Introduction to Stochastic Inventory Models and Supply Contracts

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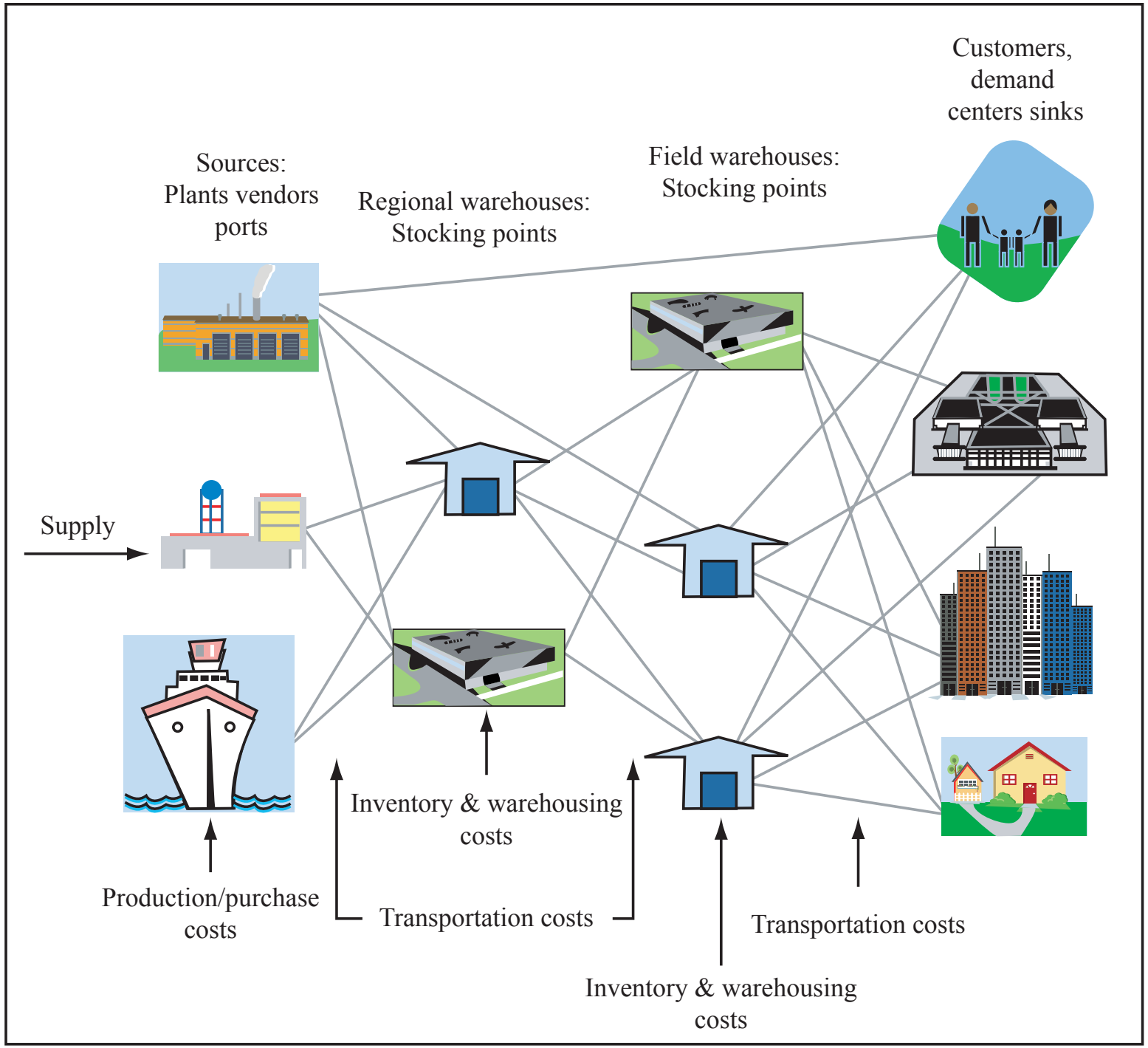
Outline of the Presentation

