Lecture 20 - Laser Diodes 1 - Outline

- **Stimulated emission and optical gain**
  - Absorption, spontaneous emission, stimulated emission
  - Threshold for optical gain
- **Laser diode basics**
  - Lasing and conditions at threshold
  - Threshold current density
  - Differential quantum efficiency
- **In-plane laser cavity design** (as far as we get; to be cont. in Lect. 21)
  - Vertical structure: homojunction
    - double heterojunction
    - quantum well, wire, dot; quantum cascade
  - Lateral definition: stripe contact
    - buried heterostructure
    - shallow rib
  - End-mirror design: cleaved facet
    - etched facet
    - distributed feedback, Bragg reflector

Laser diodes: comparing LEDs and laser diodes

Light emitting diodes vs. Laser diodes

LEDs are based on *spontaneous* emission, and have

1. A broad output beam that is hard to capture and focus
2. A relatively broad spectral profile
3. Low to moderate overall efficiency
4. Moderate to high speed ($\approx 1/\tau_{min}$)

Laser Diodes are based on *stimulated* emission, and have the opposite characteristics

1. Narrow, highly directed output
2. Sharp, narrow emission spectrum
3. High differential and overall efficiency
4. High to very high speed

Stimulated emission occurs when a passing photon triggers the recombination of an electron and hole, with emission of a second photon with the same frequency (energy), momentum, and phase.

Laser diodes: achieving stimulated gain

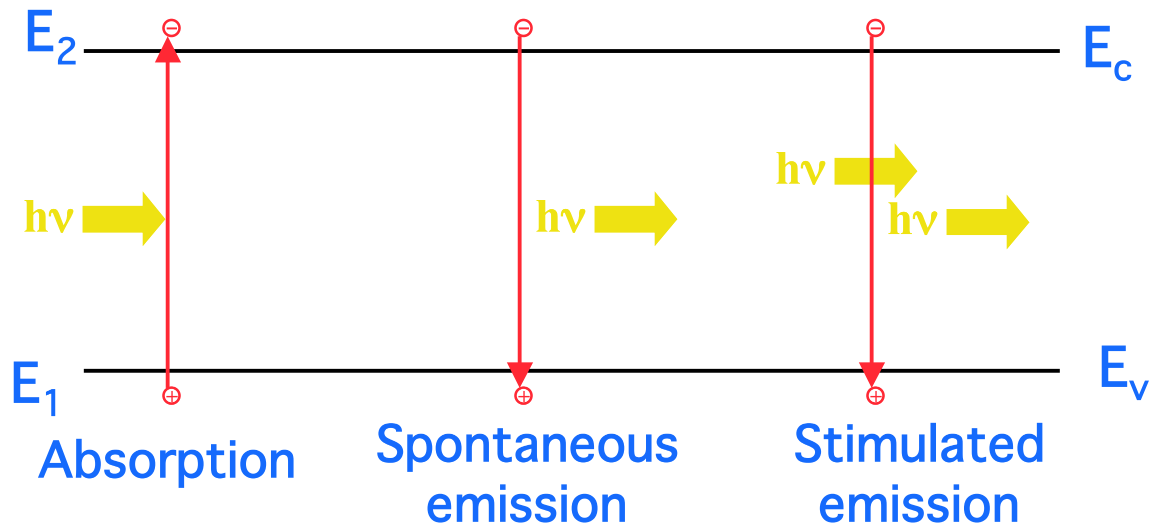
To understand what is necessary to obtain net optical gain, rather than net absorption, we consider optical transitions between two levels in a solid (E_1 and E_2), and we look at three transitions ocurring with the absorption or emission of photons:

1. **from E_1 to E_2 due to absorption**
2. **from E_2 to E_1 due to spontaneous emission**
3. **from E_2 to E_1 due to stimulated emission**

We model the rate of each process using the Einstein A and B coefficients, and then find when the probability is higher that a photon passing will stimulate emission than be absorbed.

Laser diodes: achieving stimulated gain, cont.

In a semiconductor we consider one state, E_1, to be in the valence band, and the other, E_2 to be in the conduction band:



The rates these processes occur depend on the populations:

Absorption rate, R_{ab}: photon pop. x E_1 pop. x E_2 empty state pop.

