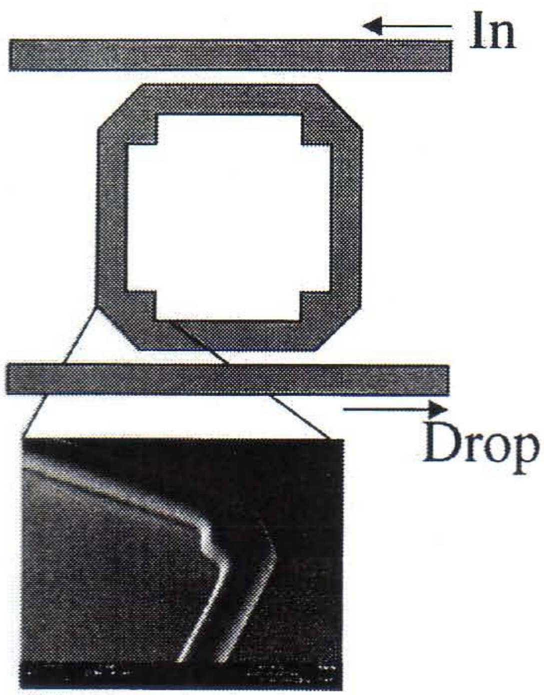
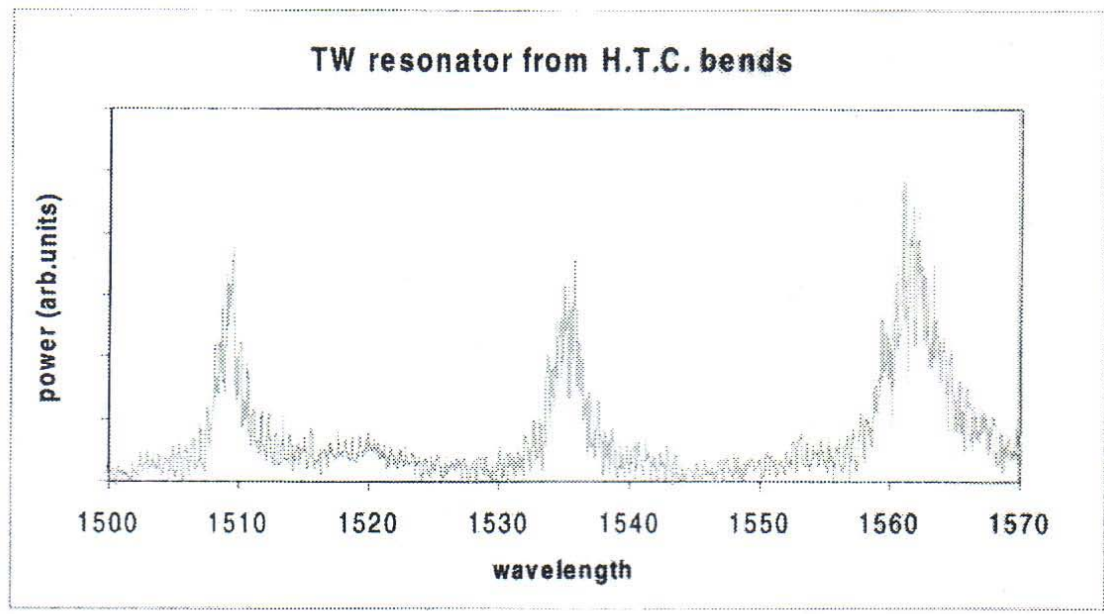
Lecture 19 - Laser Diodes, 1 - Outline

- **Final Waveguide and LED comments** (continued from Lect. 18)
  - Haus resonant corners - experimental data
  - New LED foils for Lect. 18
- **Stimulated emission and optical gain**
  - Absorption, spontaneous emission, stimulated emission
  - Threshold for optical gain
- **Laser diode basics** (as far as we get; to be cont. in Lect. 20)
  - Lasing and condition at threshold
  - Threshold current density
  - Differential quantum efficiency
  - Cavity design (in-plane geometries)
    - Vertical structure: homojunction
      - double heterojunction
      - quantum well
    - Lateral definition: stripe contact
      - buried heterostructure
      - shallow rib
    - End-mirror design: cleaved facet
      - etched facet
      - distributed feedback, Bragg reflector