- **Introduction**
- **The Effect of Demand Uncertainty**
  - **(s,S) Policy**
  - **Supply Contracts**
  - **Risk Pooling**
- **Practical Issues in Inventory Management**



Goals: *Reduce Cost, Improve Service*

- By effectively managing inventory:
  - Xerox eliminated $700 million inventory from its supply chain
  - Wal-Mart became the largest retail company utilizing efficient inventory management
  - GM has reduced parts inventory and transportation costs by 26% annually

Goals:
Reduce Cost, Improve Service

- By not managing inventory successfully
  - In 1994, “IBM continues to struggle with shortages in their ThinkPad line” (WSJ, Oct 7, 1994)
  - In 1993, “Liz Claiborne said its unexpected earning decline is the consequence of higher than anticipated excess inventory” (WSJ, July 15, 1993)
  - In 1993, “Dell Computers predicts a loss; Stock plunges. Dell acknowledged that the company was sharply off in its forecast of demand, resulting in inventory write downs” (WSJ, August 1993)

Understanding Inventory

- The inventory policy is affected by:
  - Demand Characteristics
  - Lead Time
  - Number of Products
  - Objectives
    - Service level
    - Minimize costs
  - Cost Structure

The Effect of Demand Uncertainty

- Most companies treat the world as if it were predictable:
  - Production and inventory planning are based on forecasts of demand made far in advance of the selling season
  - Companies are aware of demand uncertainty when they create a forecast, but they design their planning process as if the forecast truly represents reality

Demand Forecast

- **The three principles of all forecasting techniques:**
  - Forecasting is always wrong
  - The longer the forecast horizon the worst is the forecast
  - Aggregate forecasts are more accurate

