CONTROL CHART DESIGN

$\bar{x}$ Chart

- **Case 1**: μ, σ known: $x_i \sim N(\mu, \sigma^2)$

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \quad , \quad \sigma_{\bar{x}}^2 = \frac{\sigma^2}{n} \qquad \text{Sample size } n$$

Limits UCL: $\mu + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$ to achieve desired

LCL: $\mu - z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$ α-level of confidence (e.g. $\alpha = 0.002$)

Or, conventionally a "3 sigma" rule gives $\alpha = 0.0027$

UCL: $\mu + 3\sigma/\sqrt{n}$ Upper control limit
LCL: $\mu - 3\sigma/\sqrt{n}$ Lower control limit
CL: μ Center line

- **Case 2**: μ, σ unknown: Base on $\underline{m}$ runs

(1) Estimator for μ? $\Rightarrow$ $\boxed{\hat{\mu} = \bar{\bar{x}}} = \frac{1}{m}\sum_{j=1}^{m} \bar{x}_j$ Grand mean

(2) Estimator for σ? Can use either RANGE or SAMPLE STD. DEV.

Consider $R \triangleq x_{max} - x_{min}$ ... a random variable with known distribution when sampling from a normal distribution

$w = R/\sigma$ ... a new r.v., the "relative range"

$\mu_w \triangleq d_2(n)$ ... Mean of w depends on n; see charts

$\Rightarrow$ $\boxed{\hat{\sigma} = \bar{R}/d_2}$ ESTIMATE for process std. dev.

where $\bar{R} \triangleq \frac{1}{m}\sum_{i=1}^{m} R_i$ $\qquad A_2 \triangleq \frac{3}{d_2\sqrt{n}} = f(n)$

$$\therefore \quad \begin{aligned} UCL &= \hat{\mu} + 3\hat{\sigma}/\sqrt{n} = \bar{\bar{x}} + 3\bar{R}/d_2\sqrt{n} = \bar{\bar{x}} + A_2\bar{R} \\ CL &= \hat{\mu} = \bar{\bar{x}} = \bar{\bar{x}} \\ LCL &= \hat{\mu} - 3\hat{\sigma}/\sqrt{n} = \bar{\bar{x}} - 3\bar{R}/d_2\sqrt{n} = \bar{\bar{x}} - A_2\bar{R} \end{aligned}$$

R Chart

We also want to monitor the process variability.

$R = W\sigma$ Define $\sigma_W \triangleq d_3$ (also a known function of n that is tabulated)

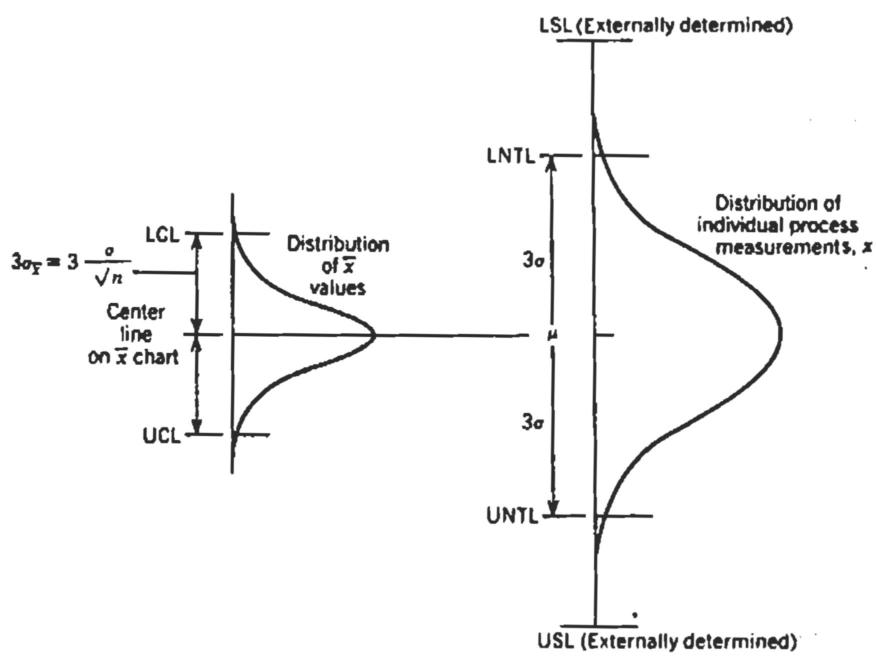
$\sigma_R = d_3\,\sigma$

$$\boxed{\hat{\sigma}_R = d_3 \frac{\bar{R}}{d_2}}$$ using our estimate $\hat{\sigma}$

$$
\begin{aligned}
\therefore\quad UCL &= \hat{\mu}_R + 3\hat{\sigma}_R = \bar{R} + 3\frac{d_3}{d_2}\bar{R} = \bar{R}D_4, \quad D_4 \triangleq 1 + 3\frac{d_3}{d_2} \\
CL &= \hat{\mu}_R = \bar{R} = \bar{R} \\
LCL &= \hat{\mu}_R - 3\hat{\sigma}_R = \bar{R} - 3\frac{d_3}{d_2}\bar{R} = \bar{R}D_3, \quad D_3 \triangleq 1 - 3\frac{d_3}{d_2}
\end{aligned}
$$

Control vs. Specification Limits

- Caution: Spec Limits on $\bar{x}$, R charts are related to the INDIVIDUALS values, while Control limits relate to the SAMPLING DISTRIBUTION



Relationship of natural tolerance limits, control limits, and specification limits.