Achieving compact rectangular waveguide layouts

- **Resonator made using new 90˚ corners - example fabricated using polysilicon on silicon dioxide**





Q of resonance indicates 0.3 dB loss per corner

[Unpublished data reported in PhD thesis of Desmond Lim Siong, EECS, MIT, June 2000; figure taken from LEOS Tutorial "High Density Optical Integration," by Hermann A. Haus, presented at LEOS, Glasgow, 2002.

Light emitting diodes: radiative efficiency, cont

Calculation of lifetimes for Si and GaAs using some representative parameter values:

Density of non-radiative centers, N_{nr} = 10^{15} cm^{-3}
Non-radiative center cross-section, σ_{nr} = 10^{-15} cm^{2}
Thermal velocity, v_{th} = 10^{7} cm s^{-1}
Doping level, $N_{Ap} = p_o$ = 10^{17} cm^{-3}

Quantity	GaAs	Si
B	7.2×10^{-10} cm^3 s^{-1}	1.8×10^{-15} cm^3 s^{-1}
τ_r [$= 1/Bp_o$]	1.4×10^{-8} s	5.6×10^{-3} s
τ_{nr} [$= 1/r_{nr}v_{th}N_{nr}$]	1.0×10^{-7} s	1.0×10^{-7} s
η_r [$= 1/(1+\tau_r/\tau_{nr})$]	0.88	1.8×10^{-5}

Light emitting diodes: current efficiency, η_i

Evaluating η_i for several different diodes:

1. Long-base homojunction
2. Long-base heterojunction
3. Double heterojunction

$$\eta_i \equiv \frac{A}{i_D}\left[J_e\left(0^+\right) - J_e\left(w_p\right)\right]$$

1. Long-base, n⁺-p homojunction

$$i_D = qAn_i^2\left[\frac{D_e}{N_{Ap}w_p^*} + \frac{D_h}{N_{Dn}w_n^*}\right]\left(e^{qv_{AB}/kT} - 1\right)$$

If $w_p >> L_e$, then $AJ_e\left(0^+\right) \approx qAn_i^2\left[\frac{D_e}{N_{Ap}L_e}\right]\left(e^{qv_{AB}/kT} - 1\right)$

and $AJ_e\left(w_p\right) \approx 0$ □

Using these results we find:

$$\eta_i = \frac{1}{1+\delta_e}, \text{ where we introduce } \delta_e \equiv \frac{D_h}{D_e}\cdot\frac{L_e}{w_n^*}\cdot\frac{N_{Ap}}{N_{Dn}}$$

Note: we want δ_e small, to make η_i near one.

Light emitting diodes: current efficiency, η_i

2. Long-base N-p heterojunction

Continue with assumption that $w_p \gg L_e$

Assume no conduction band spike (i.e., spike graded out)

In this case:

$$i_D = qA\left[\frac{D_e n_{iNBG}^2}{N_{Ap} L_e} + \frac{D_h n_{iWBG}^2}{N_{Dn} w_n^*}\right]\left(e^{qv_{AB}/kT} - 1\right)$$

and

$$AJ_e(0^+) \approx qA\left[\frac{D_e n_{iNBG}^2}{N_{Ap} L_e}\right]\left(e^{qv_{AB}/kT} - 1\right), \quad AJ_e(w_p) \approx 0$$

Thus in this device:

$$\delta_e = \frac{D_h}{D_e} \cdot \frac{L_e}{w_n^*} \cdot \frac{N_{Ap}}{N_{Dn}} \cdot \frac{n_{iWBG}^2}{n_{iNBG}^2} = \frac{D_h}{D_e} \cdot \frac{L_e}{w_n^*} \cdot \frac{N_{Ap}}{N_{Dn}} \cdot e^{-\Delta E_g/kT}$$

Note: If there is a spike ΔE_g is replaced by ΔE_v.

