1.270J/ESD.273J

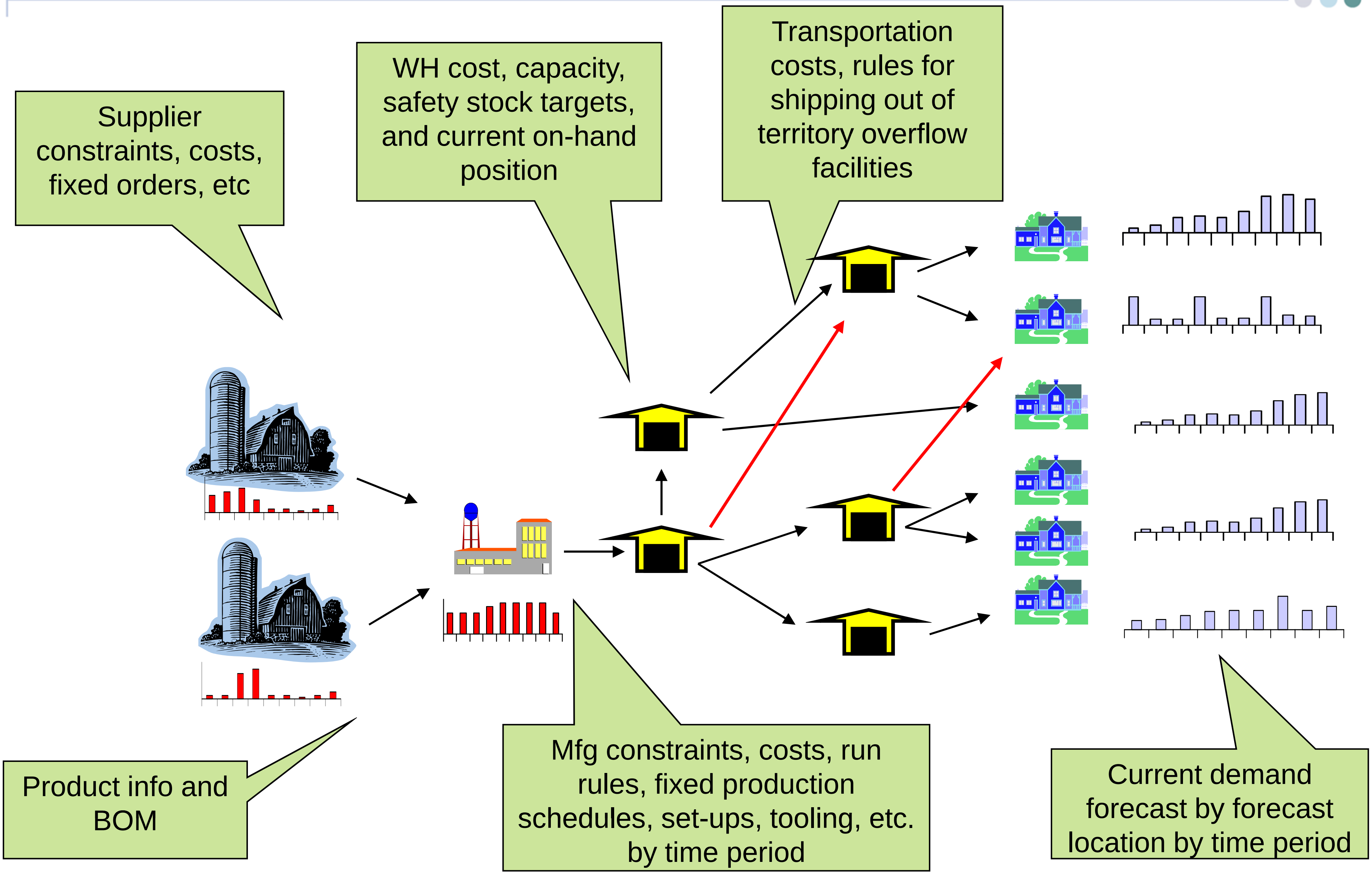
Logistics and Distribution Systems

Dynamic Economic Lot Sizing Model

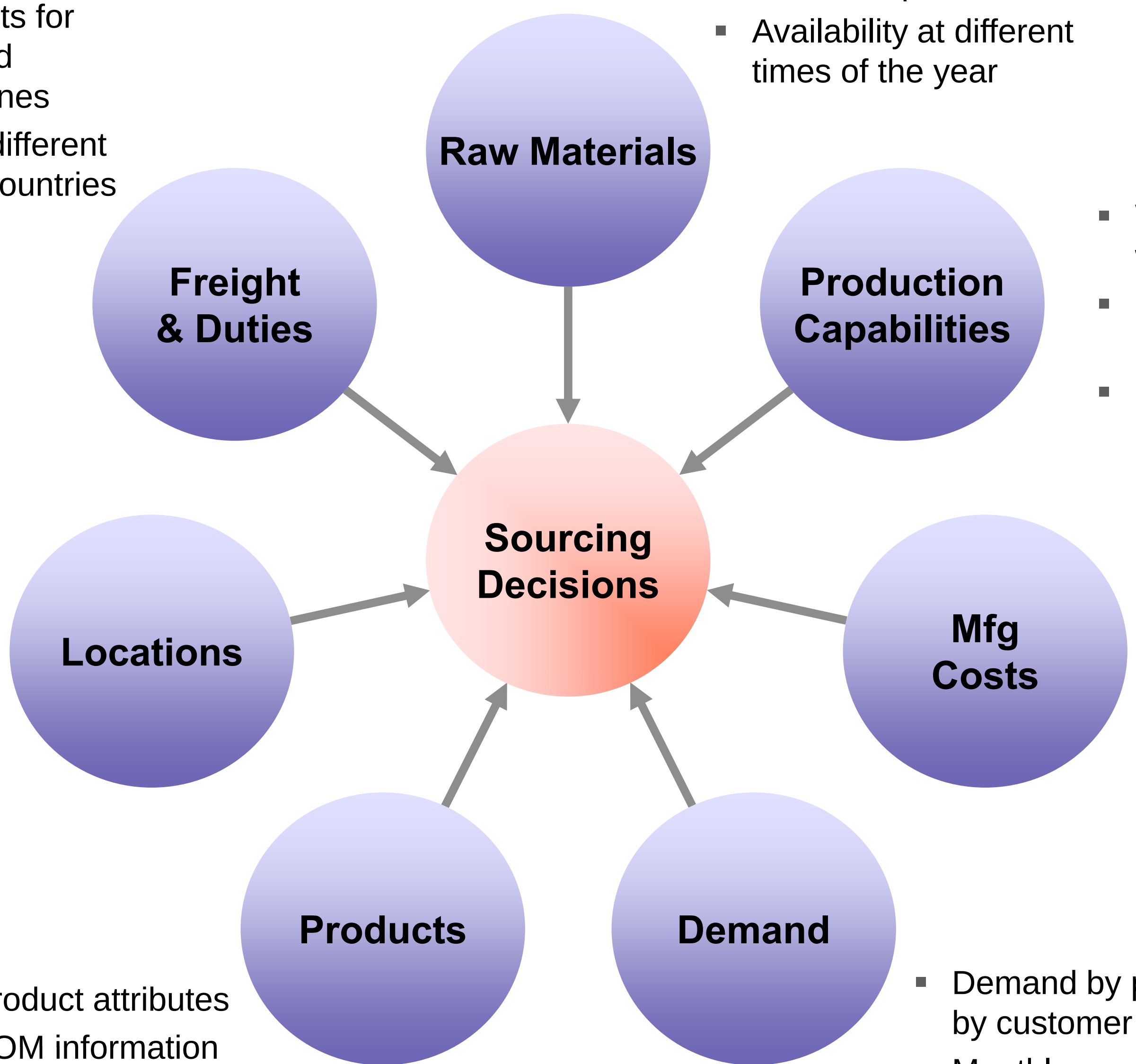
Outline

- The Need for DELS
- DELS without capacity constraints:
  - ZIO policy;
  - Shortest path algorithm.
- DELS with capacity constraints:
  - Capacity constrained production sequences;
  - Shortest path algorithm.

