Lecture 17: Flux Flow & Pinning

Outline

1. **Review of Vortices**
2. **Flux Flow**
3. **Pinning**
4. **Critical State Model**

November 8, 2005

Fluxoid Quantization and Type II Superconductors

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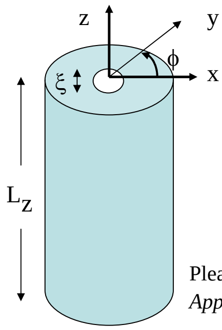
Please see: Figure 6.1, page 259, from Orlando, T., and K. Delin. *Foundations of Applied Superconductivity*. Reading, MA: Addison-Wesley, 1991. ISBN: 0201183234.

Vortex in a cylinder κ >> 1



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Please see: Figure 6.5, page 271, from Orlando, T., and K. Delin. *Foundations of Applied Superconductivity*. Reading, MA: Addison-Wesley, 1991. ISBN: 0201183234.

$$\mathrm{B}(\mathbf{r}) = \begin{cases} \dfrac{\Phi_o}{2\pi\lambda^2} K_0\left(\dfrac{r}{\lambda}\right) \mathbf{i}_z & \text{for } r \geq \xi \\ \dfrac{\Phi_o}{2\pi\lambda^2} K_0\left(\dfrac{\xi}{\lambda}\right) \mathbf{i}_z & \text{for } r < \xi \end{cases}$$

$$\mathbf{J}_s(\mathbf{r}) = \begin{cases} \dfrac{\Phi_o}{2\pi\mu_o\lambda^3} K_1\left(\dfrac{r}{\lambda}\right) \mathbf{i}_\phi & \text{for } r \geq \xi \\ 0 & \text{for } r < \xi \end{cases}$$

The Vortex State

$$\langle B \rangle = n_V \Phi_V$$

n_V is the areal density of vortices, the number per unit area.

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Please see: Figure 6.2a, page 262, from Orlando, T., and K. Delin. *Foundations of Applied Superconductivity*. Reading, MA: Addison-Wesley, 1991. ISBN: 0201183234.

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Please see: "A current-carrying type II superconductor in the mixed state" from http://phys.kent.edu/pages/cep.htm

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Vortices in High-Field Magnets

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Please see: Figure 7.7, page 353, from Orlando, T., and K. Delin. *Foundations of Applied Superconductivity*. Reading, MA: Addison-Wesley, 1991. ISBN: 0201183234.

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Please see: Figure 7.8, page 354, from Orlando, T., and K. Delin. *Foundations of Applied Superconductivity*. Reading, MA: Addison-Wesley, 1991. ISBN: 0201183234.

Why use Type-II for Magnets?

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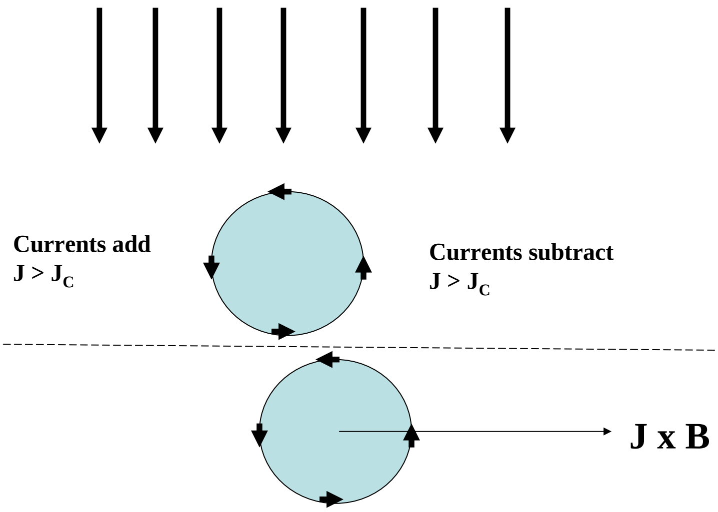
Please see: Figure 7.9, page 355, from Orlando, T., and K. Delin. *Foundations of Applied Superconductivity*. Reading, MA: Addison-Wesley, 1991. ISBN: 0201183234.