Light emitting diodes: current efficiency, η_i

3. Double heterojunction, N-p-P

In the single heterojunction device the current efficiency is already essentially 100%. Adding a second heterojunction makes $J_e(w_p) = 0$ even if the narrow bandgap p-region is narrow, and makes it possible to reach high level injection uniformly throughout the p-region.

With a heterojunction on the p-side of the device the electrons injected into this side will be blocked at the p-P heterojunction, and will "pile-up" in the narrow bandgap p-region.

We can write:

$$i_D = qA\frac{\int_0^{w_p} n'(x)dx}{\tau_e} \approx qA\frac{n'(0)w_p}{\tau_e}$$

$$= qAw_p\frac{n_{iWBG}^2}{N_{Ap}\tau_e}\left(e^{qv_{AB}/kT}-1\right)$$

Light emitting diodes: current efficiency, η_i

3. Double heterojunction, N-p-P, cont.

One interesting consequence of using a second heterojunction can be seen by comparing the diode currents in cases 2 and 3:

$$\frac{i_{D,DH}}{i_{D,SH}} = \frac{qAw_p \dfrac{n_{iWBG}^2}{N_{Ap}\tau_e}\left(e^{qv_{AB}/kT}-1\right)}{qA\dfrac{D_e n_{iWBG}^2}{N_{Ap}L_e}\left(e^{qv_{AB}/kT}-1\right)} = \frac{w_p L_e}{D_e \tau_e} = \frac{w_p}{L_e}$$

We see from this result that the current will be smaller for a given applied bias in the DH diode.

Another interesting and useful result we can obtain from the double heterojuction analysis, is an expression for the excess population in the p-region in terms of the diode current:

$$n' \approx p' \approx \frac{\tau_e}{qAw_p} i_D$$

C. G. Fonstad, 4/03

Light emitting diodes: current efficiency, η_i

3. Double heterojunction, N-p-P, cont.

The next step we should take is to recognize that τ_e becomes a function of p' at high injection levels, so we should write:

$$p' \approx \frac{i_D}{qV_p\left[A + B\left(p_o + p'\right)\right]}$$

(We have introduced V_p, the volume of the p-region(= $w_p A$), to avoid confusing the A in the lifetime with the area A.)

Solving for p' we have:

$$p' = \sqrt{\frac{i_D}{qV_p B} + \left(\frac{A + Bp_o}{2B}\right)^2} - \left(\frac{A + Bp_o}{2B}\right)$$

(You might want to check that this result is consistent with our LLI result that p' is linearly proportional to i_D at LLI. It is.)

Finally, we could use this result to quantify the increase in η_{rad} at HLI:

$$\eta_{rad} = \frac{1}{1 + A/\left[B\left(p_o + p'\right)\right]}$$

Light emitting diodes: fighting total internal reflection

Transferred substrate technology

LED heterostructure etched free of its GaAs substrate, and a GaP.

Comparisons of emission and structures of conventional and transferred substrate LEDs.

(Images deleted)