Strategic Sourcing Inputs



Key Drivers in Sourcing Decisions



Raw Materials
- Purchase prices
- Availability at different times of the year

Freight & Duties
- Freight costs for existing and potential lanes
- Duties for different customer countries

Production Capabilities
- What can be made where, tooling, etc.
- Mfg speeds & capacities
- Setups, batches, yield loss, etc.

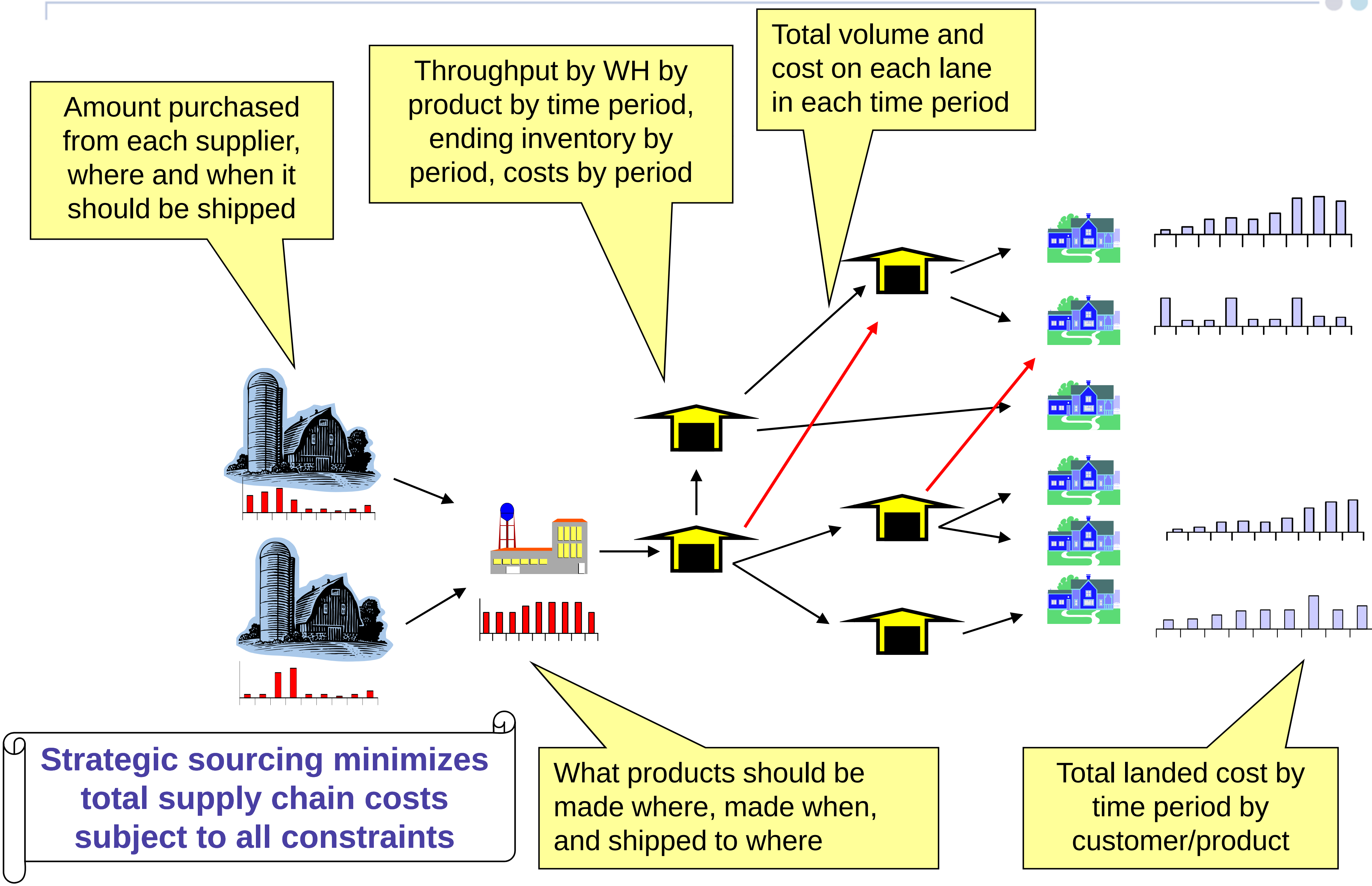
Locations
- Suppliers
- Plants
- Warehouses
- Customers

Mfg Costs
- Production costs
- Allocation of fixed & variable overheads
- Economies of scale

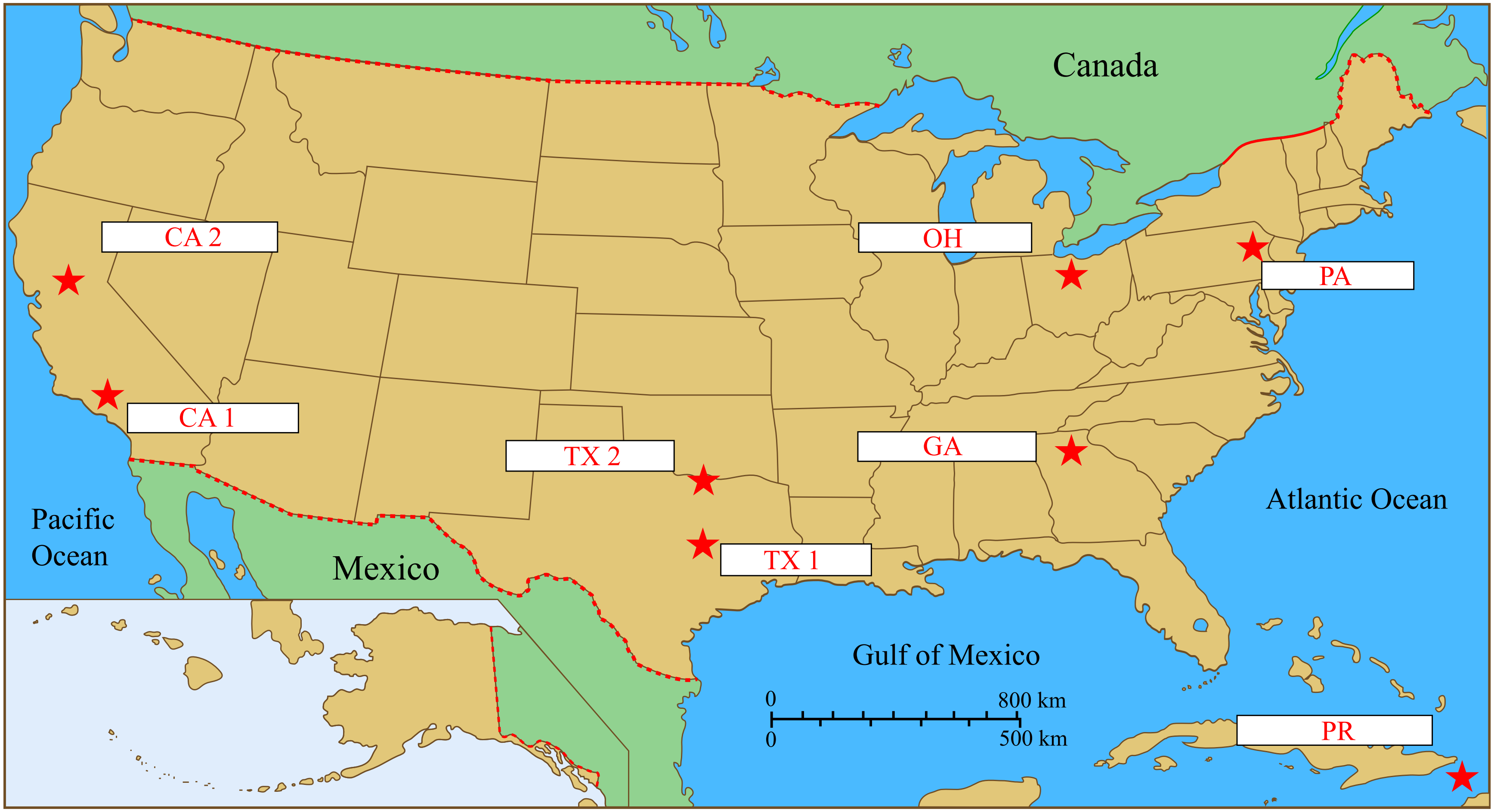
Products
- Product attributes
- BOM information
- Special rules and constraints