The Effect of Demand Uncertainty

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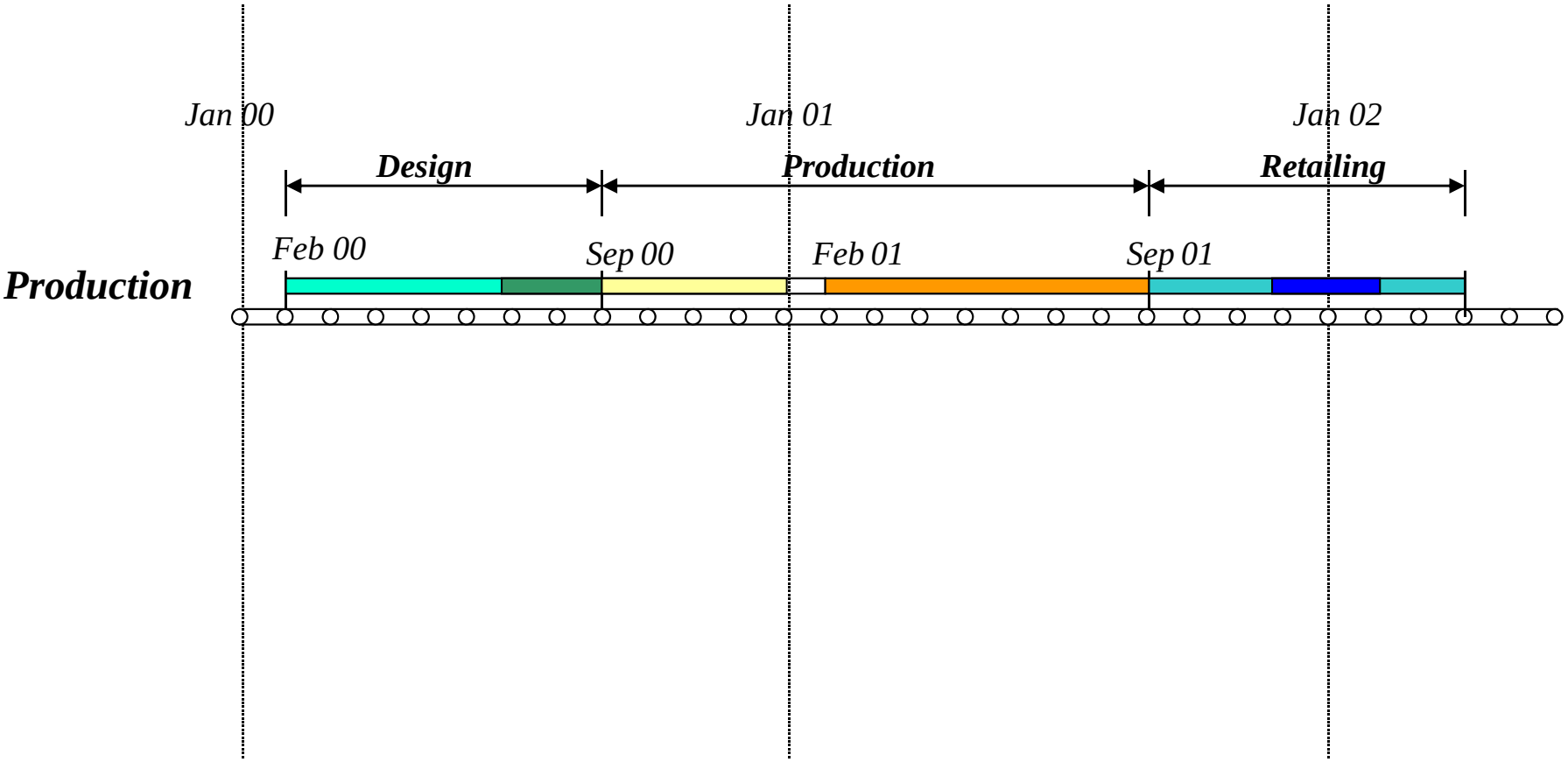
  - Production and inventory planning are based on forecasts of demand made far in advance of the selling season
  - Companies are aware of demand uncertainty when they create a forecast, but they design their planning process as if the forecast truly represents reality
- Recent technological advances have increased the level of demand uncertainty:
  - Short product life cycles
  - Increasing product variety

SnowTime Sporting Goods

- Fashion items have short life cycles, high variety of competitors
- SnowTime Sporting Goods
  - New designs are completed
  - One production opportunity
  - Based on past sales, knowledge of the industry, and economic conditions, the marketing department has a probabilistic forecast
  - The forecast averages about 13,000, but there is a chance that demand will be greater or less than this.