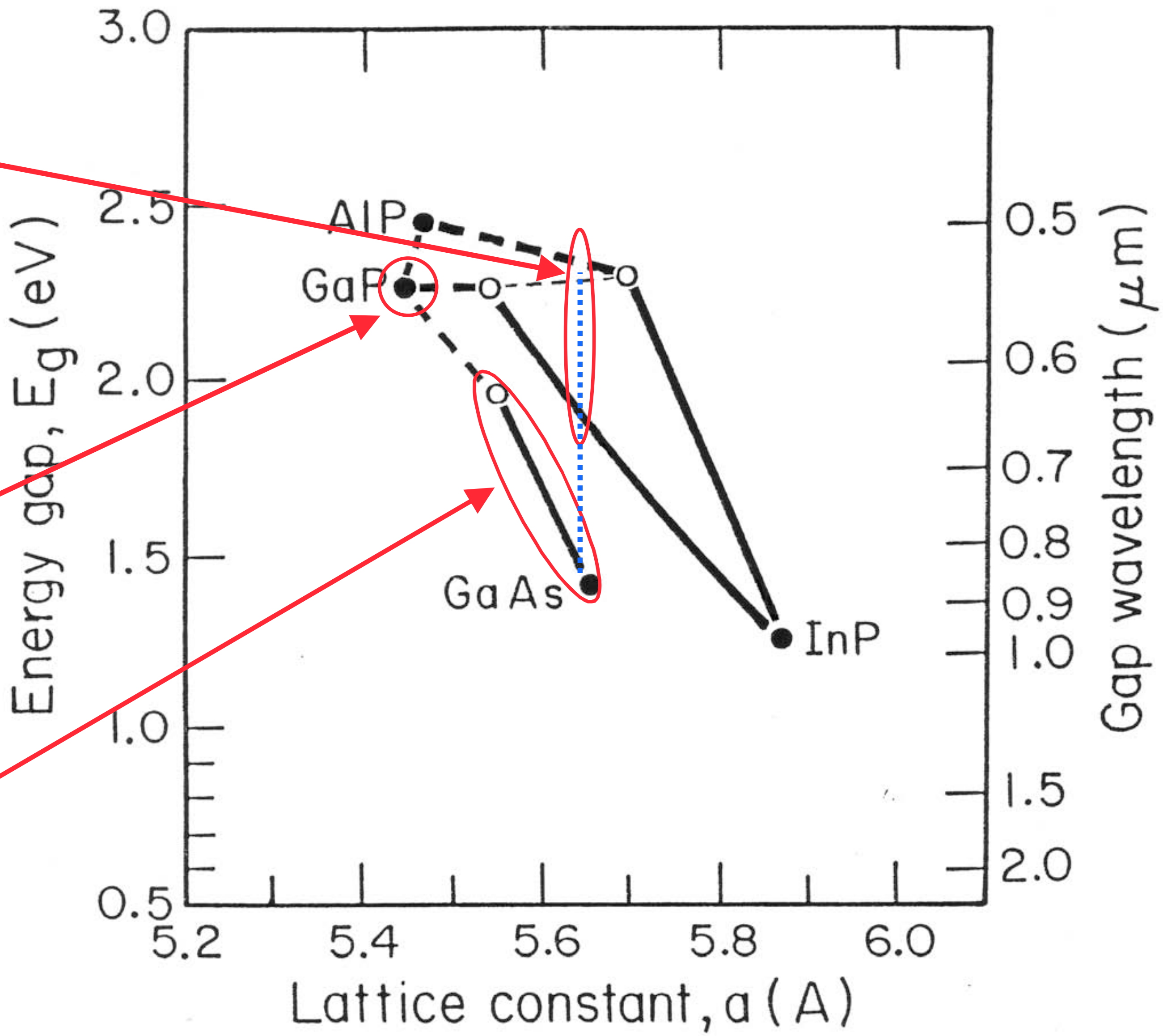
See Kish et al, Appl. Phys. Lett. 64 (1994) 2839-2841.

Materials for Red LEDs: GaAsP, AlInGaP, and GaP

Modern AlInGaP red LEDs grown lattice-matched on GaAs, and then transferred to GaP substrates
- Kish, et al, APL 64 (1994) 2838.

GaP red LEDs grown GaP and based on Zn-O pair transitions

Early GaAsP red LEDs grown on a linearly graded buffer on GaAs
- Holonyak and Bevacqua, APL 1 (1962) 82.



Laser diodes: comparing LEDs and laser diodes

Light emitting diodes vs. Laser diodes

LEDs are based on *spontaneous* emission, and have

1. A broad output beam that is hard to capture and focus
2. A relatively broad spectral profile
3. Low to moderate overall efficiency
4. Moderate to high speed ($\approx 1/\tau_{min}$)

Laser Diodes are based on *stimulated* emission, and have the opposite characteristics

1. Narrow, highly directed output
2. Sharp, narrow emission spectrum
3. High differential and overall efficiency
4. High to very high speed

Stimulated emission occurs when a passing photon triggers the recombination of an electron and hole, with emission of a second photon with the same frequency (energy), momentum, and phase.