Demand
- Demand by product by customer
- Monthly or weekly forecasts

Strategic Sourcing Outputs



Dase Study: Strategic Sourcing



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Background

- Sold product throughout US to variety of customers
  - Direct to customers/distributors
  - Through their own stores
  - Through retailers
- Wide variety of product
  - 4,000 different SKU's
  - 1500 different base products (could be labeled differently)
  - Many low-volume products
- Batch Manufacturing
  - Manufacturing done in batch, so there significant economies of scale if a single product is made in one location
- Mfg Capability
  - Each plant had many different processes
  - Many plants can produce the same problem

Business Problems

- Are products being made in the right location?
- Should plants produce a lot of products to serve the local market or should a plant produce a few products to minimize production costs?
- Should we close the high cost plant?
- How should we manage all the low volume SKU's?

Key Driver - Data Collection

Key Drivers

Raw Material Costs
- Invoice cost
- Freight cost
- % shrinkage
- Difficulty - Medium

Variable Mfg. Costs
- Variable Mfg. costs by process/by site
- Difficulty - High

Technical Capability
- Product Family
- Difficulty - High

Manufacturing Capability
- Demonstrated Capacity
- Difficulty - High

Yield Loss
- % of lost volume per 100 units
- Site Specific
- Difficulty - Low

Interplant Freight Costs
- Average run rates in from site to site
- Difficulty - Low

Process Change

Existing Process	New Process
Sourcing decisions currently made in isolation ⇨ Decisions are made for small groups of products	Sourcing decisions made in context of entire supply chain ⇨ Decisions are made considering the entire supply chain
Excel is the primary decision tool – Many drawbacks ⇨ Excel does not capture the many different trade-offs that exist in the supply chain ⇨ Excel can <u>only</u> calculate the costs for a given decision; it cannot make decisions	Sourcing decisions are made using Master Planning ⇨ Optimization model provides global optimization capability
No formal data collection process ⇨ Data not collected systematically across the supply chain to make these decisions	Model automatically updates for on-going decision making ⇨ Developed process for automatically updating and maintaining the model so the decisions can be made on an on-going basis

Built 2 Models

1. Model for all the base products
2. Model for low volume SKU's

Results

- Savings
  - Identified immediate low volume SKU moves
  - Identified $4-$10M in savings for moving base products
  - Identified negotiation opportunities for raw materials

- Details
  - Moved 20% more volume into the high cost plant
  - 80% of savings were from 10% of the production moves

- Implementation
  - Implementation done in phases, starting with the easiest and highest value changes first
  - Expect 3-5 months to complete analysis, another 3-6 months to implement
  - Expect to adjust plans as you go forward

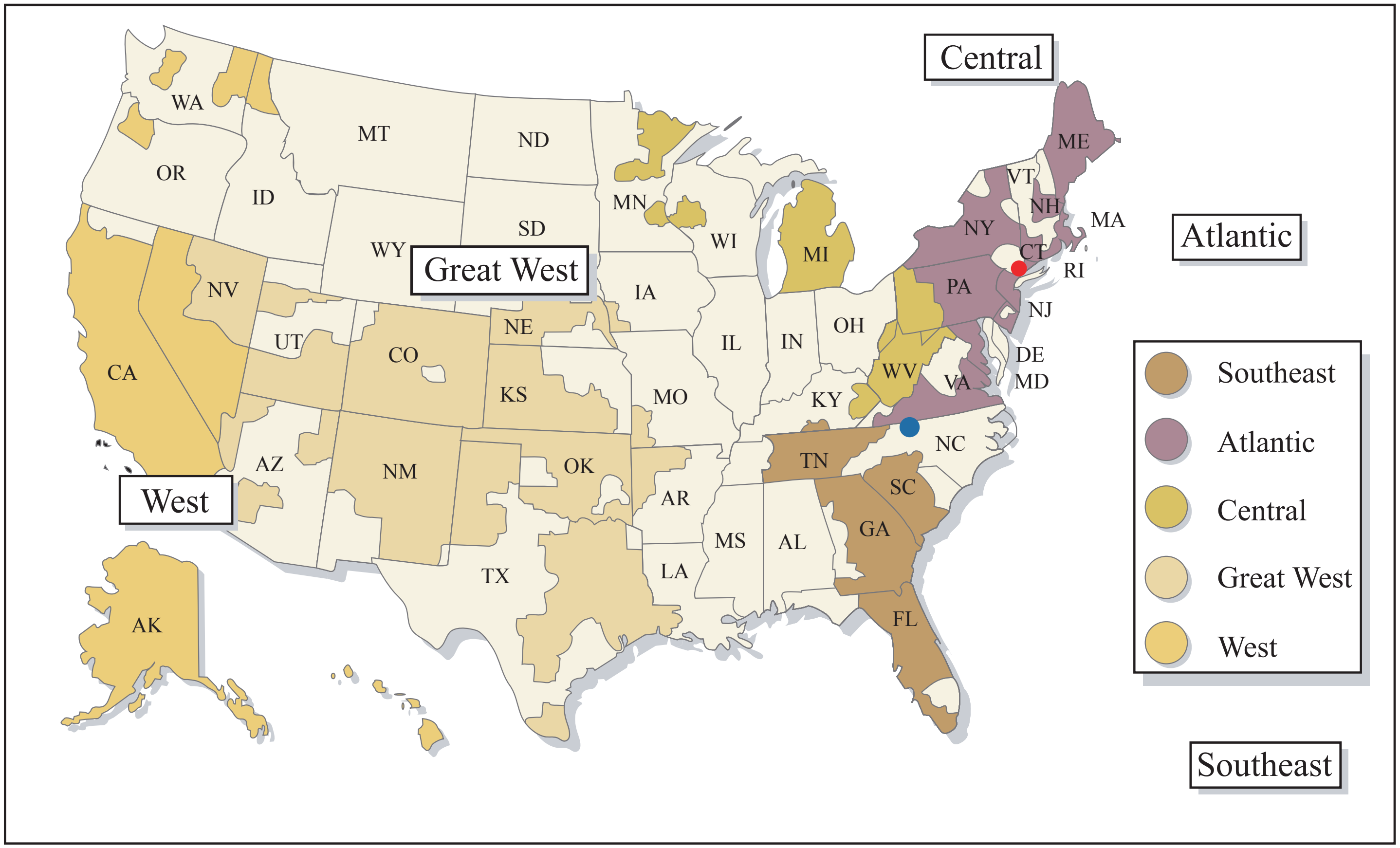
More Volume to High Cost Plant

- Baseline
  - Product A: 20% of the volume, 45% of the variable cost
  - Product B: 80% of the volume, 55% of the variable cost
- Optimization
  - Product A: 5% of the volume, 15% of the variable cost
  - Product B: 95% of the volume, 85% of the variable cost
- Net change was an increase in total volume