Spontaneous emission rate, R_{sp}: E_2 pop. x E_1 empty state pop.

Stimulate emission rate, R_{st}: E_2 pop. x E_1 empty state pop. x photon pop.

Laser diodes: achieving stimulated gain, cont

Absorption rate:

$$R_{ab} = B_{12} \cdot f_1 \cdot N_v(E_1) \cdot \left(1 - f_2\right) \cdot N_c(E_2) \cdot \rho_p\left(E_2 - E_1\right)$$

where

- B_{12}: transition probability for absorption
- N_v: valence band density of states at E_1
- N_c: conduction band density of states at E_2
- $\rho_p(E_2-E_1)$: density of photons with correct energy
- f_i: Fermi function evaluated at E_i

$$f_i = 1 \Big/ \left(e^{E_i - E_{fi}} + 1\right)$$

where

- E_{fi}: quasi-Fermi level for level i

Spontaneous emission rate:

$$R_{sp} = A_{21} \cdot f_2 \cdot N_c(E_2) \cdot \left(1 - f_1\right) \cdot N_v(E_1)$$

<u>Laser diodes</u>: achieving stimulated gain, cont

In the last equation we introduced:

A_{21}: transition probability for spontaneous emission

<u>Stimulated emission rate</u>:

$$R_{st} = B_{21} \cdot f_2 \cdot N_c(E_2) \cdot \left(1 - f_1\right) \cdot N_v(E_1) \cdot \rho_p\left(E_2 - E_1\right)$$

where

B_{21}: transition probability for stimulated emission

Note, finally, that in these expressions the Fermi function is evaluated either in the conduction band (i = 2) or valence band (i =1):

$$f_1 = 1\Big/\left(e^{E_1 - E_{fv}} + 1\right), \quad f_2 = 1\Big/\left(e^{E_2 - E_{fc}} + 1\right)$$

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The coefficients, A_{21}, B_{12}, and B_{21}, are related, as we can see by looking at thermal equilibrium, where

$$R_{ab} = R_{sp} + R_{st}, \quad E_{fv} = E_{fc}, \quad \rho_p(E_i) = \frac{8\pi r_o^3}{h^3 c^3} E_i^2 \frac{1}{\left(e^{E_i/kT} - 1\right)}$$

Laser diodes: achieving stimulated gain, cont

Proceeding in this we we find:

$$B_{12} = B_{21}, \text{ and } A_{21} = \frac{8\pi r_o^3 E_i^2}{h^3 c^3} B_{21}$$

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Now we are ready to find the condition for optical gain, which we take as when the probability of stimulated emission is greater than that for absorption. Looking back at our equations, we find $R_{st} > R_{ab}$ leads to:

$$B_{21} \cdot f_2 N_c \cdot (1 - f_1) N_v \cdot \rho_p (E_2 - E_1) > B_{12} \cdot f_1 N_v \cdot (1 - f_2) N_c \cdot \rho_p (E_2 - E_1)$$

Canceling equivalent terms yields:

$$f_2 (1 - f_1) > f_1 (1 - f_2)$$

and substituting the appropriate Fermi functions gives us:

$$E_{fc} - E_{fv} > (E_2 - E_1) = h\upsilon \geq E_g$$

Laser diodes: achieving stimulated gain, cont.

Our conclusion is that we will have net optical gain, i.e., more stimulated emission than absorption, when we have the quasi-Fermi levels separated by more than the band gap. This in turn requires high doping and current levels. It is the equivalent of population inversion in a semiconductor:

$$E_{fc} - E_{fv} > E_g$$

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Next we relate the absorption coefficient, α, to R_{ab}, R_{st}, and R_{sp}. A bit of thought shows us that we can say:

$$R_{ab}(E) > \left[R_{st}(E) + R_{sp}(E)\right] \approx R_{st}(E) \rightarrow \alpha(E) > 0 \qquad \text{Net loss}$$

$$R_{ab}(E) < \left[R_{st}(E) + R_{sp}(E)\right] \approx R_{st}(E) \rightarrow \alpha(E) < 0 \qquad \text{Net gain}$$

$$R_{ab}(E) = \left[R_{st}(E) + R_{sp}(E)\right] \approx R_{st}(E) \rightarrow \alpha(E) = 0 \rightarrow E = E_{fc} - E_{fv}$$

Notes: Spontaneous emission is negligible because it is randomly directed. It starts the lasing process, but it does not sustain it.