Laser diodes: achieving stimulated gain □

To understand what is necessary to obtain net optical gain, rather than net absorption, we consider optical transitions between two levels in a solid (E_1 and E_2), and we look at three transitions occuring with the absorption or emission of photons:

from E_1 to E_2 due to absorption

from E_2 to E_1 due to spontaneous emission

from E_2 to E_1 due to stimulated emission

We model the rate of each process using the Einstein A and B coefficients, and then find when the probability is higher that a photon passing will stimulate emission than be absorbed.

In a semiconductor we consider one state, E_1, to be in the valence band, and the other, E_2 to be in the conduction band.

Laser diodes: achieving stimulated gain, cont □

Absorption rate:

$$R_{ab} = B_{12} \cdot f_1 \cdot N_v(E_1) \cdot \left(1 - f_2\right) \cdot N_c(E_2) \cdot \rho_p\left(E_2 - E_1\right)$$

where

- B_{12}: transition probability for absorption
- N_v: valence band density of states at E_1
- N_c: conduction band density of states at E_2
- $\rho_p(E_2\text{-}E_1)$: density of photons with correct energy
- f_i: Fermi function evaluated at E_i

$$f_i = 1\Big/\left(e^{E_i - E_{fi}} + 1\right)$$

where □

- E_{fi}: quasi-Fermi level for level i □

Spontaneous emission rate:

$$R_{sp} = A_{21} \cdot f_2 \cdot N_c(E_2) \cdot \left(1 - f_1\right) \cdot N_v(E_1)$$

In the last equation we introduced:

A_{21}: transition probability for spontaneous emission

Stimulated emission rate:

$$R_{st} = B_{21} \cdot f_2 \cdot N_c(E_2) \cdot \left(1 - f_1\right) \cdot N_v(E_1) \cdot \rho_p\left(E_2 - E_1\right)$$

where

B_{21}: transition probability for stimulated emission

Note, finally, that in these expressions the Fermi function is evaluated either in the conduction band (i = 2) or valence band (i =1):

$$f_1 = 1\Big/\left(e^{E_1 - E_{fv}} + 1\right), \quad f_2 = 1\Big/\left(e^{E_2 - E_{fc}} + 1\right)$$

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The coefficients, A_{21}, B_{12}, and B_{21}, are related, as we can see by looking at thermal equilibrium, where

$$R_{ab} = R_{sp} + R_{st}, \quad E_{fv} = E_{fc}, \quad \rho_p(E_i) = \frac{8\pi r_o^3}{h^3 c^3} E_i^2 \frac{1}{\left(e^{E_i/kT} - 1\right)}$$

Laser diodes: achieving stimulated gain, cont

Proceeding in this we we find:

$$B_{12} = B_{21}, \quad \text{and} \quad A_{21} = \frac{8\pi\, r_o^3 E_i^2}{h^3 c^3} B_{21}$$

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Now we are ready to find the condition for optical gain, which we take as when the probability of stimulated emission is greater than that for absorption. Looking back at our equations, we find $R_{st} > R_{ab}$ leads to:

$$B_{21} \cdot f_2 N_c \cdot (1 - f_1) N_v \cdot \rho_p (E_2 - E_1) > B_{12} \cdot f_1 N_v \cdot (1 - f_2) N_c \cdot \rho_p (E_2 - E_1)$$

Canceling equivalent terms yields:

$$f_2 (1 - f_1) > f_1 (1 - f_2)$$

and substituting the appropriate Fermi functions gives us:

$$E_{fc} - E_{fv} > (E_2 - E_1) = h\upsilon \geq E_g$$

Laser diodes: achieving stimulated gain, cont.

Our conclusion is that we will have net optical gain, i.e., more stimulated emission than absorption, when we have the quasi-Fermi levels separated by more than the band gap. This in turn requires high doping and current levels. It is the equivalent of population inversion in a semiconductor: $E_{fc} - E_{fv} > E_g$

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Next we relate the absorption coefficient, α, to R_{ab}, R_{st}, and R_{sp}. A bit of thought shows us that we can say:

$$R_{ab}(E) > \left[R_{st}(E) + R_{sp}(E)\right] \approx R_{st}(E) \rightarrow \alpha(E) > 0 \qquad \text{Net loss}$$

$$R_{ab}(E) < \left[R_{st}(E) + R_{sp}(E)\right] \approx R_{st}(E) \rightarrow \alpha(E) < 0 \qquad \text{Net gain}$$

$$R_{ab}(E) = \left[R_{st}(E) + R_{sp}(E)\right] \approx R_{st}(E) \rightarrow \alpha(E) = 0 \rightarrow E = E_{fc} - E_{fv}$$

Notes: Spontaneous emission is negligible because it is randomly directed. It starts the lasing process, but it does not sustain it.

