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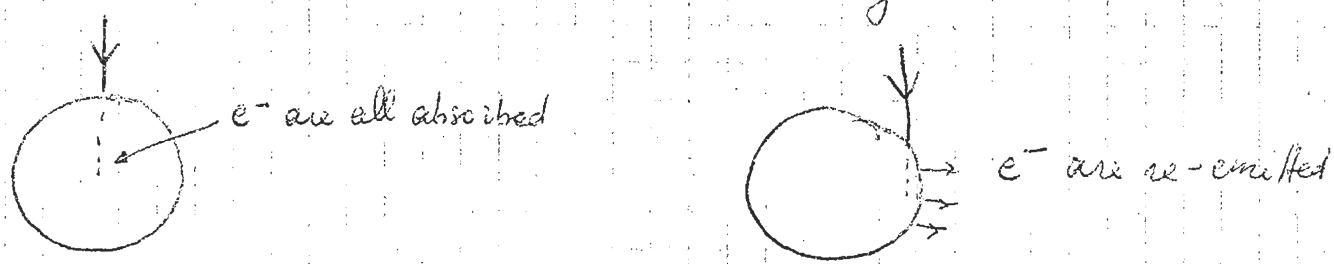
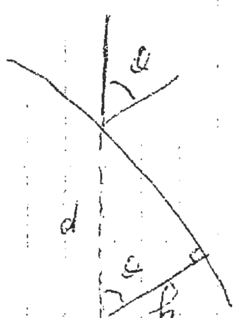
Scanning Electron Microscopes

The contrast is reversed w/ traditional images



→ This is called SEM model.



$$d = \frac{h}{\cos\theta_e}$$

Lambert	Hapke	SEM
$\cos\theta_i$	$\sqrt{\frac{\cos\theta_i}{\cos\theta_e}}$	$\frac{1}{\cos\theta_i}$
$k = \frac{1}{2}$	$k = 0$	$k = -\frac{1}{2}$

$$\left(\cos\theta_i^{\,k+1/2} \cos^{k-1/2}\theta_e\right)$$

Chapter 11.

Ex: Hapke type surface

$$R(p,q) = f(ap + bq)$$

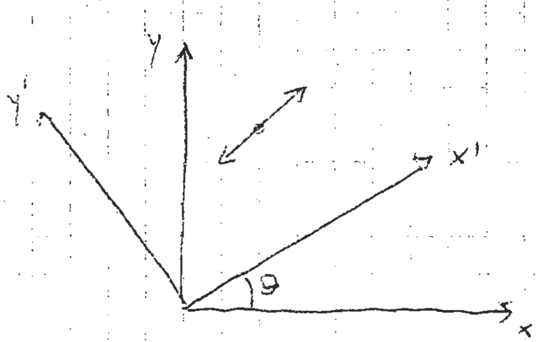
Image irradiance equation: $E(x,y) = R(p,q)$

We want to solve for p, q: $ap + bq = f^{-1}(E(x,y))$

$$\frac{a}{\sqrt{a^2+b^2}}\, p + \frac{b}{\sqrt{a^2+b^2}}\, q = \frac{1}{\sqrt{a^2+b^2}}\, f^{-1}(E(x,y))$$

$$\cos\theta \cdot p + \sin\theta \cdot q = \frac{1}{\sqrt{a^2+b^2}}\, f^{-1}(E(x,y))$$

If we use a rotated coordinate system:



$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\frac{\partial f}{\partial x'} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \partial f/\partial x \\ \partial f/\partial y \end{pmatrix}$$

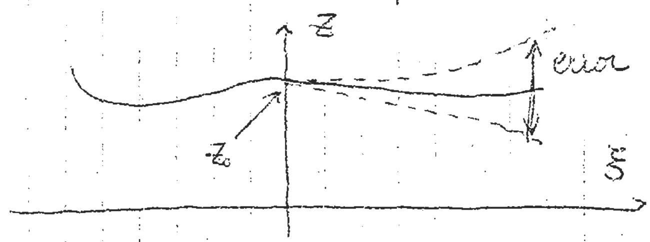
Thus if $p' = p\cos\theta + q\sin\theta$

$$\frac{\partial z}{\partial x'} = \frac{1}{\sqrt{a^2+b^2}} f^{-1}(E(x,y))$$

$$\cos\theta = \frac{a}{\sqrt{a^2+b^2}} \qquad \sin\theta = \frac{b}{\sqrt{a^2+b^2}}$$

If we take a small step $\delta\xi \rightarrow \delta z = \frac{1}{\sqrt{a^2+b^2}} f^{-1}(E(x,y))$

Then $z(\xi) = z_0 + \int_0^{\xi} \frac{1}{\sqrt{a^2+b^2}} f^{-1}(E(x,y))\, d\xi$



accumulation of error along ξ.