Case Study 3: Optimizing S&OP at PBG

Make	Sell	Deliver	Service
PBG Operates 57 Plants in the U.S. and 103 Plants Worldwide	Over 125 Million 8 oz. Servings are Enjoyed by Pepsi Customers Each Day!	240,000 Miles are Logged Every Day to Meet the Needs of Our Customers	Strong Customer Service Culture Identified as “Customer Connect”

PBG Structure - The PBG Territory



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- PBG accounts for 58% of the domestic Pepsi Volume...the other 42% is
- generated through a network of 96 Bottlers

- Each BUs act independently and meet local needs

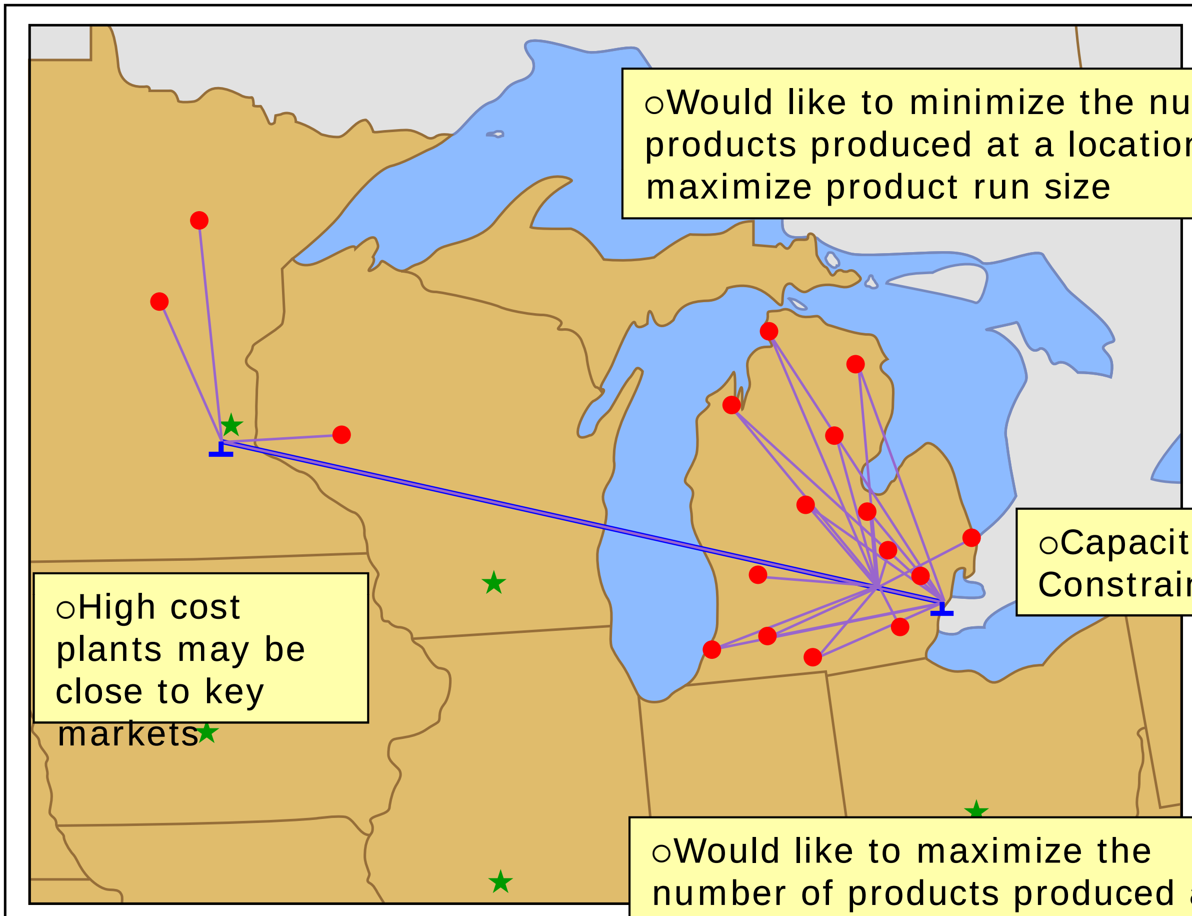
Challenges and Objective

- Problem: How should the firm source its products to minimize cost and maximize availability?

- Objective: Determine where products should be produced

Optimization Basics

- Tradeoffs associated with optimizing a network…



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Optimization Scenario

- Optimized Central BU model:

Total Cost

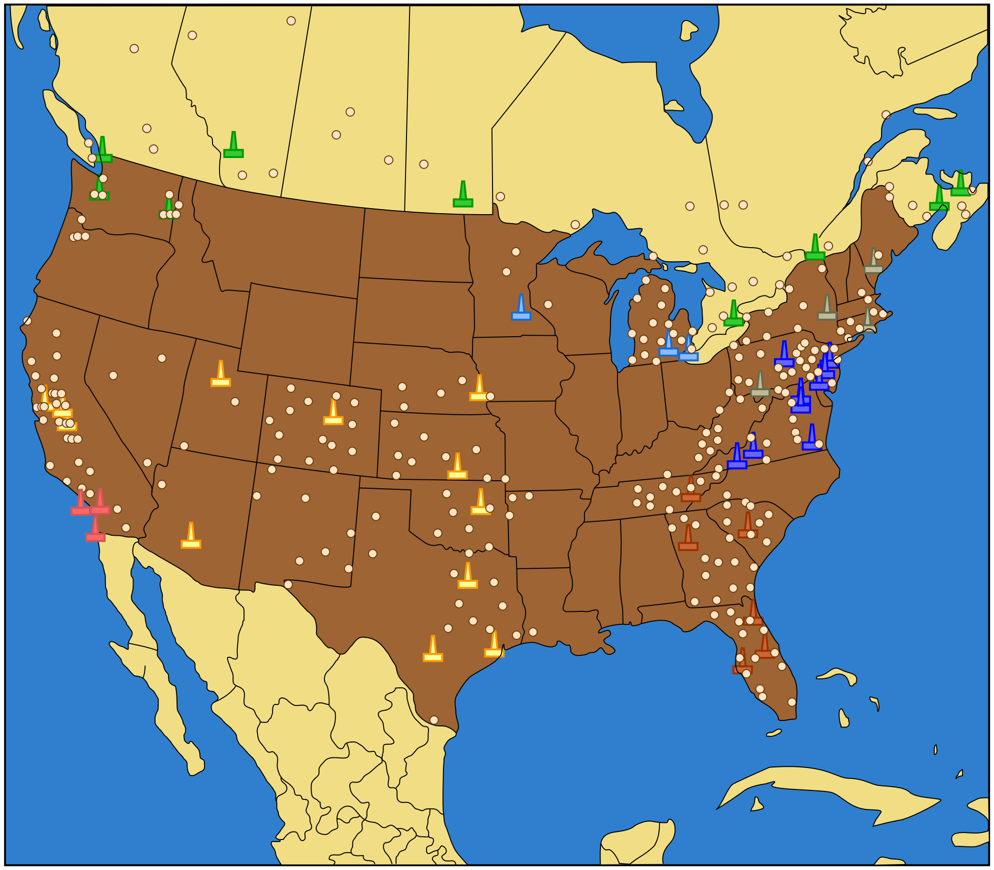
Category	Baseline	Optimized	Difference	% savings
MFG Cost	$ 2,610,361.00	$ 2,596,039.00	$ 14,322.00	0.6%
Trans Cost	$ 934,920.00	$ 857,829.00	$ 77,091.00	9.0%
Total Cost	$ 3,545,281.00	$ 3,453,868.00	$ 91,413.00	2.6%

