Josephson Circuits I.

Outline

1. RCSJ Model Review
2. Response to DC and AC Drives
   - Voltage standard
3. The DC SQUID
4. Tunable Josephson Junction

October 27, 2005

JJ RCSJ Model as Circuit Element

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Please see: Figure 9.6, page 459, from Orlando, T., and K. Delin. *Foundations of Applied Superconductivity*. Reading, MA: Addison-Wesley, 1991. ISBN: 0201183234.

$$i = I_c \sin\varphi + \frac{v}{R} + C\frac{dv}{dt}$$ and $$v = \frac{\Phi_o}{2\pi}\frac{d\varphi}{dt}$$

Therefore,

$$i = I_c \sin\varphi + \frac{\Phi_o}{2\pi R}\frac{d\varphi}{dt} + C\left(\frac{\Phi_o}{2\pi}\right)^2\frac{d^2\varphi}{dt^2}$$

DC Current Drive

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Please see: Figure 9.6, page 459, from Orlando, T., and K. Delin. *Foundations of Applied Superconductivity*. Reading, MA: Addison-Wesley, 1991. ISBN: 0201183234.

$$\beta_c = \frac{\tau_{RC}}{\tau_J} = \frac{R^2 C}{L_J} = \frac{2\pi I_c R^2 C}{\Phi_o}$$

A. Static Solution: $\varphi = \sin^{-1} \frac{i}{I_c}$ for $i \leq I_c$

B. Dynamical Solution

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Please see: Figure 9.10, page 464 from Orlando, T., and K. Delin. *Foundations of Applied Superconductivity*. Reading, MA: Addison-Wesley, 1991. ISBN: 0201183234.

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Please see: Figure 9.11, page 465, from Orlando, T., and K. Delin. *Foundations of Applied Superconductivity*. Reading, MA: Addison-Wesley, 1991. ISBN: 0201183234.

DC Voltage Drive

The voltage source is DC with $v=V_0$, so that

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Please see: Figure 9.6, page 459, from Orlando, T., and K. Delin. *Foundations of Applied Superconductivity*. Reading, MA: Addison-Wesley, 1991. ISBN: 0201183234.

$$\varphi(t) = \varphi(0) + \frac{2\pi}{\Phi_o} V_o t$$

The resulting current across the JJ is ac

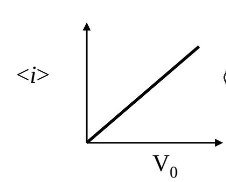
$$i_J = I_c \sin\left(\frac{2\pi}{\Phi_o} V_o t + \varphi(0)\right)$$

The current across the resistor is dc

$$i_R = \frac{V_o}{R}$$

The total current is then

$$i = \frac{V_0}{R} + I_c \sin\left(\frac{2\pi}{\Phi_o} V_o t + \varphi(0)\right)$$



$$\langle i \rangle = \frac{V_o}{R}$$

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AC Voltage Drive

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Please see: Figure 9.13, page 467, from Orlando, T., and K. Delin. *Foundations of Applied Superconductivity*. Reading, MA: Addison-Wesley, 1991. ISBN: 0201183234.

The voltage source $v(t) = V_o + V_s \cos \omega_s t$

Then the guage-invariant phase is

$$\varphi(t) = \varphi(0) + \frac{2\pi}{\Phi_o} V_o t + \frac{2\pi V_s}{\Phi_o \omega_s} \sin \omega_s t$$

The resulting current across the JJ is

$$i_J = I_c \sin\left(\varphi(0) + \frac{2\pi}{\Phi_o} V_o t + \frac{2\pi V_s}{\Phi_o \omega_s} \sin \omega_s t\right)$$

The current across the resistor is

$$i_R(t) = \frac{V_o}{R} + \frac{V_s}{R} \cos \omega_s t$$

The current across the capacitor is $i_C(t) = -C\, V_s \omega_s \sin \omega_s t$

The total current is then $i(t) = i_R + i_J + i_C$

AC Voltage Drive

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$$\langle i \rangle = \frac{V_o}{R} + I_c \sum_{n=-\infty}^{+\infty} (-1)^n J_n\left(\frac{2\pi V_s}{\Phi_o \omega_s}\right) \sin \varphi(0)\, \delta_{2\pi f_J, n\omega_s}$$

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Please see: Figure 9.14, page 468, from Orlando, T., and K. Delin. *Foundations of Applied Superconductivity*. Reading, MA: Addison-Wesley, 1991. ISBN: 0201183234.