Rational Subgroups

- Sample data subgroups should be selected so that
  (1) chances for assignable differences **BETWEEN** subgroups is *maximized*
  (2) chances for assignable differences **WITHIN** a subgroup is *minimized*

Example: May be tracking LOT-to-LOT variation.
- A rational subgroup might be 3-5 wafers from within lot
- A *poor* sample would be last + first wafer from sequential lots

Approaches: (1) Sample units produced at same time
→ e.g. lots, batches, small sequence in time

(2) Sample randomly from *all* units produced since last sample
→ *rarely* done in semiconductor fab

- Thus, $\bar{x}$ monitors BETWEEN sample variability

R monitors WITHIN sample variability

∴ $\hat{\sigma}$ estimated based *only* on within-sample data, dangerous to "POOL" data & use

$$s^2 = \sum_{i=1}^{m} \sum_{j=1}^{n} (x_{ij} - \bar{\bar{x}})^2 \frac{1}{mn-1}$$

because this overestimates if $\bar{x}_j$ are different and thus combines both between & with sample variability.

Guidelines for Control Chart Design

- Control chart design is ITERATIVE:
  - examine initial data (eg. 20 - 25 runs)
  - consider TRIAL limits based on chart design methods
  - examine data against limits
    - * identify assignable causes for "outliers" & eliminate

- Generally best to consider R Chart first
  - if process variability not in control, then likely that $\bar{x}$ won't be meaningful

- How about sampling frequency?
  - tradeoff between (a) cost of measurements, and (b) risk of missing deviation ⇒ more frequent small samples better than less freq. large samples

- Economic Control Chart design
  - determine α, β risks, n sample size based on COST issues, e.g.
    - cost of sampling
    - cost of investigating signals
    - costs of out-of-control product

⇒ Not typically done in semiconductor manufacturer; an interesting project!

Average Run Length (ARL)

ARL = Average # runs between false alarms when process is IN-CONTROL $= \dfrac{1}{\alpha}$

e.g. for $\pm 3\sigma$ limits, $\alpha = 0.0027$, ARL = 370

Sample Size and Operating Characteristic Curves

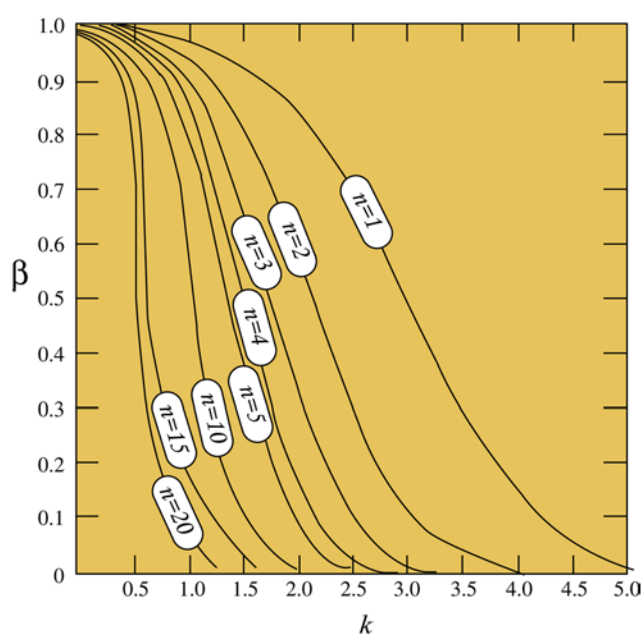
- How choose the sample size?
  - economic and ease of acquisition
  - tradeoff in false alarm rate on $\bar{x}$
  - risk in missing real shifts

- $\bar{x}$: looking for $k\sigma$ shifts in mean
  β = risk miss shift in any one sample

Pr (detect on 1st sample) $= 1-\beta \triangleq$ "power" of chart
Pr (detect on 2nd sample) $= \beta(1-\beta)$
Pr (detect on k^{th} sample) $= \beta^{k-1}(1-\beta)$

Expected # of runs before shift in mean detected $= \sum_{k=1}^{\infty} k\,\beta^{k-1}(1-\beta) = \frac{1}{1-\beta}$