Production Breakdown

	Units Produced		
Plant	Baseline	Optimized	% change
Burnsville	2,444,277.00	2,457,688.00	1%
Howell	3,509,708.00	3,828,727.50	8%
Detroit	2,637,253.00	2,304,822.50	-14%

Multi Stage Approach

- Stage 1-2: 2005-6 POC
  - 6 months
  - 2 Business Units
- Stage 3: 2007 Annual Operating Plan (AOP)
  - Model USA
  - Full year model
- Stage 4: Q1 – Q4 2007
  - Quarter based model
  - Package / Category



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Impact

- Creation of regular meetings bringing together Supply chain, Transport, Finance, Sales and Manufacturing functions to discuss sourcing and pre-build strategies
- Reduction in raw material and supplies inventory from $201 to $195 million
- A 2 percentage point decline in in growth of transport miles even as revenue grew
- An additional 12.3 million cases available to be sold due to reduction in warehouse out-of-stock levels

To put the last result in perspective, the reduction in warehouse out-of-stock levels effectively added one and a half production lines worth of capacity to the firm's supply chain without any capital expenditure.

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Assumptions

- Finite horizon: T periods;
- Varying demands: d_t, t=1,…,T;
- Linear ordering cost: $K_t\delta(y_t)+c_t y_t$;
- Linear holding cost: h_t ;
- Inventory level at the end of period t, I_t
- No shortage;
- Zero lead time;
- Sequence of Events: Review, Place Order, Order Arrives, Demand is Realized

Wagner-Whitin (W-W) Model

$$\begin{aligned}
\text{min} \quad & \sum_{t=1}^{T} [K_t \delta(y_t) + c_t y_t] + \sum_{t=1}^{T} h_t I_t \\
\text{s.t.} \quad & I_t = I_{t-1} + y_t - d_t, t = 1, 2, \ldots, T \\
& I_0 = 0 \\
& I_t, y_t \geq 0, t = 1, 2, \ldots, T.
\end{aligned}$$

Zero Inventory Ordering Policy (ZIO)

- Any optimal policy is a ZIO policy, that is,

$$I_{t-1} \cdot y_t = 0, \text{ for } t = 1, \ldots, T.$$

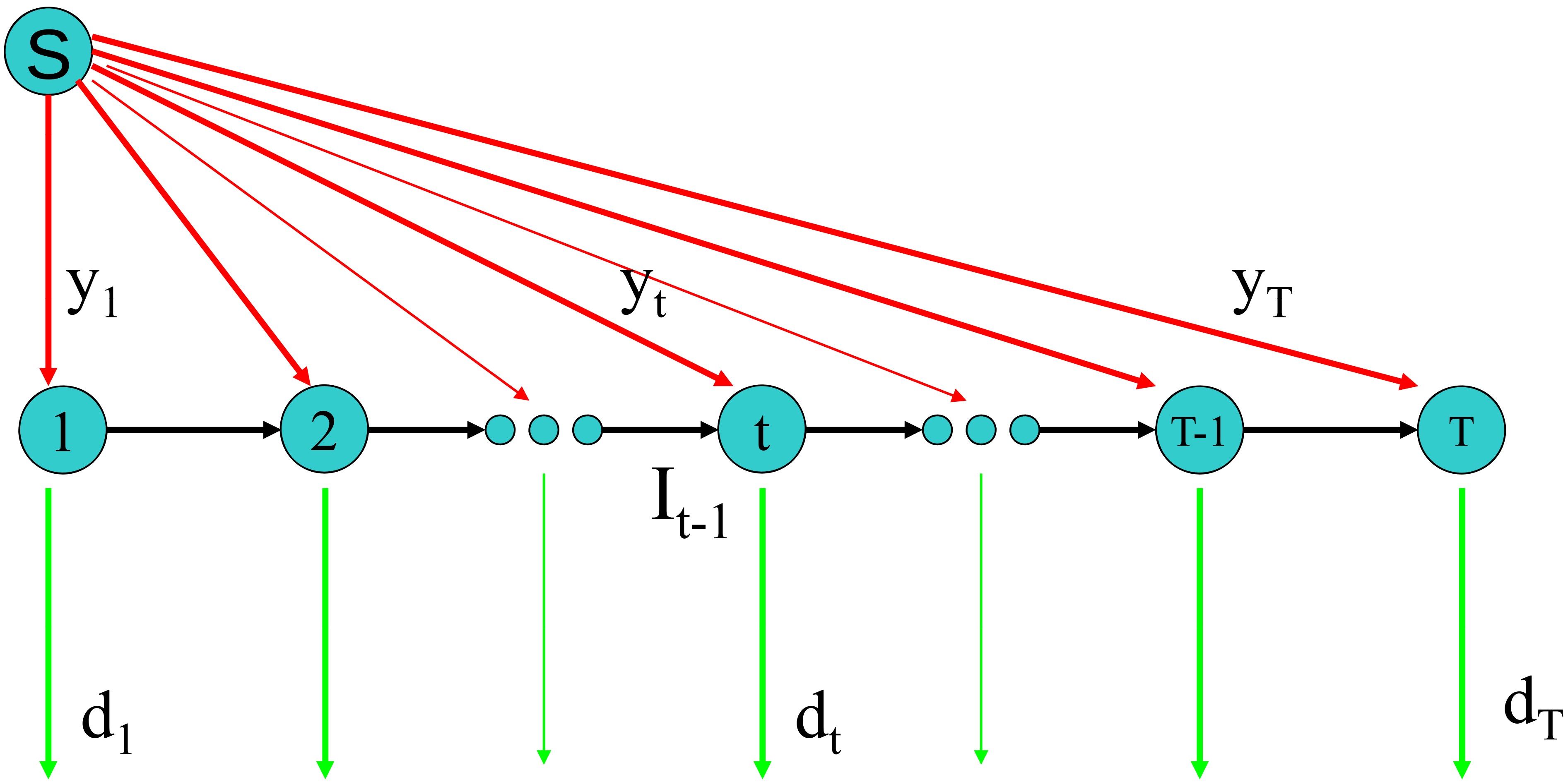
- Time independent costs: c, h.
- Time dependent costs: c_t, h_t.