Supply Chain Time Lines



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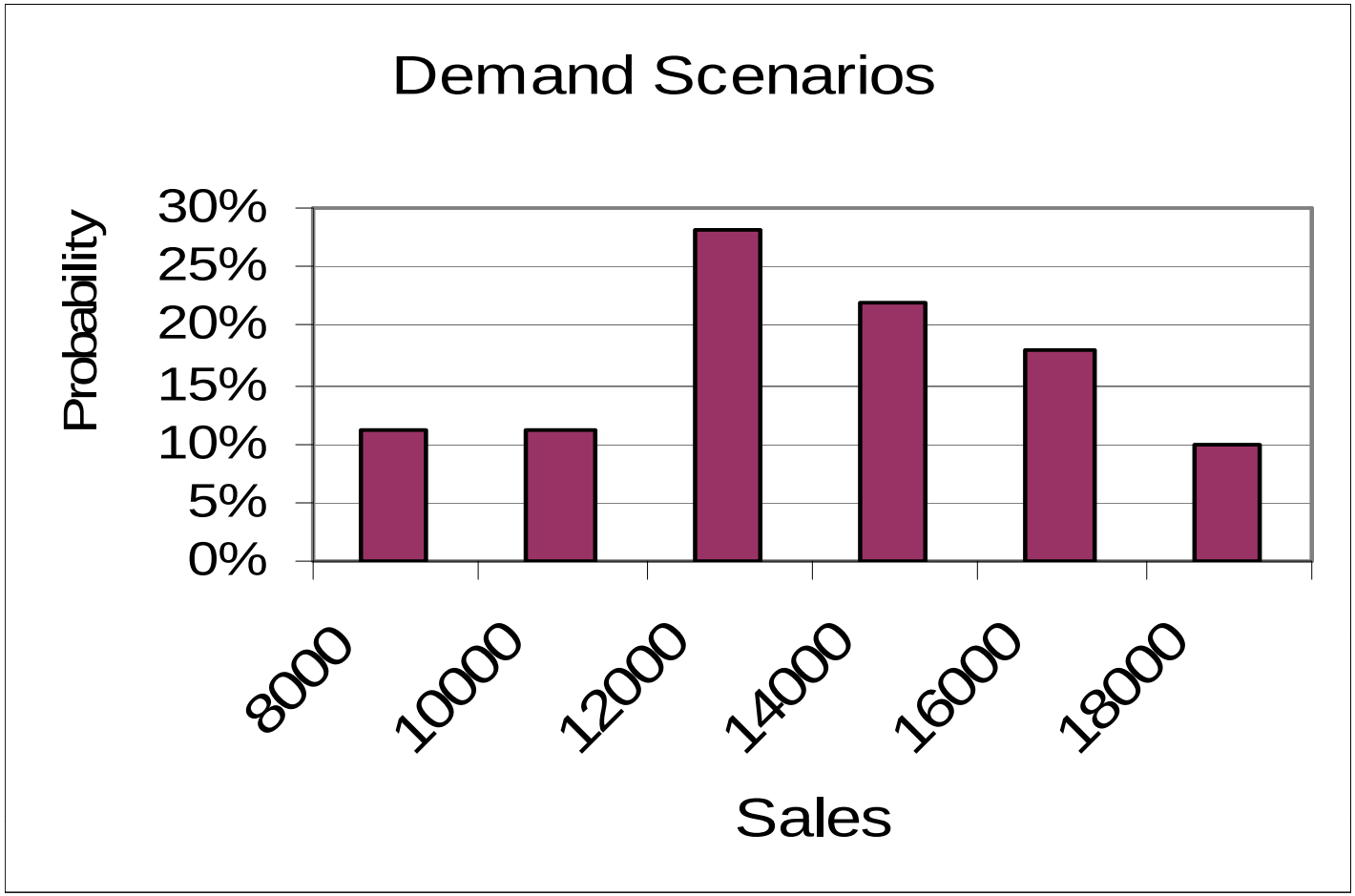
SnowTime Demand Scenarios



SnowTime Costs

- Production cost per unit (C): $80
- Selling price per unit (S): $125
- Salvage value per unit (V): $20
- Fixed production cost (F): $100,000
- Q is production quantity, D demand

- Profit =
  Revenue - Variable Cost - Fixed Cost + Salvage

SnowTime Best Solution

- Find order quantity that maximizes weighted average profit.
- Question: Will this quantity be less than, equal to, or greater than average demand?