ARL when OUT OF CONTROL $\triangleq 1/(1-\beta)$

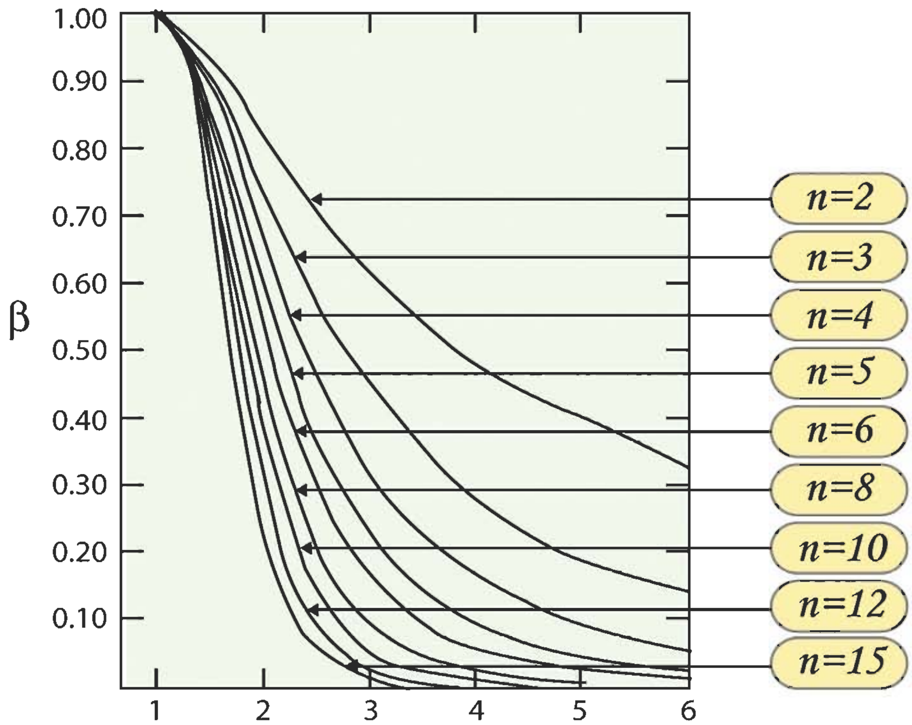


Operating-characteristic curves for the $\bar{x}$ chart with 3-sigma limits. $\beta = P$(not detecting a shift of $k\sigma$ in the mean on the first sample following the shift).

E.g. Detect 2σ shift with $n=4$, $\beta = 0.2$, ARL= 1.25
1σ shift with $n=4$, $\beta = 0.9$, ARL= 10

- R : looking for changes in process variability, i.e. $\sigma_1 > \sigma_0$,

Use $\lambda = \sigma_1 / \sigma_0$ = relative size of process variation increase to detect



$\lambda = \sigma_1 / \sigma_o$, ratio of new to old process standard deviation

Operating-characteristic curves for the *R* chart with 3-sigma limits. (Adapted from A.J. Duncan, "Operating Characteristics of *R* Charts," *Industrial Quality Control*, vol. 7, no. 5, pp. 40-41, 1951)

Example: To find a DOUBLING in std. deviation ($\lambda = 2$), $n = 5$, $\beta = 0.6$, so $1 - \beta = 0.4$

$\Rightarrow$ only 40% chance of detecting on each run!

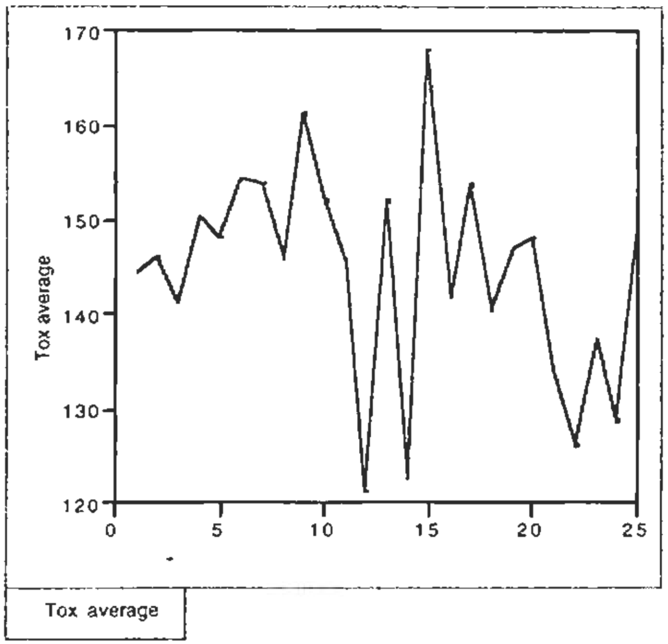
For $n \geq 10$-12, better to use S chart ...

(that is, if want to detect smaller variance changes)

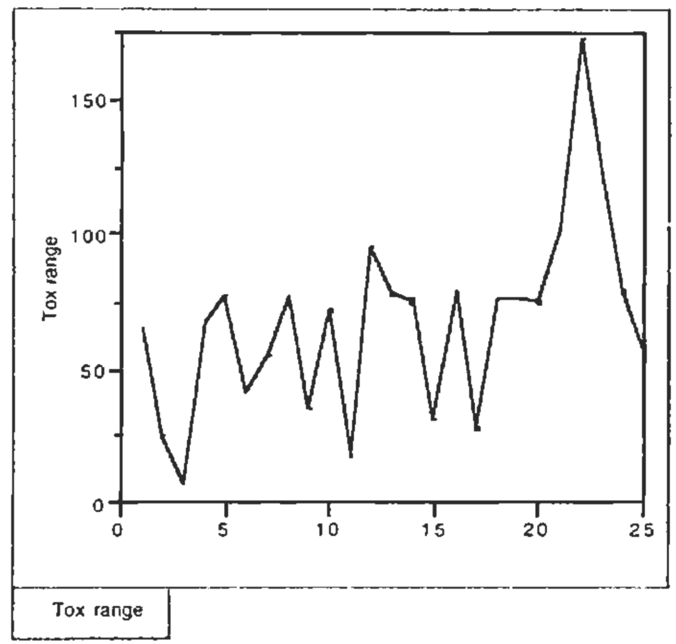
Example: Oxide Thickness Monitoring

Oxide growth monitoring -- desire 150 anstrom oxide thickness.
Measure thickness on 4 wafers out of each lot.