The point at which α = 0 is called the transparency point.

Laser diodes: optical gain coefficient, g(E)

The negative of the absorption coefficient is defined as the gain coefficient:

$$g(E) \equiv -\alpha(E)$$

Writing the light intensity in terms of g(E) we have:

$$L(E,x) = L_o(E)e^{-\alpha(E)x} = L_o(E)e^{g(E)x}$$

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Stimulated recombination is proportional to the carrier populations, and in a semiconductor one carrier is usually in the minority and its population is the one that changes significantly with increasing current injection. If we assume p-type material, we have:

$$g > 0 \rightarrow n > n_{tr}$$

To first order, the gain will be proportional to this population, to the extent that it exceeds the transparency level:

$$g \cong G(n - n_{tr})$$

Laser diodes: threshold current

We not look at a laser diode and calculating the threshold current for lasing, and the light-current relationship

Consider the following cavity:

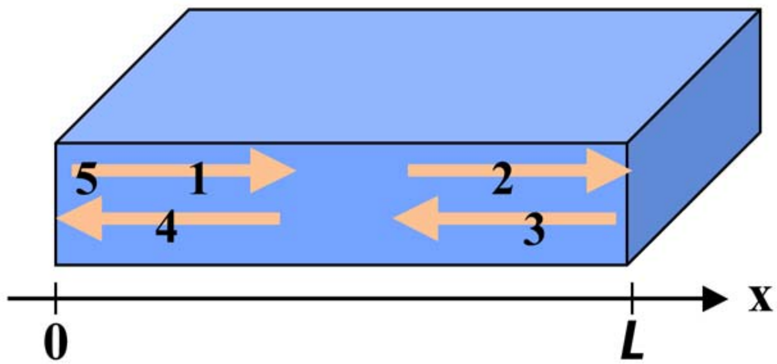
Lasing will be sustained when the optical gain exceeds the optical losses for a round-trip in the cavity.

The threshold current is the current level above which this occurs.

Laser diodes: threshold current, cont.

Track the light intensity on a full circuit, beginning with I_o just inside the facet at $x = 0^+$, and directed to the right:

1. At x = 0^+, directed to the right, $I(0^+) = I_o$
2. At x = L^-, directed to the right, $I(L^-) = I_o e^{(g-\alpha_l)L}$
3. At x = L^-, directed to the left, $I(L^-) = R_2 I_o e^{(g-\alpha_l)L}$
4. At x = 0^+, directed to the left, $I(0^+) = R_2 I_o e^{(g-\alpha_l)2L}$
5. At x = 0^+, directed to the right, $I(0^+) = R_1 R_2 I_o e^{(g-\alpha_l)2L}$



For sustained lasing we must have the intensity after a full circuit (5) be equal to, or greater than, the initial intensity (1):

$$R_1 R_2 I_o e^{(g-\alpha_l)2L} \geq I_o$$

Laser diodes: threshold current, cont.

This leads us to identify the threshold gain, g_{th}:

$$g_{th} \equiv \alpha_l + \frac{1}{2L}\ln\left(\frac{1}{R_1 R_2}\right)$$

To relate this threshold gain to current we recall that the gain is proportional to the carrier population in excess of the transparency value,

$$g \approx G(n - n_{tr}) = G'\Gamma(n - n_{tr})$$

where G': the portion of G due to material parameters alone

Γ: the portion of G due to geometrical factors (i.e., the overlap of the optical mode and the active medium)

and that the population will in general be proportional to the current:

$$n \approx K i_D$$

where K: a proportionality factor that depends on the device structure, which we will determine in specific situations later

Laser diodes: threshold current, cont.

Writing g in terms of i_D, and setting it equal to g_{th}, yields:

$$g_{th} = G'\Gamma\left(Ki_D - n_{tr}\right) = \alpha_l + \frac{1}{2L}\ln\left(\frac{1}{R_1 R_2}\right)$$

The diode current that corresponds to this threshold gain is defined to be the threshold current, I_{th}:

$$i_D @ g_{th} \equiv I_{th} = \frac{1}{K}\left(\frac{1}{G'\Gamma}\left[\alpha_l + \frac{1}{2L}\ln\left(\frac{1}{R_1 R_2}\right)\right] + n_{tr}\right)$$

(This will take on more meaning as we look at specific laser diode geometries and quantify the various parameters.)