AC Voltage vs Current Drives

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Please see: Figure 9.15, page 469, from Orlando, T., and K. Delin. *Foundations of Applied Superconductivity*. Reading, MA: Addison-Wesley, 1991. ISBN: 0201183234.

Voltage Standard

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Please see: NIST (public domain) http://www.boulder.nist.gov/div814/div814/whatwedo/volt/dc/JVS.html

10 V conventional Josephson voltage standard chip. The chip is 1 cm x 2 cm and contains 20,208 series connected Nb-AlOx -Nb junctions.

http://www.boulder.nist.gov/div814/div814/whatwedo/volt/dc/JVS.html

HYPRES: Voltage Standard Chip

HYPRES is the only commercial manufacturer of the superconducting integrated circuit used in Primary Voltage Standard Systems. HYPRES chips are used in the primary voltage standards in national laboratories around the world including Italy, France, United Kingdom, Australia, China, Malaysia, Japan, England, Canada, Norway, United States, Netherlands and Mexico.
The HYPRES Josephson Junction Array Voltage Standard circuits provide the ultimate accuracy for realizing and maintaining the SI Volt.

Features/Specifcations

- Niobium/Aluminum Oxide/Niobium, SiO2 dielectric, Niobium wiring technology.
- All Niobium technology. Refractory. Impervious to moisture and thermal cycling.
- 20,208 Josephson junctions (10 V chip) 3,660 Josephson junctions (1 V chip)
- 18 x 38 micrometers junction area.
- Installed in a FR-4 epoxy glass mount.
- RF input WR-12 waveguide flange.
- RF Distribution - 16 way parallel x 1263 cells in series (10 V) - 4 way parallel x 915 cells in series (1
- Designed for a frequency range of 72-78 GHz
- Operating temperature of 4.2 K
- Common DC terminal resistance is < 1 Ohm - typical
- Approximately 10 mW operating power at the input flange for 10 V chip (2 mW for 1 V)
- -11V to +11 V range for 10 V chip.
- -2.5 V to + 2.5 V range for 1 V chip.
- Stability time is typically 1 hour for the 10 V chip, 5 hours for the 1V chip
- 0.005 PPM accuracy at 10 V (10 V chip)
- 0.05 PPM accuracy at 1V (1V chip)
- Calibration certificate supplied with each chip.
- Two (2) year warranty

http://www.hypres.com/

Parametric Inductor

Please see: Figure 9.6, page 459, from Orlando, T., and K. Delin. *Foundations of Applied Superconductivity*. Reading, MA: Addison-Wesley, 1991. ISBN: 0201183234.

$$i(t) = I_c \sin\varphi(t) + \frac{v(t)}{R} + C\frac{d}{dt}v(t)$$

Take the time derivative of the currents, and for the Josephson term:

$$\frac{d}{dt} i_J(t) = \left[\frac{2\pi I_c}{\Phi_o}\cos\varphi(t)\right] v(t)$$

The parameteric (time-dependent) inductance can be defined as

$$L_J = \frac{\Phi_o}{2\pi I_c \cos\varphi(t)}$$

*On the zero-voltage branch, for $I_s << I_0$ and $I_0 + I_s << I_{c,}$ then $I_0 \sim I_c \sin\phi$, so that

$$L_J \approx \frac{\Phi_o}{2\pi I_c\sqrt{1-(I_o/I_c)^2}}$$

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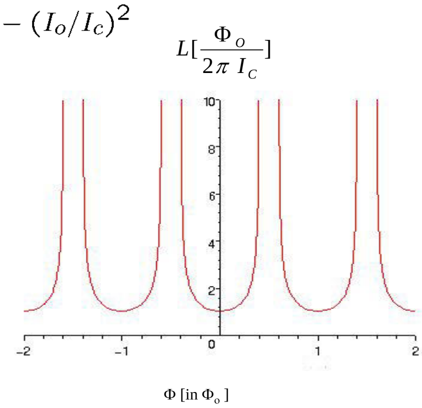
Inductance along V=0 branch

$$L_J \approx \frac{\Phi_o}{2\pi I_c\sqrt{1-(I_o/I_c)^2}}$$



The DC SQUID (damped)

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Please see: Figure 9.16, page 471, from Orlando, T., and K. Delin. *Foundations of Applied Superconductivity*. Reading, MA: Addison-Wesley, 1991. ISBN: 0201183234.

Our goal is to show that the DC SQUID circuit in (a) is equivalent to the circuit for a single junction with an effective I_c and an effective resistance. The inductance of the SQUID loop in (a) is considered negligible.