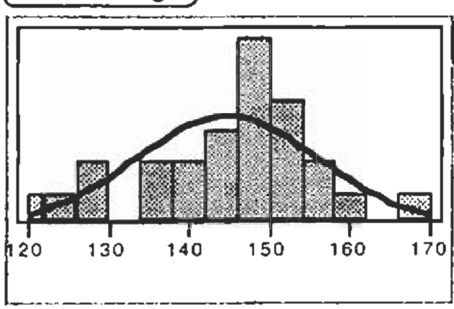
Tox Average - Trial Data



Tox Range - Trial Data



Tox average

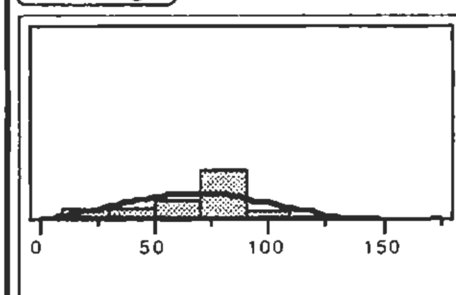


Quantiles

Moments

Mean	144.7918
Std Dev	11.3926
Std Error Mean	2.2785
Upper 95% Mean	149.4944
Lower 95% Mean	140.0892
N	25.0000
Sum Weights	25.0000

Tox range



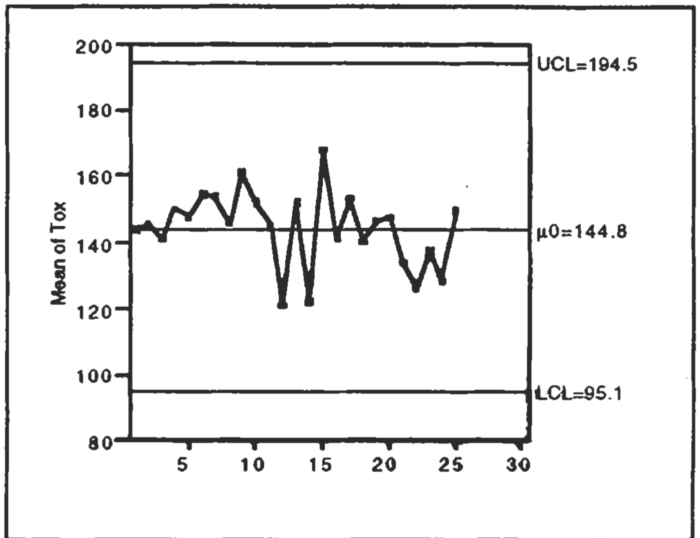
Quantiles

Moments

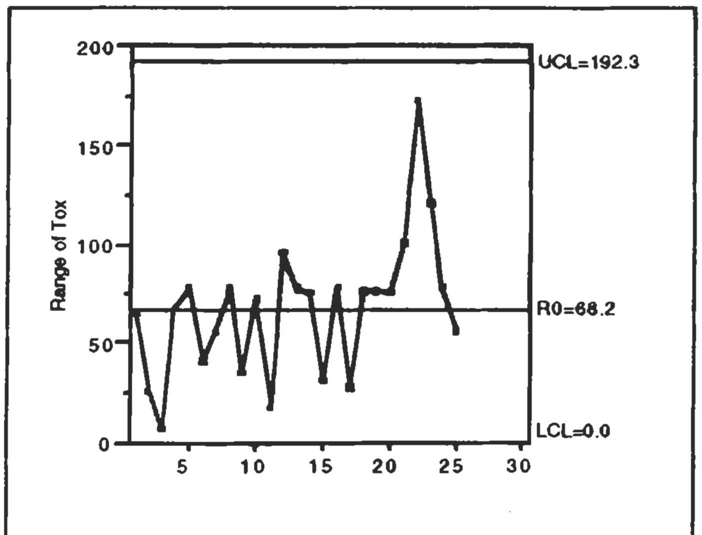
Mean	68.20842
Std Dev	35.02755
Std Error Mean	7.00551
Upper 95% Mean	82.66697
Lower 95% Mean	53.74988
N	25.00000
Sum Weights	25.00000

