6.777J/2.751J Design and Fabrication of Microelectromechanical Devices
Spring Term 2007

Massachusetts Institute of Technology

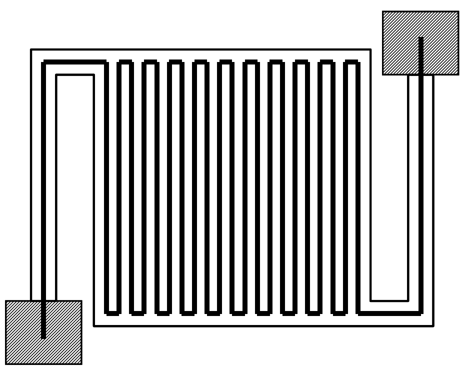
PROBLEM SET 5 (16 pts)

Issued: Lecure 13 Due: Lecture 14

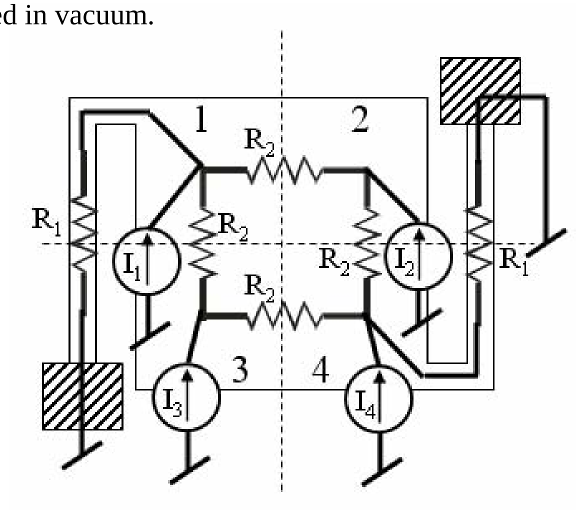
Problem 12.6 (4 pts): Heating a thermal source to a uniform temperature

In class we examined a bolometer that detects incoming radiation by means of the temperature rise that it causes in a suspended structure; an array of such structures acts as a MEMS thermal imager. In this problem we will examine an array of similar structures, which could be used as a thermal source for calibrating an imager. In this case, resistive heaters would be used to heat up an array of suspended structures. For this application, separate pixels can be heated up to different temperatures, but the temperature at different points on a given pixel should be as uniform as possible.

The pixel body is a square of silicon-rich silicon nitride (assume κ = 25 W/m-K) with a 50 μm edge length, a 1 μm thickness, and a 3 μm gap between it and the substrate surface. The pixel is anchored to the substrate by 1 μm thick silicon nitride tethers that are 5 μm in width and 50 μm long. Heating resistor(s) are patterned on the surface of the pixel body. The details of the resistor layout may be arranged to minimize thermal non-uniformities across the pixel. The pixels are packaged in vacuum.



Schematic top view of pixel supported on tethers, with a simple meandering resistor covering the pixel body



Equivalent circuit overlaid on the pixel diagram.

a) Write expressions for and calculate the thermal resistances that govern heat loss from the pixel, assuming that it is in air and the substrate is maintained at 300 K. What is the dominant heat removal mechanism (i.e. radiation, conduction, etc.)? We can also define a characteristic thermal resistance for heat flow through the pixel body. As an approximation, we will define this as the thermal resistance between centers of the individual quadrants of the pixel body. Write an expression for and then calculate this characteristic thermal resistance (ie. Find an expression for R_2 as shown the thermal circuit.) . In what follows, the term relative temperature will refer to the difference between the absolute temperature and that of the substrate (300K).