In fact, we will also show later that there is an equivalent statement even for the underdamped cass.

DC SQUID Equivalent Circuit

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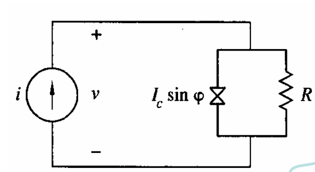
Please see: Figure 9.16, page 471, from Orlando, T., and K. Delin. *Foundations of Applied Superconductivity*. Reading, MA: Addison-Wesley, 1991. ISBN: 0201183234.



$$\begin{aligned} i &= i_{J_1} + i_{J_2} + i_{R_1} + i_{R_2} \\ &= I_{c1}\sin\varphi_1 + I_{c1}\sin\varphi_2 + \frac{v}{R_1} + \frac{v}{R_2} \end{aligned}$$

Use flux quantization, $\varphi_2 - \varphi_1 = \frac{2\pi\Phi}{\Phi_o}$ and the fact that the junctions are identical

$$i = \underbrace{2I_{c1}\cos\left(\frac{\pi\Phi_{\text{ext}}}{\Phi_o}\right)}_{I_c} \sin\underbrace{\left(\varphi_1 + \frac{\pi\Phi_{\text{ext}}}{\Phi_o}\right)}_{\varphi} + \underbrace{\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}_{1/R} \underbrace{v(t)}_{\frac{\Phi_0}{2\pi}\frac{d}{dt}\varphi}$$

$$I = I_c \sin \varphi + 1/R \; \frac{\Phi_0}{2\pi}\frac{d}{dt}\varphi$$

DC SQUID Equivalent Circuit

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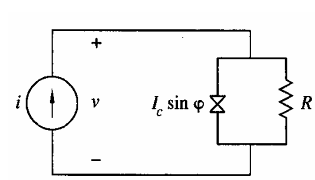

$$i = I_c \sin\varphi + \frac{1}{R}\frac{\Phi_o}{2\pi}\frac{d\varphi}{dt} \quad \text{with} \quad I_c = 2I_{c1}\cos\left(\frac{\pi\Phi_{\text{ext}}}{\Phi_o}\right)$$

Therefore, for this overdamped equivalent circuit, for $i > I_C$

$$\langle v(t)\rangle = iR\sqrt{1 - \left[\frac{2I_{c1}}{i}\cos\left(\frac{\pi\Phi_{\text{ext}}}{\Phi_o}\right)\right]^2} \qquad \langle v(t)\rangle = iR\sqrt{1 - \left(\frac{i_{\text{max}}}{i}\right)^2}$$

DC SQUID Voltage Modulation

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Please see: Figure 5.11a, page 272, from Van Duzen and Turner.

DC SQUID Voltage Modulation

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DC SQUID Sensitivity

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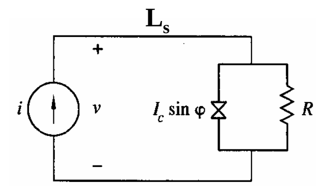
Please see: Figure 9.16, page 471, from Orlando, T., and K. Delin. *Foundations of Applied Superconductivity*. Reading, MA: Addison-Wesley, 1991. ISBN: 0201183234.



Assume that $L_s > L_J = \Phi_0/2\pi I_c(\Phi_{ext})$, then the range of modulation of the current is about

$$\delta i = i^{+}_{\max} - i^{-}_{\max} \approx \Phi_o/L$$

The sensitivity of the output voltage to the input flux is

$$\frac{\delta \langle v \rangle}{\delta \Phi_{ext}} \approx \frac{R_D \delta i}{\frac{1}{2}\Phi_o} = \frac{2R_D}{L}$$

With $R_D \sim R = 1$ Ohm and $L \sim 1$ nH, then the sensitivity is about one microvolt per flux quantum, so that small fractions of a flux quantum can be measured.

DC SQUID and Thermal Noise

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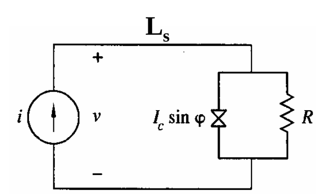
Please see: Figure 9.16, page 471, from Orlando, T., and K. Delin. *Foundations of Applied Superconductivity*. Reading, MA: Addison-Wesley, 1991. ISBN: 0201183234.