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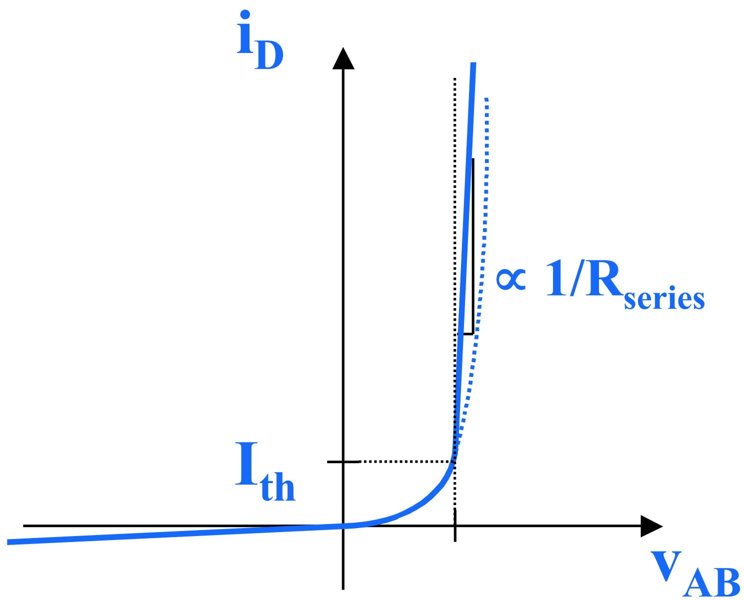
A final useful observation is that the mirror reflectivity term in these equations can be viewed as a mirror loss coefficient, α_m:

$$\alpha_m \equiv \frac{1}{2L}\ln\left(\frac{1}{R_1 R_2}\right)$$

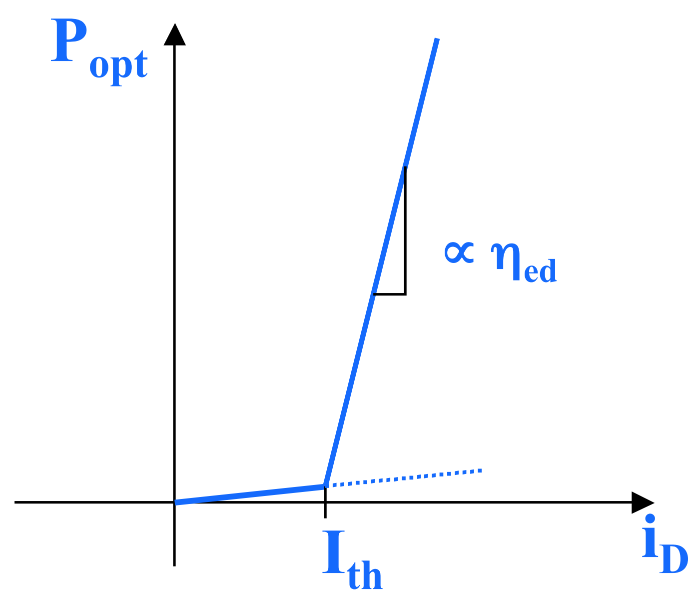
Laser diodes: threshold current, cont.

Above threshold, essentially all of the additional excitation fuels stimulated recombination, and n' stays fixed at its threshold value. So too does $E_{fn} - E_{fp}$, which implies that the junction voltage is also pinned.

The current-voltage and power-current characteristics of a laser diode thus have the following forms:



current-voltage



power-current

Laser diodes: output power, P_{opt}, and external differential quantum efficiency, η_{ed}

To calculate the optical output power, P_{opt}, we begin with several points:

First, we recall that a particle flux can be written in terms of a particle density times their velocity. This holds for photons as well, and the velocity is the mode, or "group" velocity:

$$F_{ph}(x,y,z) = v_g \, N_{ph}(x,y,z)$$

Second, we recall that the rate of change of a photon population with time at a given point, is the photon flux times the absorption coefficient, α:

$$\frac{\partial N_{ph}(x,y,z)}{\partial t} = -\alpha \, v_g \, N_{ph}(x,y,z)$$

Finally, we note that the output power will be the flux of photons emitted times the energy per photon, $h\nu$.

$$P_{out} = h\nu \, F_{ph,out,tot}$$

Laser diodes: P_{opt} and η_{ed}, cont.