Xbar - R Control Charts --   xide Thickness Monitoring

Variable Control Charts



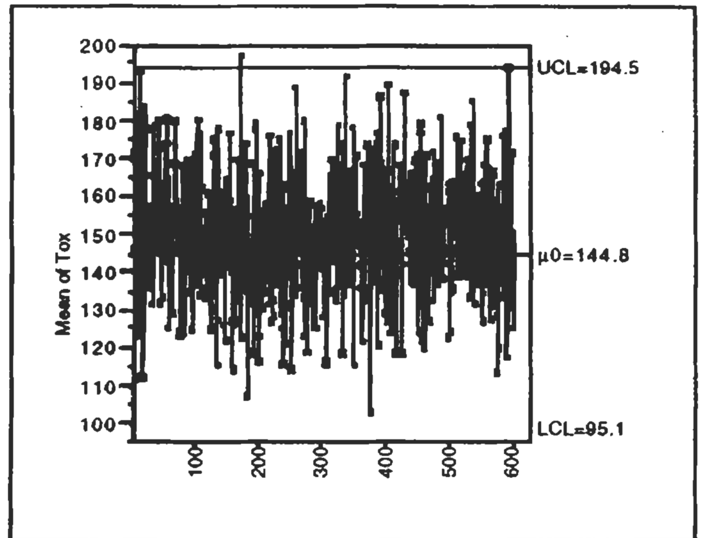
Mean of Tox



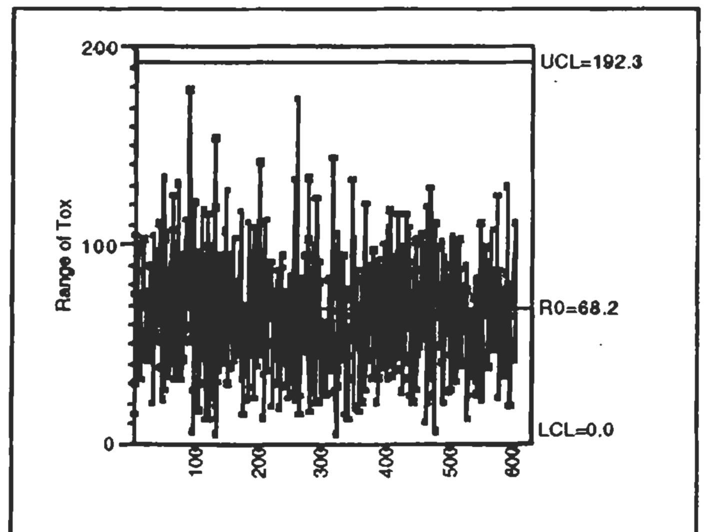
Range of Tox

Variable Control Charts

Tox -- Run Data



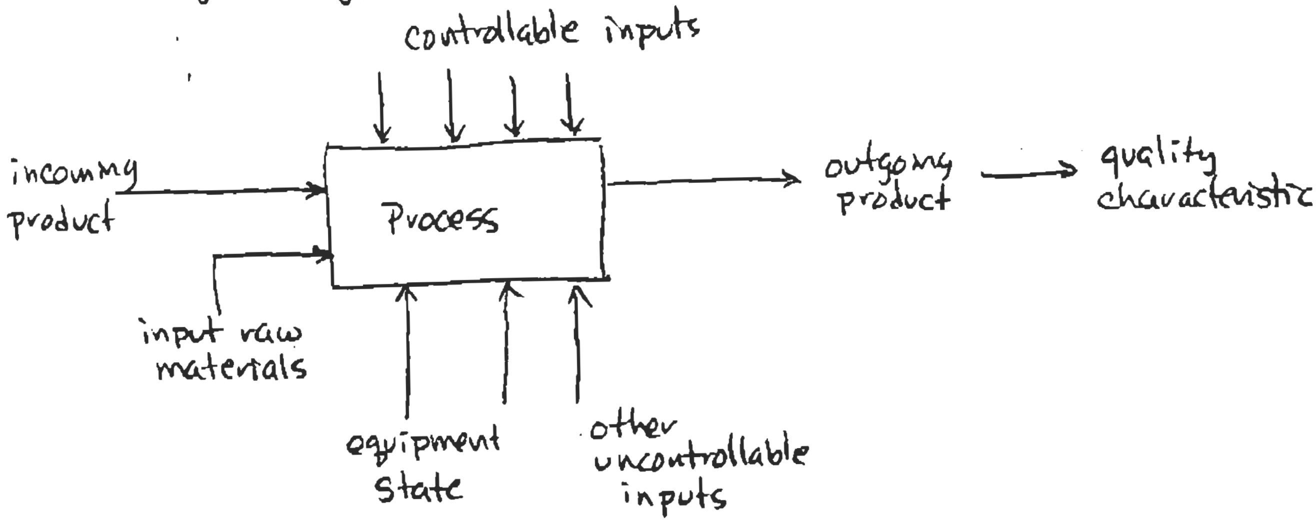
Mean of Tox



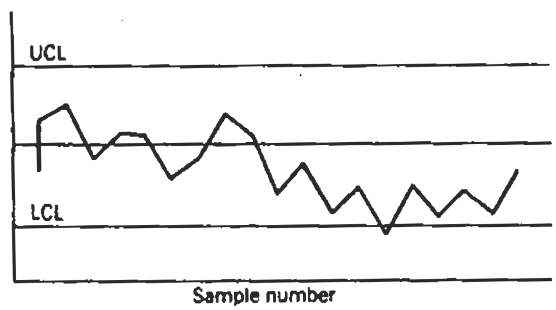
Range of Tox

Identifying Assignable Causes

- An engineering/detective exercise!

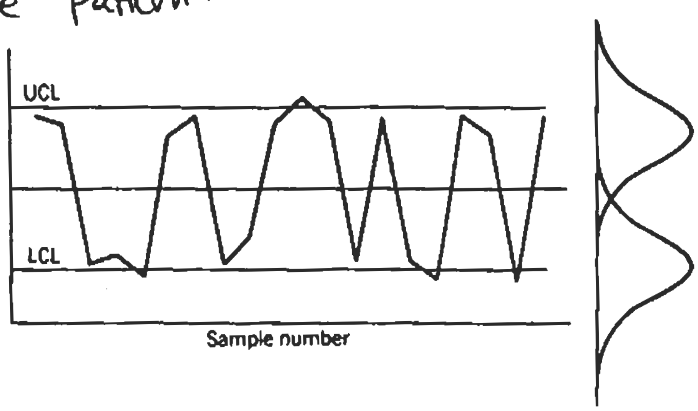


- Use information in chart itself ⟹ patterns ; trends
  - cycles ~ cyclic behavior
  - time of occurence (of course)
  - shift in process level



A shift in process level.

  - mixture pattern:



A mixture pattern.

$\bar{x}$ and S Control Charts

- Use when sample size is large, $n > 10\text{-}12$

Why? R is a poor estimator as n increases, as it does not make effective use of data in the entire range.