ZIO Policy ↔ Extreme Point

- **Definition:** Given a polyhedron **P**, A vector ***x*** is an extreme point if we cannot find two other vectors ***y***, ***z*** in **P**, and a scalar $\lambda, 0 \leq \lambda \leq 1$, such that $\boldsymbol{x} = \lambda \boldsymbol{y} + (1-\lambda)\mathbf{z}$.

- **Theorem:** Consider the linear programming problem of minimizing ***c'x*** over a polyhedron *P*, then either the optimal cost is equal to $-\infty$, or there exists **an extreme point** which **is optimal**.

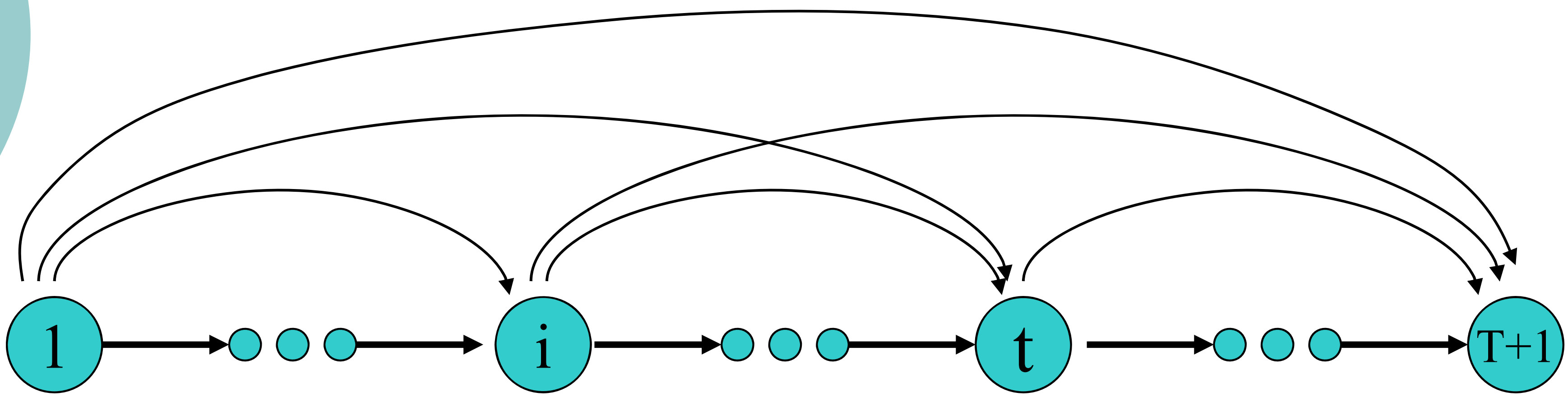
Min-Cost Flow Problem



Implication of ZIO

- Each order covers exactly the demands of several consecutive periods.
- Order times sufficient to decide on order quantities.

Network Representation



$$l_{ij} = \begin{cases} K_i + c_i \sum_{t=i}^{j-1} d_t + \sum_{k=i}^{j-1} h_k \sum_{t=k}^{j-1} d_t, & 1 \le i < j \le T+1 \\ +\infty & o.w. \end{cases}$$

Shortest Path Algorithm

- Let $V(i)$ be the cost-to-go starting from period i with zero initial inventory level. Then

$$V(i) = \min_{i<t\leq T+1} l_{it} + V(t), i = 1, 2, \ldots, T,$$

where $V(T+1) = 0$

- Complexity of the shortest path algorithm: $O(T^2)$.
- With a sophisticated algorithm, $O(T\ lnT)$.

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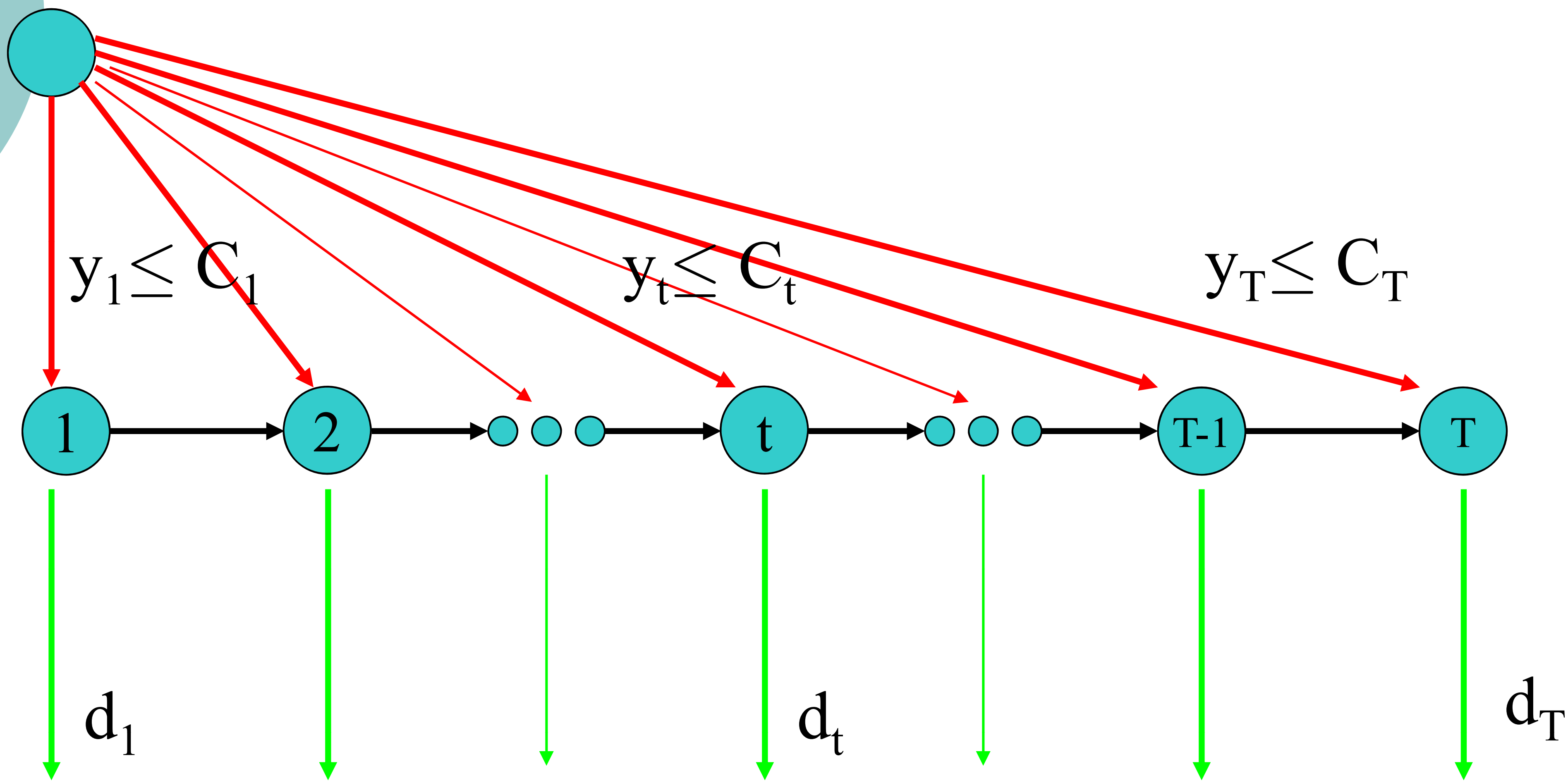
DELS with Capacity Constraints

$$
\begin{aligned}
\text{min} \quad & \textstyle\sum_{t=1}^{T} [K_t \delta(y_t) + c_t y_t] + \sum_{t=1}^{T} h_t I_t \\
\text{s.t.} \quad & I_t = I_{t-1} + y_t - d_t, t = 1, 2, \ldots, T \\
& I_0 = 0 \\
& I_t, y_t \geq 0, t = 1, 2, \ldots, T, \\
& y_t \leq C_t, t = 1, 2, \ldots, T.
\end{aligned}
$$