Vortex Driven by a Uniform Current



Power Dissipation due to Flux Flow

Let the vortices move in unison with velocity u_X

$$\Delta\Phi = \Phi_o n_V h u_x \, \Delta t$$

Faraday's law gives a voltage in direction of current

$$v = \frac{d\Phi}{dt} = \Phi_o n_V h u_x$$

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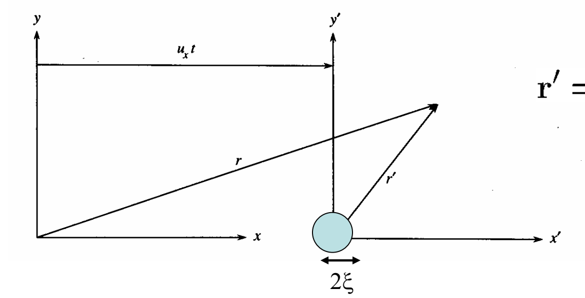
The power dissipated is then

$$P_{\text{dis}} = iv = i\Phi_o n_V h u_x$$

Please see: Figure 7.9b, page 355, from Orlando, T., and K. Delin. *Foundations of Applied Superconductivity*. Reading, MA: Addison-Wesley, 1991. ISBN: 0201183234.

We now seek a more microscopic description of the dissipation. This will allow us to find the velocity. So we look at one vortex to see where the power is dissipated.

Microscopic Picture of Dissipation

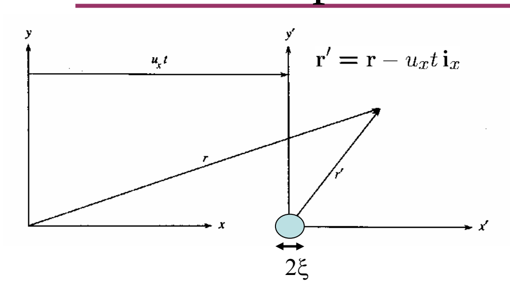


$$\mathbf{r}' = \mathbf{r} - u_x t\,\mathbf{i}_x$$

Strategy:

1. **J** in Superconductor near the core is known, **J**(moving vortex)
2. Use this to find **E** in superconductor near the core
3. Use continuity of **E** to find **E** inside the core
4. **J** inside the core is just given by ohm's law **J** = σ **E**
5. All the dissipation takes place in the core as an ohmic loss.
6. Compare this result with the previous macroscopic power to get u_X

Microscopic Picture of Dissipation



Near the core,

$$\mathbf{J}_V(\mathbf{r}) = \frac{\Phi_o}{2\pi\lambda^2}\frac{1}{\sqrt{(x-u_x t)^2 + y^2}}\,\mathbf{i}_{\phi'}$$

The electric field in the superconductor is

$$\mathbf{E}^{\text{out}}(\mathbf{r}) = \frac{\partial}{\partial t}\left(\mu_o\lambda^2\mathbf{J}(\mathbf{r})\right)$$

So that from the chain rule

$$\mathbf{E}^{\text{out}}(\mathbf{r}) = u_x\frac{\partial}{\partial x'}\left(\mu_o\lambda^2\mathbf{J}_V(\mathbf{r})\right)$$

A little algebra gives a dipole field!

$$\mathbf{E}^{\text{out}}(\mathbf{r}) = -u_x\frac{\Phi_o}{2\pi}\frac{1}{r'^2}\left(\mathbf{i}_{r'}\sin\phi' - \mathbf{i}_{\phi'}\cos\phi'\right)$$

Therefore, the field inside the core must be a uniform field:

$$\mathbf{E}^{\text{in}}(\mathbf{r}) = u_x\frac{\Phi_o}{2\pi}\frac{1}{\xi^2}\mathbf{i}_y$$ and $$\mathbf{J}^{\text{in}}(\mathbf{r}) = \sigma_o\mathbf{E}^{\text{in}}(\mathbf{r})$$

Power dissipated in the Core

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Please see: Figure 7.16, page 365, from Orlando, T., and K. Delin. *Foundations of Applied Superconductivity*. Reading, MA: Addison-Wesley, 1991. ISBN: 0201183234.

$$\mathbf{E}^{\text{in}}(\mathbf{r}) = u_x \frac{\Phi_o}{2\pi}\frac{1}{\xi^2}\mathbf{i}_y$$

$$\mathbf{J}^{\text{in}}(\mathbf{r}) = \sigma_o \mathbf{E}^{\text{in}}(\mathbf{r})$$

The power dissipated is $P_{\text{dis}}^{\text{core}} = \int_{\text{core}} \mathbf{E}^{\text{in}}(\mathbf{r}) \cdot \mathbf{J}^{\text{in}}(\mathbf{r})\, dv$

$$P_{\text{dis}}^{\text{core}} \approx \frac{u_x^2 \Phi_o^2 \sigma_o}{4\pi\xi^2} L_z \underset{\text{GL}}{=} \frac{u_x^2 \Phi_o^2 \sigma_o}{2\pi\xi^2} L_z$$