- Problem: while S^2 is an unbiased estimator for σ^2, S is biased estimator for σ

$$S = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}}$$

$$\mu_S = c_4 \sigma$$, where c_4 is constant, function of n

and

$$\sigma_S = \sigma \sqrt{1 - c_4^2}$$

- S chart design: assume we will be computing sample S on each sample.

$\therefore$ Set limits as:

$$\begin{aligned} UCL &= \mu_S + 3\sigma_S &&= c_4\sigma + 3\sigma\sqrt{1-c_4^2} &&= B_6\sigma \\ CL &= \mu_S &&= c_4\sigma &&= c_4\sigma \\ LCL &= \mu_S - 3\sigma_S &&= c_4\sigma - 3\sigma\sqrt{1-c_4^2} &&= B_5\sigma \end{aligned}$$

- Typically we don't know σ either ...

$$\hat{\sigma} = \frac{\bar{S}}{c_4} = \frac{1}{c_4} \cdot \frac{1}{m} \sum_{i=1}^{m} S_i$$ is unbiased estimator for σ

$$\begin{aligned} UCL &= \bar{S} + 3\frac{\bar{S}}{c_4}\sqrt{1-c_4^2} &&= \bar{S}\, B_4 \\ CL &= \bar{S} &&= \bar{S} \\ LCL &= \bar{S} - 3\frac{\bar{S}}{c_4}\sqrt{1-c_4^2} &&= \bar{S}\, B_3 \end{aligned}$$

where c_4, B_5, B_6, B_3, B_4 are TABULATED as $f(n)$

- Using $\hat{\sigma} = \bar{S}/c_4$, may also use in $\bar{x}$ chart limits:

$$\begin{aligned} UCL &= \bar{\bar{x}} + 3\bar{S}/c_4\sqrt{n} &&= \bar{\bar{x}} + A_3\bar{S} \\ CL &= \bar{\bar{x}} &&= \bar{\bar{x}} \\ LCL &= \bar{\bar{x}} - 3\bar{S}/c_4\sqrt{n} &&= \bar{\bar{x}} - A_3\bar{S} \end{aligned}$$

with $A_3 = \dfrac{3}{c_4\sqrt{n}}$

S^2 Control Chart

- Since S^2 is an unbiased estimator of variance, can be easier to design a chart to a desired α-confidence using S^2 rather than S

$$UCL = \frac{\bar{S}^2}{n-1}\chi^2_{\alpha/2,\, n-1}$$

$$CL = \bar{S}^2$$

$$LCL = \frac{\bar{S}^2}{n-1}\chi^2_{1-\alpha/2,\, n-1}$$

$\Rightarrow$ cleaner from a hypothesis testing / confidence interval perspective

$\Rightarrow$ Rarely used (why? - non-intuitive units?)

Moving Estimate of R

- In cases where it is easy to do, we may measure <u>every</u> unit. How monitor variability?

- Approach: Use $R = |x_{prev} - x_{current}|$

(1) Individuals chart (a) Based on historical (standard) σ
- or (b) Use moving range for 15-20 runs

Case b:

$$UCL = \bar{x} + 3\bar{R}/d_2 \qquad \text{where } d_2 \text{ found for } n=2$$
$$CL = \bar{x}$$
$$LCL = \bar{x} - 3\bar{R}/d_2$$

(2) Moving R chart: calculate using R, and $n=2$

$$UCL = D_4\bar{R} \qquad \text{for } n=2, \quad D_4 = 3.267$$
$$CL = \bar{R} \qquad D_3 = 0$$
$$LCL = D_3\bar{R}$$

- Cautions: - Moving ranges are correlated, which may induce patterns on the R chart
  - x values uncorrelated, so patterns should be investigated

- <u>Alternatives</u>: CUSUM, EWMA charts for case where every unit is sampled.

Process Capability

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- Note that a process may be:
  - in control but NOT meeting specs
  - out of control but meeting specs,
  - ...

- Process Cabability Ratio

$$PCR = C_p \triangleq \frac{USL - LSL}{6\sigma}$$ when process is centered

$$C_{pk} \triangleq \min\left\{ \frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma} \right\}$$ when process is not centered

- Process Fallout : defective parts-per-million (ppm)

Values of the Process Capability Ratio (PCR) and Associated Process Fallout for a Normally Distributed Process (in defective ppm)

	Process Fallout (in defective ppm)	
PCR	One-Sided Specifications	Two-Sided Specifications
0.50	66,800	133,600
0.75	12,200	24,400
1.00	1,350	2,700
1.10	483	966
1.20	159	318
1.30	48	96
1.40	13	26
1.50	3.40	6.80
1.60	0.80	1.60
1.70	0.17	0.34
1.80	0.03	0.06
2.00	0.0009	0.0018

- Estimating C_p, C_{pk} from control charts

  - An advantage of control charting is that one develops over time a good estimate of the natural variation in the process $\Rightarrow$ use $\bar{x}$ & S

    (A) $\bar{x}$, S charts : use $\hat{\mu} = \bar{\bar{x}}$, $\hat{\sigma} = \bar{S}/c_4$ for sample size n

    (B) $\bar{x}$, R charts : use $\hat{\mu} = \bar{\bar{x}}$, $\hat{\sigma} = \bar{R}/d_2$

  - Also good practice to look at historical samples
    (a) histograms
    (b) normal-probability plots

    For unusual effects, (e.g. mixtures)