DELS model with capacity: description



Feasibility of DELS with capacity

For DELS model with capacity, a feasible solution exists if and only if

$$\sum_{j=1}^{i} C_j \geq \sum_{j=1}^{i} d_j, \text{ for } i = 1, 2, \ldots, T.$$

Inventory Decomposition Property

Theorem Suppose that the constraint

$$I_k = 0, \text{ for some } k \in \{1, \ldots, k-1\}$$

is added to DELS problem and

$$\sum_{j=k+1}^{i} C_j \geq \sum_{j=k+1}^{i} d_j, \text{ for } i = k+1, \ldots, T.$$

holds. Then an optimal solution to the original problem can be found by independently finding solutions to te problems for the first k periods and for the last $T-k$ periods.

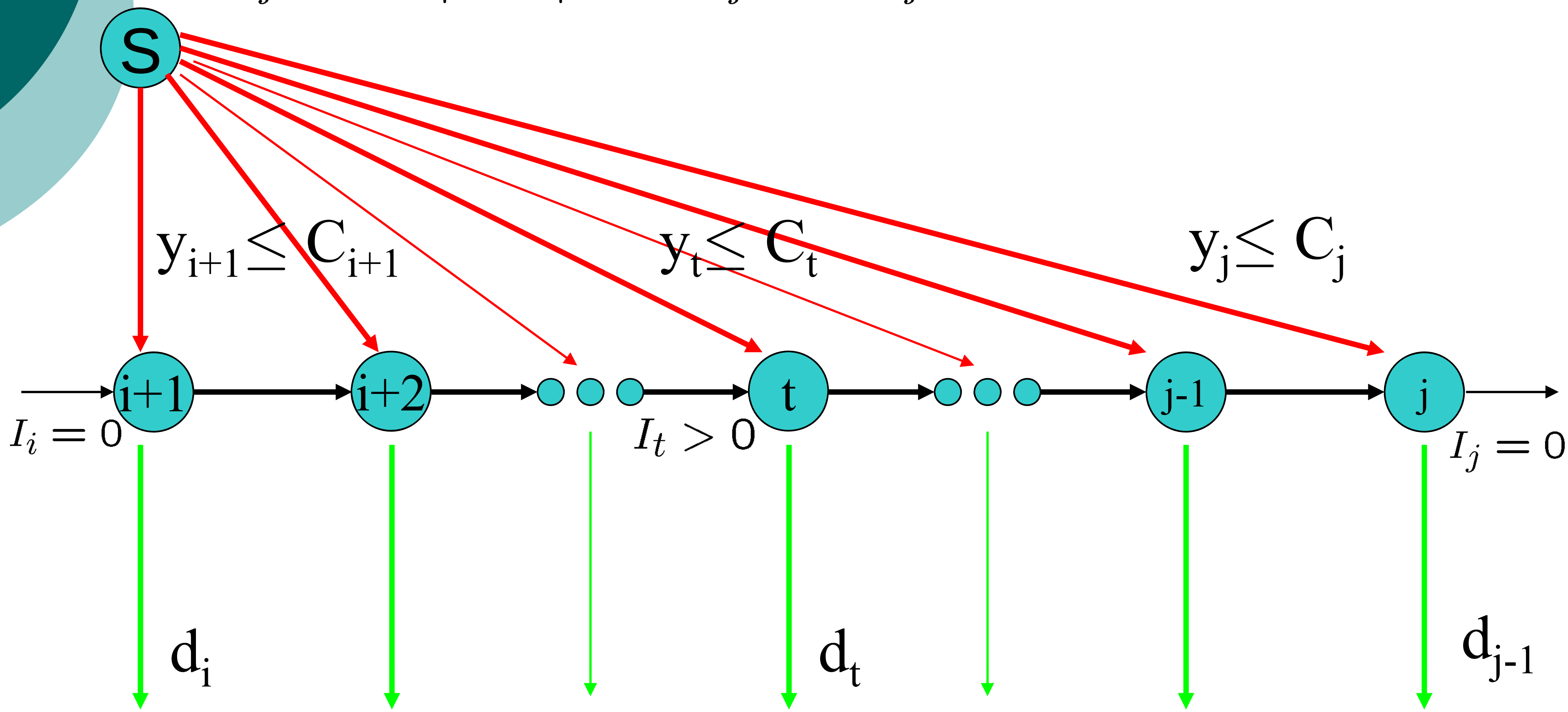
Structure of Optimal Policy

Define a production sequence S_{ij} to be capacity constrained if the production level in at most one period k $(i+1 \leq k \leq j)$ satisfies $0 < y_k < C_k$ and all other production levels are either zero or at their capacities.

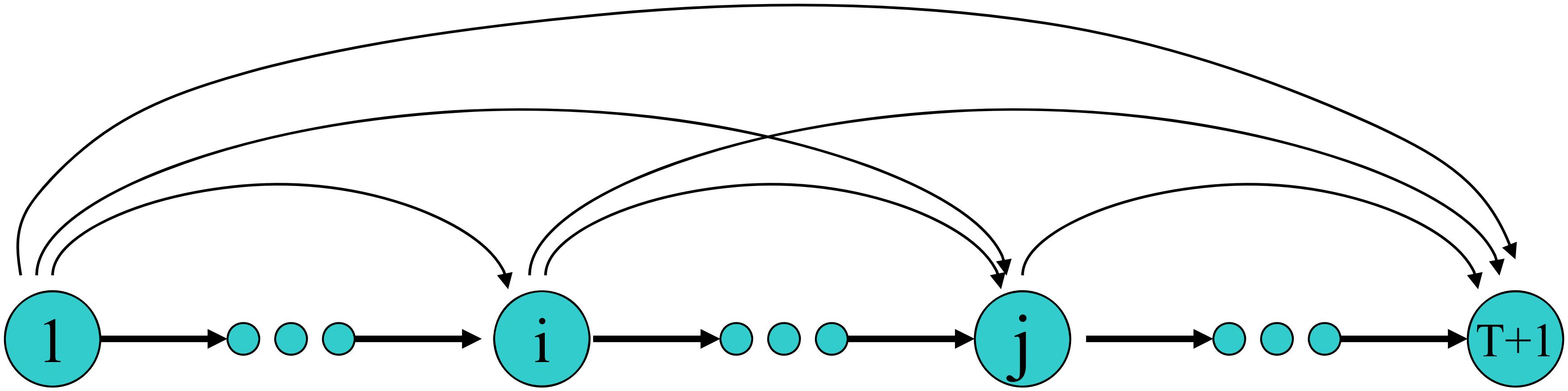
Theorem There exists an optimal solution which consists of capacity constrained production sequences only.