Adding the power from each vortex gives the total power dissipated

$$P_{\text{dis}} = \frac{u_x^2 \Phi_o^2 \sigma_o}{4\pi\xi^2} n_V\, 2ahL_z$$

Flux-Flow Resistance

$$P_{\text{dis}} = i\Phi_o n_V h u_x$$

Macroscopic picture

$$P_{\text{dis}} = \frac{u_x^2 \Phi_o^2 \sigma_o}{4\pi\xi^2} n_V\, 2ahL_z$$

Microscopic picture

Comparing these results gives: $u_x = \frac{2\pi\xi^2}{\Phi_o \sigma_o}\frac{1}{2aL_z} i$

Therefore, we find that the power dissipated can be written as

$$P_{\text{dis}} = i^2 R_{\text{ff}} \quad \text{where} \quad R_{\text{ff}} = \frac{h}{2aL_z}\underbrace{\frac{1}{\sigma_o} 2\pi\xi^2 n_V}_{\text{Flux-flow resistivity}}$$

Flux-flow resistivity

$$\rho_{\text{ff}} = \rho_o 2\pi\xi^2 n_V = \rho_o \frac{\mathcal{B}}{\mu_o H_{c2}}$$

as if current goes thru the cores

Flux-flow viscosity

Total force can be thought of as a current driving force plus a viscous drag force

$$\mathbf{f}'_{\text{tot}} = \mathbf{f}'_L + \mathbf{f}'_d$$

f' is per unit length

$$\mathbf{f}'_L \to \mathbf{J} \times \Phi_o \mathbf{i}_z \qquad \mathbf{f}'_d \to -\eta \mathbf{u}$$

The forces balance in steady state, so that

$$u_x = \frac{J\Phi_o}{\eta}$$

We found before that $u_x = \dfrac{2\pi\xi^2}{\Phi_o\sigma_o}\dfrac{1}{2aL_z}i$ so that $\eta = \dfrac{\Phi_o^2}{2\pi\xi^2}\sigma_o$

How do we keep vortices from moving and dissipating energy?

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Please see: "A current-carrying type II superconductor in the mixed state" from http://phys.kent.edu/pages/cep.htm

Pinning a Vortex

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Please see: Figure 7.17, page 369, from Orlando, T., and K. Delin. *Foundations of Applied Superconductivity*. Reading, MA: Addison-Wesley, 1991. ISBN: 0201183234.

a) Free energy decrease if core is in the normal region: $\Delta G_{\text{core}} = \frac{1}{2}\mu_o H_c^2 \pi \xi^2 L_z$

b) Free energy core is partly in the normal region: $\Delta G_{\text{core}} = \frac{1}{2}\mu_o H_c^2 \pi \xi \ell L_z$

Therefore, there is a force restraining the vortex in the normal region given by

$$f_p = -\frac{1}{2}\mu_o H_c^2 \pi \xi L_z = \frac{\Phi_o^2}{16\pi\mu_o\lambda^2\xi} L_z$$

Critical Current Density

When the force of the applied current density equals the pinning force, vortices move:

$$\mathbf{f}_L = \mathbf{J}_{\text{ext}} \times \Phi_o \mathbf{i}_z L_z \qquad f_p = -\frac{\Phi_o^2}{16\pi\mu_o\lambda^2\xi} L_z$$

$$J_c^{\text{max}} = \frac{\Phi_o}{16\pi\mu_o\lambda^2\xi}$$

Depinning critical current processing dependent

$$J_{\text{pair}} = \frac{\Phi_o}{3\sqrt{3}\pi\mu_o\lambda^2\xi}$$

Depairing critical current material specific

$$J_c^{\text{max}} \approx 3 \times 10^7 \text{ A/cm}^2$$

Phase Diagram of a Type II Superconductor

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Please see: Phase Diagram from http://www.futurescience.com/manual/sc1000.html#C

Critical State Model of Bean and Livingston

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Please see: Figure 7.22, page 378, from Orlando, T., and K. Delin. *Foundations of Applied Superconductivity*. Reading, MA: Addison-Wesley, 1991. ISBN: 0201183234.

Critical State Model of Bean and Livingston

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Please see: Figure 7.24, page 380, from Orlando, T., and K. Delin. *Foundations of Applied Superconductivity*. Reading, MA: Addison-Wesley, 1991. ISBN: 0201183234.

Superconducting Wire

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Please see: Figure 7.26, page 383, from Orlando, T., and K. Delin. *Foundations of Applied Superconductivity*. Reading, MA: Addison-Wesley, 1991. ISBN: 0201183234.