Suppose we have a solution $z(x,y)$

Then we have $\frac{\partial z}{\partial x} = p$ and $\frac{\partial z}{\partial y} = q$ and $R(p,q) = E(x,y)$

Then $z' = z + g(bx - ay)$ is also a solution

↳ we can add any shape to z.

↳ there is an ambiguity.

How to get Moon's surface ?

 craters are rotationally symmetric.

<u>General case:</u>

$(x, y, z) \rightarrow (x+\delta x, y+\delta y, z+\delta z)$ take a small step and continue the solution

$\delta z = p\delta x + q\delta y$ but (p,q) unknown

$(x, y, z, p, q) \rightarrow (x+\delta x, y+\delta y, z+\delta z, p+\delta p, q+\delta q)$

then $\delta p = r\delta x + s\delta y$ $\quad r = p_x = z_{xx}$ $\quad t = q_y = z_{yy}$

$\delta q = s\delta x + t\delta y$ $\quad s = p_y = z_{xy}$

↳ we need higher level derivative ... not good.

$$\begin{vmatrix} \delta p \\ \delta q \end{vmatrix} = \begin{vmatrix} r & s \\ s & t \end{vmatrix} \begin{vmatrix} \delta x \\ \delta y \end{vmatrix}$$

Hessian matrix (curvature)

If we can find H, we are done !

However: $E(x,y) = R(p,q)$

If we differentiate: $\begin{vmatrix} E_x \\ E_y \end{vmatrix} = \begin{vmatrix} r & s \\ s & t \end{vmatrix} \cdot \begin{vmatrix} R_p \\ R_q \end{vmatrix}$

→ this gives a way to estimate H.

But first lets see that,

Brightness ⟷ Surface Orientation

Brightness gradient ⟷ Surface Curvature

Thus, we want to solve for H:

we can estimate E_x, E_y in the image and R_p / R_q.

3 unknowns (r, s, t) and 2 equations ...

we need a 3rd constraint on H, for ex: $\det(H) = 0$ (plane, cylinder)

Then, how do we do ?

Take a small step $\begin{vmatrix} dx \\ dy \end{vmatrix} = \begin{vmatrix} R_p \\ R_q \end{vmatrix} d\xi$

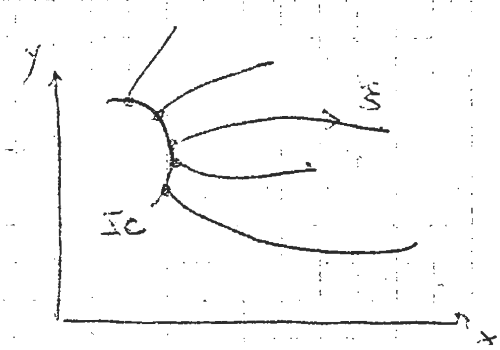
then: $\begin{vmatrix} dp \\ dq \end{vmatrix} = H \begin{vmatrix} dx \\ dy \end{vmatrix} = H \begin{vmatrix} R_p \\ R_q \end{vmatrix} d\xi = \begin{vmatrix} E_x \\ E_y \end{vmatrix} d\xi$

Finally: $$\frac{dx}{d\xi} = R_p \qquad \frac{dy}{d\xi} = R_q$$

$$\frac{dp}{d\xi} = E_x \qquad \frac{dq}{d\xi} = E_y \qquad \frac{dz}{d\xi} = pR_p + qR_q$$

Characteristic strip equations

$x(\xi)$, $y(\xi)$, $E(\xi)$ characteristic curve



Initial Conditions ? $x(\eta)$ $y(\eta)$ $z(\eta)$

$$E(x,y) = R\left(\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}\right)$$

first-order non-linear PDE.
We turned in into 5 linear ODE. Good!