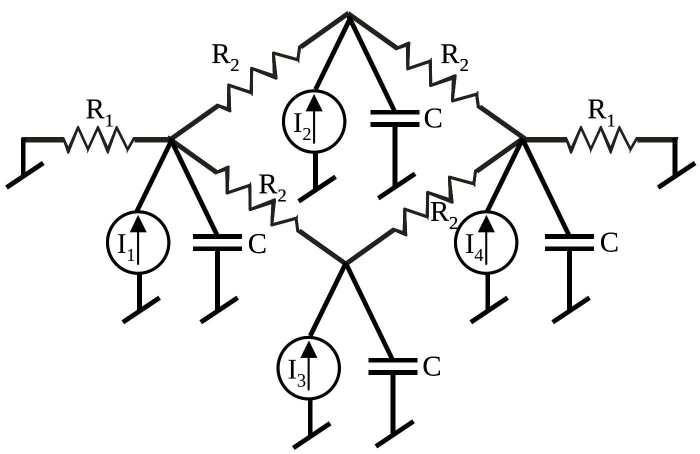
We can capture some of the physics that governs temperature distribution across the pixel by means of a simple equivalent thermal circuit, which is shown overlaid on the pixel above. In this circuit, each quadrant of the pixel has its own thermal current source (provided by the heating resistor in that quadrant) and thermal resistances between it and the neighboring quadrants (R_2, which may be approximated from the characteristic resistance calculated above). Some quadrants are also connected to the thermal resistance of the support tethers (R_1).

b) Let's start by assuming that the heaters are laid out as a single, symmetric resistor as in the figure above, so that the thermal currents to the four quadrants are identical. Create a finite difference matrix representation of the thermal circuit in the form: $\mathbf{G}\vec{T} = \vec{I}$ where **T** and **I** are column vectors representing the relative temperature of and the thermal current into each quadrant, and **G** is a matrix whose elements are functions of R_1 and R_2. Using MATLAB, find the value of the thermal current that will drive the hottest node to 500K What is the absolute temperature of each quadrant in this case?

c) Now, by assuming that the thermal currents can have different values, find those values which will give a uniform absolute temperature distribution of 500K as measured at each of the 4 nodes on the pixel. What resistor layout would produce these thermal currents? What do you think is the best way to implement variable thermal currents in each quadrant: by varying the layout of the heating resistor between quadrants, or by having the quadrants supplied by separate circuits with their own electrical currents? Why?

Problem 12.7 (4 pts): Transient analysis of the thermal source pixel

In this problem we will build upon our results from problem 12.6 and add in the transient response to determine the rate at which the display can be refreshed. To do this, we need to add thermal capacitances to our circuit as shown in the figure below to capture the heat capacity of the pixel plate.



a) Write an expression for and calculate the thermal capacitance C of each quadrant of the pixel. Assume a specific heat for SiN of about 170 J/kg-K and a density of 3300 kg/m^3.
b) Using MATLAB, create a finite-difference matrix representation of the thermal circuit shown above in the form

$$\dot{\vec{T}} = \mathbf{A}\vec{T} + \mathbf{B}\vec{I}$$

where $\vec{T}$ is a vector composed of the node temperatures of the thermal circuit relative to "ground", $\vec{I}$ is a vector of current source inputs, and **A** and **B** are matrices derived by discretizing the heat flow equation.

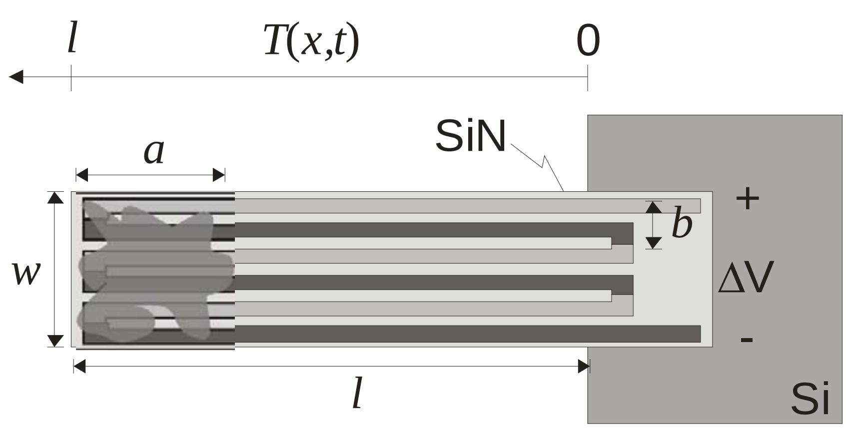
c) Now we want to solve this equation to get the transient response. Noticing that the relation is in state-space form, we can use MATLAB's control system toolbox to define a state-space system using **ss**, where the **C** matrix is the identity matrix and **D** is all zeros. From this point, use **step** to determine the step response of the system and thus the transient evolution of the system when the thermal current sources are set to their optimum values, with all nodes starting at an absolute temperature T=300 K. Plot the absolute temperature of each quadrant vs. time. What is the transient time to get to 95% of the relative steady state temperature distribution that you calculated in problem 12.6?