Assume again that $L_s > L_J = \Phi_0/2\pi I_c(\Phi_{ext})$, then to prevent thermal noise from effecting the SQUID's performance, one needs

Energy stored in the SQUID >> Thermal Energy

$$\frac{1}{2}\frac{\Phi_o^2}{L} \gg \frac{1}{2} k_B T$$

Therefore, $L_s < 1$ nH at 4 K, and 0.1 nH at 100 K.

Equivalent Tunable Junction

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Please see: Figure 9.1, page 486, from Orlando, T., and K. Delin. *Foundations of Applied Superconductivity*. Reading, MA: Addison-Wesley, 1991. ISBN: 0201183234.

$$I_c = 2 I_{c1} \cos\left(\frac{\pi \Phi_{\text{ext}}}{\Phi_o}\right)$$

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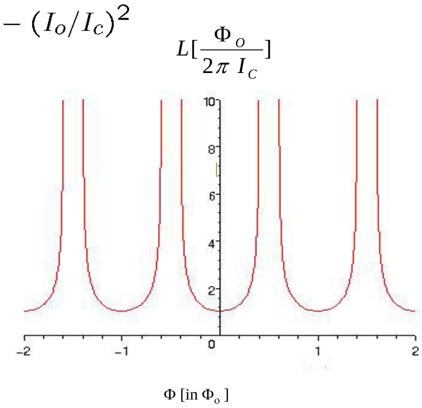
Please see: Figure 9.1, page 486, from Orlando, T., and K. Delin. *Foundations of Applied Superconductivity*. Reading, MA: Addison-Wesley, 1991. ISBN: 0201183234.

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \qquad C = C_1 + C_2$$

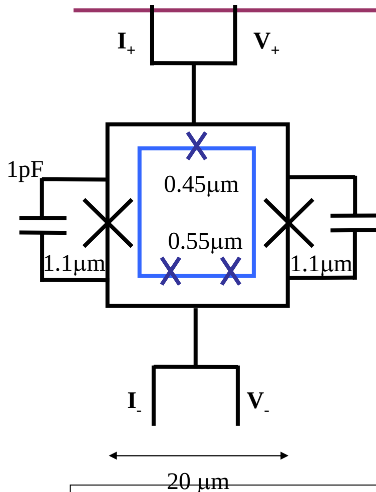
$$\beta_c = \frac{\tau_{RC}}{\tau_J} = \frac{R^2 C}{L_J} = \frac{2\pi I_c R^2 C}{\Phi_o}$$

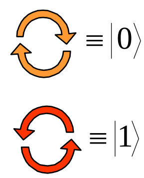
Tunable Inductance along V=0 branch

$$L_J \approx \frac{\Phi_o}{2\pi I_c \sqrt{1 - (I_o/I_c)^2}}$$



Three-Junction Loop Measurements





Three-junction Loop
Jct. Size ~ 0.45μm, 0.55μm
Loop size ~16x16μm²
$L_{3\text{-junction}}$ ~ 30pH
I_c~1 & 2μA
E_J/E_c ~ 350 & 550

DC SQUID
Shunt capacitors ~ 1pF
Jct. Size ~ 1.1μm
Loop size ~20x20μm²
L_{SQUID} ~ 50pH
I_c~10 & 20μA
M~35pH
J_c~350 & 730A/cm²

- **Measure switching current of DC SQUID**
- **Vary external flux, temperature and SQUID ramp rate**

Thermal Activation of Nb Persistent Current Qubit

5/6/03

SFQ on-chip oscillator and qubit

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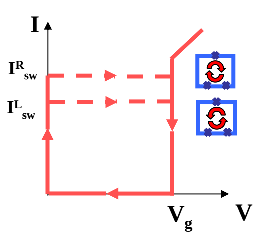
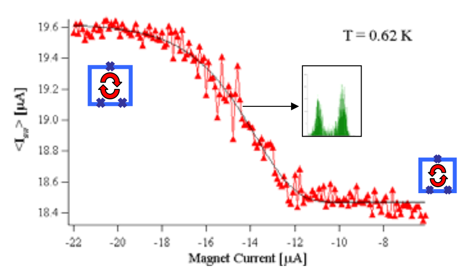
SQUID Detector





	J_c	E_J	E_J/E_c	Jct. width	α	Q
Device A:	365 A/cm²	2400 μeV	380	0.563 μm	0.613	1.0x10⁶
Device B:	730 A/cm²	4000 μeV	560	0.529 μm	0.589	1.2x10⁶

Highest Q of any submicron Nb junction

Coupling between qubits