I.C: $x(\eta)$ $x(\eta)$ $z(\eta)$
$\rightarrow$ Constraint on $p(\xi), q(\xi)$. Easy. $\frac{\partial z}{\partial \eta} = p\frac{\partial x}{\partial \eta} + q\frac{\partial y}{\partial \eta}$

We need a second constraint:

$$E(x,y) = R(p,q) \quad \text{(non-linear...)}$$

We can guaranty that there is only one solution. (ambiguity)
We need some other way to start the solution.
Another idea is the occluding boundary:

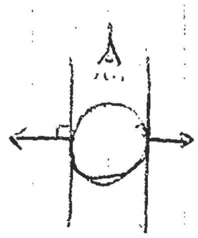
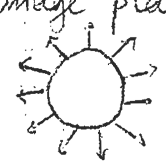
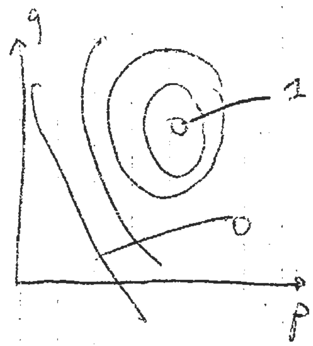


Image plane



$\rightarrow$ normal are known on the boundaries
(but $p, q \rightarrow \infty$) $\rightarrow$ hard to plug-in
however $p:q$ is finite

Other way:



extremum: unique, global

$R(p_0, q_0) > R(p,q)$ for all $(p,q) \neq (p_0, q_0)$

$p = p_s \quad q = q_s$

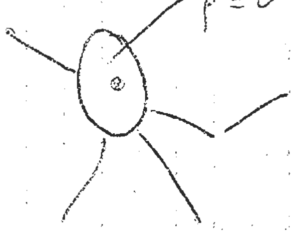
$R_p = 0 \quad R_q = 0$

$E_x = 0 \quad E_y = 0$

thus $\dot{x} = 0 \quad \dot{y} = 0 \quad \dot{z} = 0$

$\dot{p} = 0 \quad \dot{q} = 0$ Oups.

We can construct a small power series.



$\dot{p} = 0$ etc...
and then build.

ex: simple case "SEM" $R_F(p,q) = \frac{1}{2}(p^2 + q^2)$

$z = z_0 + \frac{1}{2}(ax^2 + 2bxy + cy^2) \implies \begin{cases} p = ax + by \\ q = bx + cy \end{cases}$

then $E(x,y) = \frac{1}{2}\left((ax + by)^2 + (bx + cy)^2 \right)$

$$E_x = (a^2+b^2)x + (a+c)by$$
$$E_y = (a+c)by + (c^2+b^2)y$$

$E_x = 0$ & $E_y = 0 \Rightarrow x = y = 0$

$$\left.\begin{aligned} E_{xx} &= a^2+b^2 \\ E_{xy} &= (a+c)b \\ E_{yy} &= b^2+c^2 \end{aligned}\right\}$$ $2 \times 2 \times 2 = 8$ max solutions (in fact 4)

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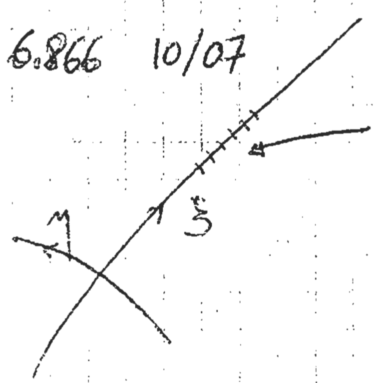

So how do we solve the ODEs

We take steps of same length which gives a constraint on $\dot{\xi}$ (either on the image or in the world)

We prefer to take same length in the world but of course, it's harder.

Another idea is to take equal steps in height ($\frac{dz}{ds} = 1$) or equal change in brightness on the image ($\frac{dE}{ds} = 1$) so we step from isophote to isophote (instead of contour to contour).

<u>Consistency of solution</u>

We might end up with crossing strips which happens if the image is noisy

A solution is to solve the strips in parallel and check consistency.