d) Now we will simulate cooling down. The easiest way to do this is to use the Matlab command **initial** using as initial conditions the node temperatures set to their steady state values as found in problem 12.6. Find the transient time to cool down the pixel to a relative temperature within 5% of the endpoint value (when all nodes are at the substrate temperature). Plot the absolute temperature of each quadrant vs. time. Is the cooling-down transient time the same as the heating up transient time? Why?

e) What is the maximum refresh rate for the thermal calibration source as described in this problem? Describe in words how you could modify the design to increase the refresh rate of the pixel.

Problem 12.8 (4 pts): Design and analysis of a thermocouple microcalorimeter

Microcalorimeters are macroscale instruments that measure the heat produced by samples (such as heats of reaction, phase change, etc.). In this problem you will design a microscale version of a microcalorimeter (*micro* in microcalorimeter comes from the amount of heat (μJ), not the size of the instrument). The microcalorimeter consists of a silicon nitride cantilever (thickness t=2 μm) connected to a silicon substrate that is thermally grounded at T=300 K. The sample—which acts as the thermal source—is placed on the end of the cantilever over length a=50 μm, and delivers 10 μW of heat. The temperature difference is converted to a voltage difference using an Al/polySi thermopile with a net Seebeck coefficient (between the two materials) of α_s=248 μV/K. Each segment of the thermopile requires b=15 μm of cantilever width. You can assume that the silicon nitride has a thermal conductivity of κ=20 W/m-K, specific heat of 170 J/kg-K, and density of 3300 kg/m^3.



a) We could model this device by using a traditional lumped-element model. However, given that the heat source area is not easily distinguished from the conduction area, we will instead lump the device in terms of modes using an eigenfunction expansion. Assuming that heat conduction is the primary mechanism for heat loss, determine the relevant boundary conditions on $T(x)$. Next, determine a set of eigenfunctions that will both solve the Poisson equation and meet these boundary conditions, along with the separation coefficients k_n. Finally, determine the coefficients of the eigenfunctions, incorporating the fact that heat is generated over only part of the cantilever (from $x=l-a$ to $x=l$).

b) Now extend your solution for $T(x)$ to time-dependent temperature distribution $T(x,t)$ by finding the natural response of the system subject to the initial condition that $T(x,0)$ is the temperature found in part a. This models suddenly turning off the heat source (after reaching steady state) and letting the temperature decay. Lump your system by assuming that we are interested in the temperature at $x=l$. Take the Laplace transform of this solution to get $T(s)$ and generate an equivalent circuit, identifying R_n, C_n, and heat current source $I_{Q,n}(s)$. A good way to determine C_n is to look at the mode volume, as we did in class.

c) Using the 1st-term of the solutions, determine R_1, C_1, and $I_{Q,1}(s)$ for our equivalent circuit.

d) We would like our microcalorimeter to have a 1 ms response time. Determine the dimensions l and w that will result in this response time. Given a maximal thermoucouple packing of $m=w/b$ thermocouples, what is the output voltage? Is it possible to increase this without affecting the time constant? How?

Problem 10.7 (4 pts): Effective mass of a cantilever beam

When we create lumped element models of electromechanical systems, we often need to include the lumped mass of the element. The lumped mass for a given structure depends on how it is supported, and in general will be different for different types of structures. In this problem, you will find the lumped mass of a cantilever beam of Young's modulus E, width b, thickness h, and length L. In particular, you will be looking at the case of a tip-loaded cantilever beam. Since we have solved this problem previously (in class), you are welcome to use the results of those calculations as a starting point.

a) First, find the elastic energy that is stored in the cantilever beam when it is deflected by a tip load F.

b) Next assume that the cantilever beam is undergoing simple harmonic motion at resonance, and find an expression for the maximum kinetic energy of the beam.

c) Based on those results, find the resonant frequency of the cantilever beam.

d) Finally, what is the lumped mass of this cantilever beam?