The point at which $\alpha = 0$ is called the transparency point.

Laser diodes: optical gain coefficient, g(E)

The negative of the absorption coefficient is defined as the gain coefficient:

$$g(E) \equiv \alpha(E)$$

Writing the light intensity in terms of g(E) we have:

$$L(E,x) = L_o(E)e^{-\alpha(E,)x} = L_o(E)e^{g(E,)x}$$

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Stimulated recombination is proportional to the carrier populations, and in a semiconductor one carrier is usually in the minority and its population is the one that changes significantly with increasing current injection. If we assume p-type material, we have:

$$g > 0 \rightarrow n > n_{tr}$$

To first order, the gain will be proportional to this population, to the extent that it exceeds the transparency level:

$$g \cong G(n - n_{tr})$$

Laser diodes: threshold current

We not look at a laser diode and calculating the threshold current for lasing, and the light-current relationship

Lasing will be sustained when the optical gain exceeds the optical losses for a round-trip in the cavity.

The threshold current is the current level above which this occurs.

Laser diodes: threshold current, cont.

Track the light intensity on a full circuit, beginning with I_o just inside the facet at x = 0^+, and directed to the right:

At x = 0^+, directed to the right, $I(0^+) = I_o$

At x = L^-, directed to the right, $I(L^-) = I_o e^{(g-\alpha_L)L}$

At x = L^-, directed to the left, $I(L^-) = R_2 I_o e^{(g-\alpha_L)L}$

At x = 0^+, directed to the left, $I(0^+) = R_2 I_o e^{(g-\alpha_L)2L}$

At x = 0^+, directed to the right, $I(0^+) = R_1 R_2 I_o e^{(g-\alpha_L)2L}$

For sustained lasing we must have the intensity after a full circuit be equal to, or greater than, the initial intensity:

$$R_1 R_2 I_o e^{(g-\alpha_L)2L} \geq I_o$$

This leads us to identify the threshold gain, g_{th}:

$$g_{th} \equiv \alpha_L + \frac{1}{2L} \ln\left(\frac{1}{R_1 R_2}\right)$$

We next relate g_{th} to the diode current to get the threshold current.

Laser diodes: threshold current, cont.

To relate this threshold gain to current we recall that the gain is proportional to the carrier population in excess of the transparency value, and that the population will in general be proportional to the current:

$$g \approx G(n - n_{tr}) = G'\Gamma(n - n_{tr})$$
$$n \approx K i_D$$

where

G': the portion of G due to material parameters alone

Γ: the portion of G due to geometrical factors (i.e., the overlap of the optical mode and the active medium)

K: a proportionality factor that depends on the device structure, which we will determine in specific situations later

Writing g in terms of i_D, and setting it equal to g_{th}, yields:

$$g_{th} = G'\,\Gamma(KI_{th} - n_{tr}) = \alpha_L + \frac{1}{2L}\ln\left(\frac{1}{R_1 R_2}\right)$$

Laser diodes: threshold current, cont.

Which we can finally solve for the current to arrive at the expression for the threshold current:

$$I_{th} = \frac{1}{K}\left(\frac{1}{G'\Gamma}\left[\alpha_L + \frac{1}{2L}\ln\left(\frac{1}{R_1 R_2}\right)\right] + n_{tr}\right)$$

This will take on more meaning as we look at specific laser diode geometries and quantify the various parameters.

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Note: Above threshold, all of the additional excitation fuels stimulated recombination and n' stays fixed at its threshold value. So to does $E_{fn} - E_{fp}$, which implies that the junction voltage is also pinned.