We next move inside the laser diode and look at the photon population there. The total number of photons inside the laser will be

$$N_{ph,in,tot} = \int N_{ph,in}(x,y,z)\,dx\,dy\,dz$$

This photon population is decreasing because photons are being absorbed internally and emitted from the ends of the cavity at a rate given by the photon flux times the effective absorption coefficient:

$$\text{Loss}: \int (\alpha_l + \alpha_m) v_g N_{ph,in}(x,y,z)\,dx\,dy\,dz = (\alpha_l + \alpha_m) v_g N_{ph,in,tot}$$

Note that the loss out the end mirrors, α_m, is the output we are looking to calculate!

and it is increasing because the diode current exceeds the threshold current, and the gain exceeds the thresh-hold gain:

$$\text{Photon generation}: \int g\, v_g N_{ph,in}(x,y,z)\,dx\,dy\,dz = \frac{(\mathrm{i_D} - \mathrm{I_{th}})}{\mathrm{q}} \eta_i$$

Laser diodes: P_{opt} and η_{ed}, cont.

In the last equation, η_i, is the current utilization efficiency, the fraction feeding stimulated recombination.

In the steady state the loss equals the generation:

$$(\alpha_l + \alpha_m)\, v_g\, N_{ph,in,tot} = \frac{(i_D - I_{th})}{q}\eta_i$$

Thus the total photon population inside the laser diode is:

$$N_{ph,in,tot} = \frac{(i_D - I_{th})}{q(\alpha_l + \alpha_m)\, v_g}\eta_i$$

The total photon flux emitted from the laser diode output mirrors is the portion of the "loss" due to α_m:

$$P_{opt} = h\nu\, \alpha_m\, v_g\, N_{ph,in,tot} = h\nu\, \frac{(i_D - I_{th})}{q}\, \frac{\alpha_m}{(\alpha_l + \alpha_m)}\eta_i$$

Laser diodes: P_{opt} and η_{ed}, cont.

We next introduce the extraction efficiency, η_e:

$$\eta_e \equiv \frac{\alpha_m}{(\alpha_l + \alpha_m)}$$

With this we write the output power as:

$$P_{opt} = h\nu \, \frac{(\mathrm{i_D} - \mathrm{I_{th}})}{\mathrm{q}} \, \eta_e \, \eta_i$$

Note that this is the total output power from both ends of the laser.

- **If the two end-faces have equal reflectivities, then half the power will come out each end.**
- **If one end is much more highly reflecting than the other, then little power will come out it, and all of the power will come out the lower reflectivity facet.**
- **If the reflectivities of the ends differ, but not by a large amount, then the division of output is complicated to calculate.**

Next we turn to the external differential quantum efficiency.

Laser diodes: P_{opt} and η_{ed}, cont.

The external differential quantum efficiency is defined as the ratio between the number of photons emitted per unit time, divided by the number of carriers crossing the diode junction per unit time:

$$\eta_{ed} = \frac{\Delta(\#\ \text{of photons out/unit time})}{\Delta(\#\ \text{of carriers across junction/unit time})}$$

In terms of the output power and diode current this is:

$$\eta_{ed} = \frac{\Delta\left(\mathrm{P_{opt}}/h\nu\right)}{\Delta\left(\mathrm{i_D/q}\right)} = \frac{q}{h\nu}\frac{dP_{opt}}{di_D}$$

Using the result on the previous foil, we find:

$$\eta_{ed} = \eta_e \eta_i$$

Laser diodes: device design and optimization

With this general background, we now go to the white board and turn to looking at specific device design and the evolution of diode laser structures over time.

- We will begin looking at evolution of active region design and the vertical device structure.
- Next we turn to lateral definition of the cavity.
- Finally we look at defining the ends of the cavity.
- This initial discussion will focus on in-plane lasers. After this we will turn to vertical cavity devices.
- At the very end we will discuss laser diode modulation.