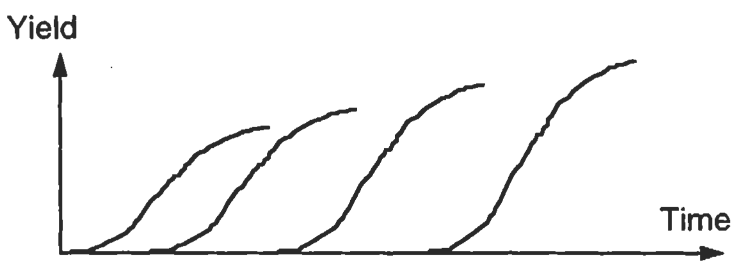
CHARTS FOR ATTRIBUTES

- When a unit's quality characteristics do not have a simple numeric representation, we can often classify as "defective" / "nonconforming" ← BAD part
  vs. "non-defective" / "conforming" ← GOOD part.

Types of charts:

1. Fraction nonconforming – p chart
2. Number of defects – c chart (single unit)
   – u chart (n units)

- MOTIVATION: YIELD
  - Often the goal is to achieve high yield as rapidly as possible: yield learning
  - Once achieved, must maintain high yield



The Yield of each new process-product combination follows a trajectory called the yield learning curve.

Time to yield for a new product can have huge implications.

Also, field reliability is often related to yield.

1 Quarter sooner => 1 billion more sales over a 10 quarter lifetime.

P CHART – Fraction Non-Conforming

- Statistical model: fraction nonconforming = $\dfrac{\#\text{ defective in POPULATION}}{\#\text{ in POPULATION}}$

Assume p = prob part defective = fraction nonconforming

In a sample of n parts, the probability mass function of the # defective, D, is that of a binomial distribution

$$D \sim B(n, p)$$

$$Pr(D) = \binom{n}{D} p^D (1-p)^{n-D}, \quad D = 1, 2, 3 \dots n$$

$$\mu_D = np$$
$$\sigma^2_D = np(1-p)$$

- Sample fraction nonconforming $\triangleq \hat{p} = \dfrac{D}{n}$

$$\mu_{\hat{p}} = p$$
$$\sigma^2_{\hat{p}} = \frac{p(1-p)}{n}$$

- Application to Control chart

CASE 1: p known

1. Sample n units; calculate $\hat{p}_i = D/n$
2. Chart using $UCL = p + 3\sigma_{\hat{p}} = p + 3\sqrt{\dfrac{p(1-p)}{n}}$

$$CL = p$$
$$LCL = p - 3\sigma_{\hat{p}} = p - 3\sqrt{\frac{p(1-p)}{n}}$$

CASE 2: p NOT known

1. Select $m \approx 20-25$ preliminary runs

$$\hat{p}_i = \frac{D_i}{n}, \quad i = 1, 2, \dots m$$

$$\bar{p} = \frac{1}{mn} \sum^{m} D_i = \frac{\sum^{m} \hat{p}_i}{m}$$

So now $\bar{p}$ is estimate of p

$$UCL: \quad \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$CL = \bar{p}$$

$$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

- Confirming Preliminary / Trial Control Limits

  - One may want to check that some collection of later data was really well-represented by the 20-25 runs of prior data

  - Formulate as hypothesis test

    p_1 = preliminary data fraction non-conforming
    p_2 = new data " "

$H_0: \quad p_1 = p_2$
$H_1: \quad p_1 \neq p_2$

Estimate these by $\hat{p}_1 = \bar{p}_k$, $\hat{p}_2 = \dfrac{\sum_{i=1}^{k} D_k}{n \cdot k}$

Test statistic: (based on null hypothesis)

If H_0 true, then $p_1 = p_2 = p$, so

$$\hat{p} = \frac{m\hat{p}_1 + k\hat{p}_2}{m+k}$$

$$Z_0 = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{m} + \frac{1}{k}\right)}}$$

Reject H_0 if $|Z_0| > Z_{\alpha/2}$

- Caution: Rational Subgrouping needed
  - For example, if believe a shift difference, avoid pooling across shifts

- p-Chart Issues

<u>Rational Subgrouping</u>: Careful in how one selects the n units. E.g. if believe a shift difference, avoid pooling data in calculating $\bar{p}$

<u>Sample Size</u>: For small p, choose larger n so have high enough prob. of finding at least one defective

E.g. $p = 0.01$, $n = 8$ $\quad UCL = 0.01 + 3\sqrt{\frac{0.01(0.99)}{8}}$

$= 0.1155$

If 1 defect is found, $\hat{p} = \frac{1}{8} = 0.125$

Approaches for selecting n:

1. Choose n so prob finding ≥ 1 nonconforming unit $\geq \gamma$
   E.g. $Pr(D \geq 1) = 0.95$
   Using Cumulative Poisson chart $\lambda = np = 3.00$ for 0.95
   $\therefore$ n for $p = 0.01$, $n = 300$

2. n large enough to detect process shift w. Prob. 0.5
   E.g. detect shift from $p = 0.01$ to $p = 0.05$
   Using Normal approximation

$$\delta = k\sqrt{\frac{p(1-p)}{n}} \Rightarrow n = \left(\frac{k}{\delta}\right)^2 p(1-p)$$

for $p = 0.01$, $\delta = 0.05 - 0.01 = 0.04$, $k = 3$ (3 sigma limits)

$$n = \left(\frac{3}{0.04}\right)^2 (0.01)(0.99) = 56$$

3. For p small, can choose n so LCL is positive
   → only useful if want to be forced to investigate low defective samples

$$LCL = p - k\sqrt{\frac{p(1-p)}{n}} > 0 \Rightarrow n > \frac{(1-p)}{p} k$$

e.g. $p = 0.05$ $\quad 3\sigma$ $\Rightarrow$ $n > 171$

- np Chart : Number Nonconforming

A simple modification is to chart the # defective rather than fraction:

$$\begin{aligned} UCL &= np + 3\sqrt{np(1-p)} \\ CL &= np \\ LCL &= np - 3\sqrt{np(1-p)} \end{aligned}$$

Use $\bar{p}$ if p not available

- Variable sample size :

Sometimes (e.g. total inspection), sample size varies for each sampling period.