Production Sequence S_{ij}

$$S_{ij} = \{(y_{i+1}, y_{i+2}, \ldots, y_j) | I_i = I_j = 0, I_k > 0, \text{ for } i < k < j\}$$



Network Representation

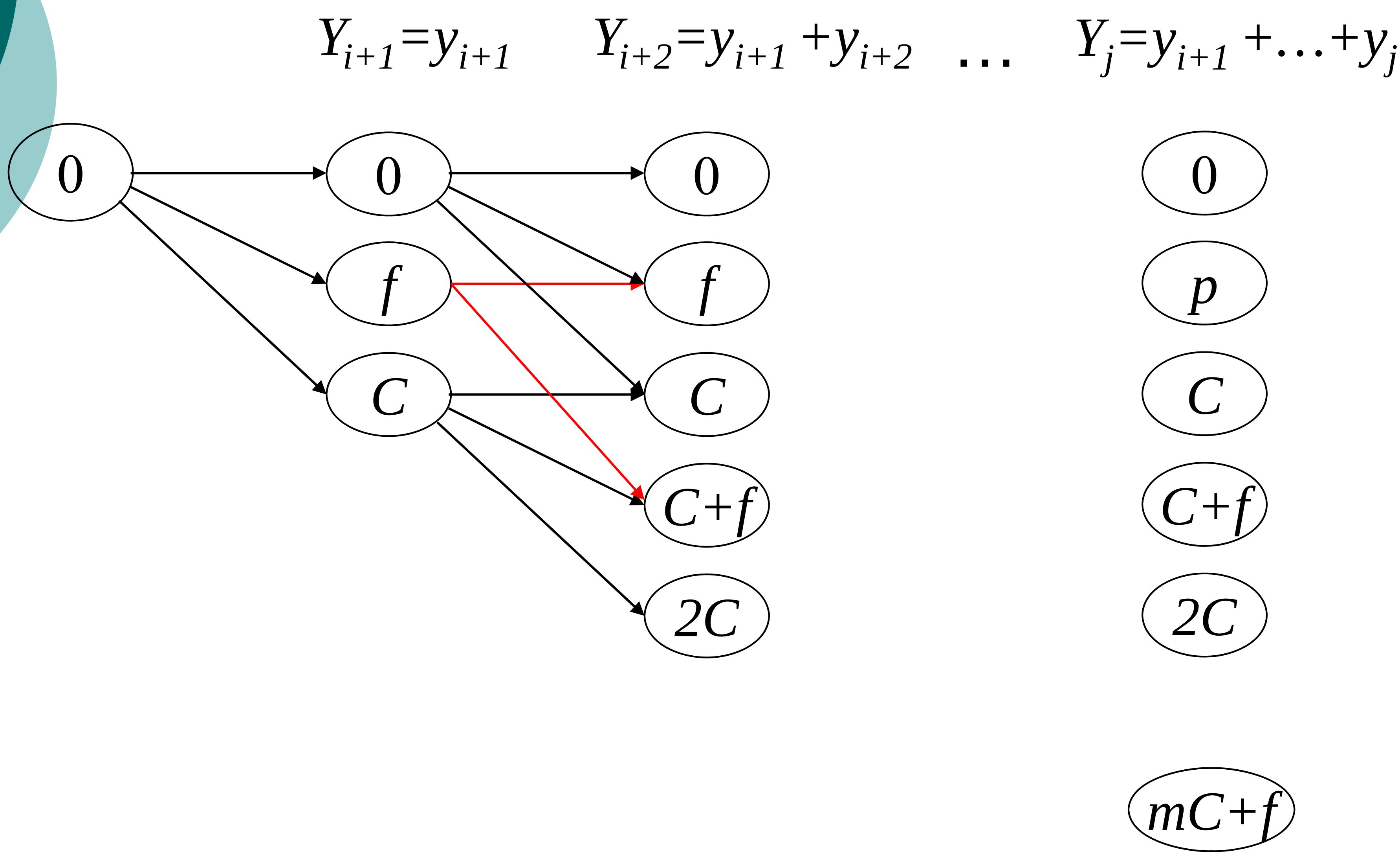


$$l_{ij}\,?$$

Calculation of Link Costs

- Time-dependent capacities: difficult;
- Time-independent capacities: C_t=C.
- Let m and f such that $mC+f = d_{i+1}+\ldots+{}_j d_t$, where m is a nonnegative integer and $0 \le f < C$, define $Y_k = \sum_{t=i+1}^{k} y_t, i < k \le j.$
  then $Y_k \in \{0, f, C, C+f, 2C, \ldots, mC+f\}.$

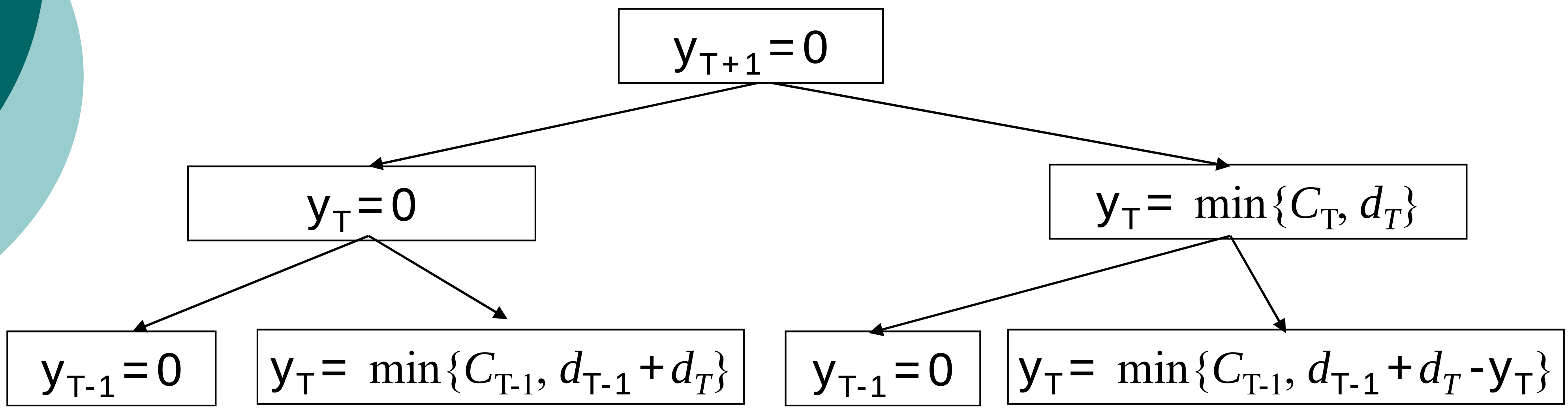
Calculation of Link Costs



Complexity: equal capacity

- Computing link cost l_{ij}: shortest path algorithm:$O((j\text{-}i)^2)$.
- Determining the optimal production sequence between all pairs of periods: $O(T^2) \times O(T^2) = O(T^4)$.
- Shortest path algorithm on the whole network : $O(T^2)$.
- Complexity for finding an optimal$_{so}$ lution for DELS model with equal capacity: $O(T^4) + O(T^2) = O(T^4)$.
- With a sophisticated algorithm: $O(T^3)$.
- Not applicable to problems with time-dependent capacity constraints, why?

Tree-Search Method



- Effective with time-dependent capacity constraints.
- Not polynomial.

ZICO Policy

- **Theorem:** Any optimal policy is a ZICO policy,

$$I_{t-1} \cdot (C_t - y_t) \cdot y_t = 0, \text{ for } t=1,\ldots,T.$$

- **Corollary:** If $(y_1, y_2, \ldots y_T)$ represents an optimal solution, and $t = \max\{j: y_j > 0\}$, then

$$y_t = \min\{C_t, d_t + \ldots + d_T\}.$$

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