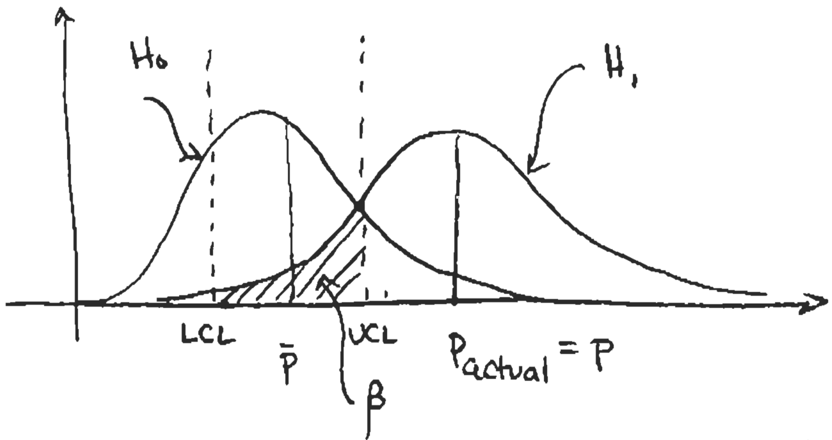
(1) Pool in trial period to get $\bar{p}$
(2) For each sample, recalculate UCL/LCL based on n_i

$$\begin{aligned} UCL &= \bar{p} + 3\hat{\sigma}_{p_i} \\ CL &= \bar{p} \\ LCL &= \bar{p} - 3\hat{\sigma}_{p_i} \end{aligned}$$

$$\hat{\sigma}_{p_i} = \sqrt{\frac{\bar{p}(1-\bar{p})}{n_i}}$$

- Operating Characteristic for p chart

β = Probability of missing true deviation in p



$$\beta = Pr(\hat{p} < UCL \mid p) - Pr(\hat{p} < LCL \mid p)$$

$$= \underbrace{Pr(D < n \cdot UCL \mid p)}_{\text{calculate using cumulative binomial distributions}} - Pr(D < n \cdot LCL \mid p)$$

c CHART - Number of Defects on a Unit

- Applicable to situations where multiple defects may be detected on a unit
  - a single defect may not necessarily imply non-conforming (e.g. cosmetic flaws on PC case, # defective welds on airplane wing, # defects on a wafer)

- Statistical Model: assume occurence of defects in constant sample size ~ Poisson

  $\Rightarrow$ ∞ opportunities for defects
  $\Rightarrow$ prob. of defect @ any site small & consta[nt]

  Assume <u>inspection unit</u> same for each sampl[e] (e.g. same "area of opportunity")

  Then $$Pr(x) = \frac{e^{-c} c^x}{x!}, \quad x = 0, 1, 2 \ldots \infty$$

  where x = # defects, c = Poisson parameter, $c > 0$

  $$\mu_x = c, \quad \sigma_x^2 = c$$

- Control Chart:

  <u>If c known</u>

  $$UCL = \mu_x + 3\sigma_x = c + 3\sqrt{c}$$
  $$CL = \mu_x = c$$
  $$LCL = \mu_x - 3\sigma_x = c - 3\sqrt{c}$$

  <u>If c unknown</u>

  Use $\bar{c} = \hat{\mu}_x$

  $$UCL = \bar{c} + 3\sqrt{\bar{c}}$$
  $$CL = \bar{c}$$
  $$LCL = \bar{c} - 3\sqrt{\bar{c}}$$

- Issues
  - c more informative than p; analyze defects by TYPE

u CHART - Nonconformities per Unit in Multiple Units

- Remove restriction of $n=1$ in the c-chart; use several inspection units to increase opportunity for nonconformity

- Statistical model $u = \frac{c}{n}$ where u is Poisson r.v. = linear comb. of n independent Poisson r.v.

$$UCL = \bar{u} + 3\sqrt{\frac{\bar{u}}{n}}$$
$$CL = \bar{u}$$
$$LCL = \bar{u} - 3\sqrt{\frac{\bar{u}}{n}}$$

where $\bar{u}$ based on preliminary set of data

- Variable sample size: if n_i on each sample, we can again adjust control limits on a sample-by-sample basis.

Roll Number, i	n_i	$UCL = \bar{u} + 3\sqrt{\bar{u}/n_i}$	$LCL = \bar{u} - 3\sqrt{\bar{u}/n_i}$
1	10.0	2.55	0.29
2	8.0	2.68	0.16
3	13.0	2.41	0.43
4	10.0	2.55	0.29
5	9.5	2.58	0.26
6	10.0	2.55	0.29
7	12.0	2.45	0.39
8	10.5	2.52	0.32
9	12.0	2.45	0.39
10	12.5	2.43	0.41