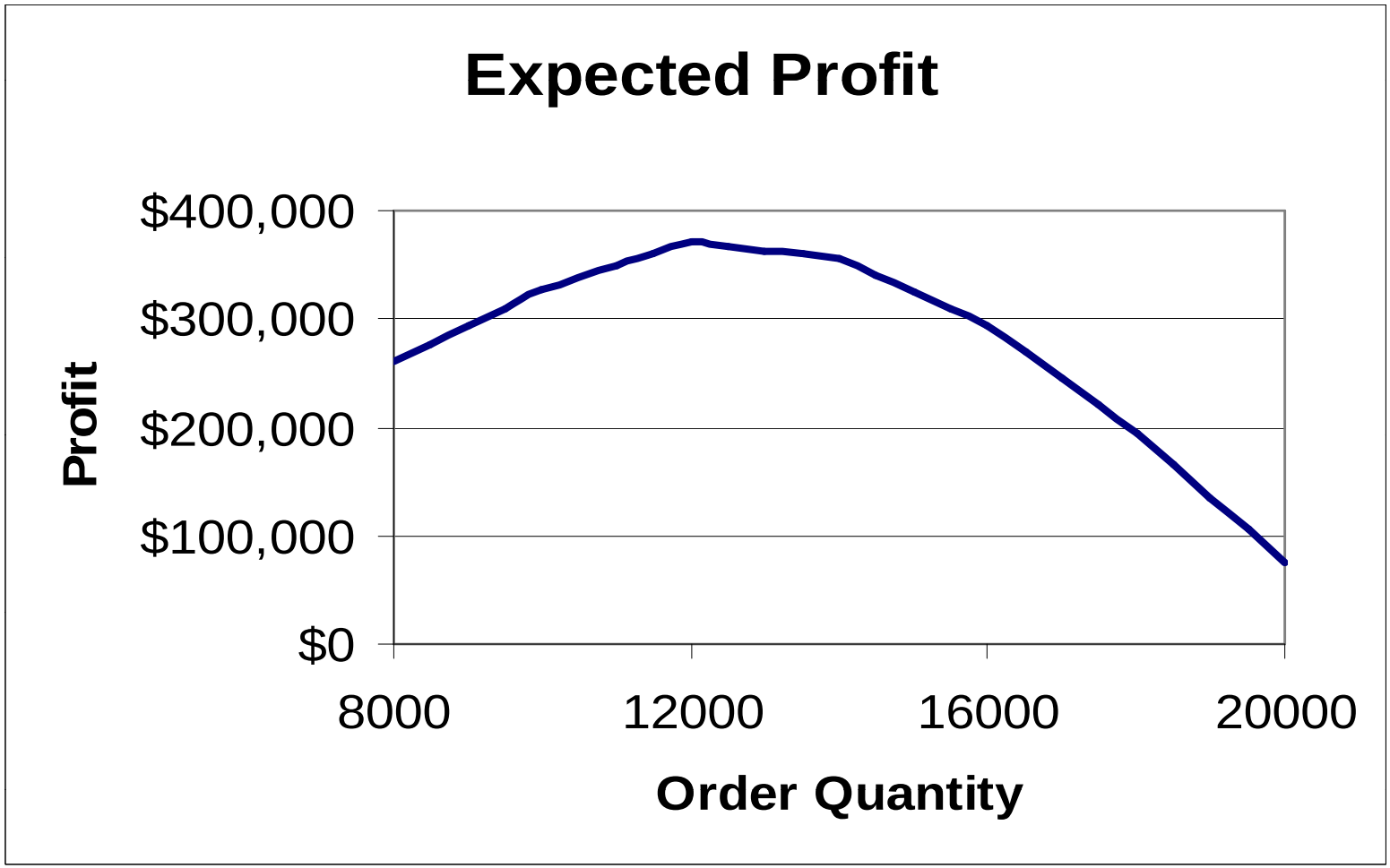
What to Make?

- Question: Will this quantity be less than, equal to, or greater than average demand?
- Average demand is 13,100
- Look at marginal cost Vs. marginal profit
  - if extra jacket sold, profit is 125-80 = 45
  - if not sold, cost is 80-20 = 60
- So we will make less than average

SnowTime Scenarios

- Scenario One:
  - Suppose you make 12,000 jackets and demand ends up being 13,000 jackets.
  - Profit = 125(12,000) - 80(12,000) - 100,000 = **$440,000**
- Scenario Two:
  - Suppose you make 12,000 jackets and demand ends up being 11,000 jackets.
  - Profit = 125(11,000) - 80(12,000) - 100,000 + 20(1000) = $ **335,000**

SnowTime Expected Profit



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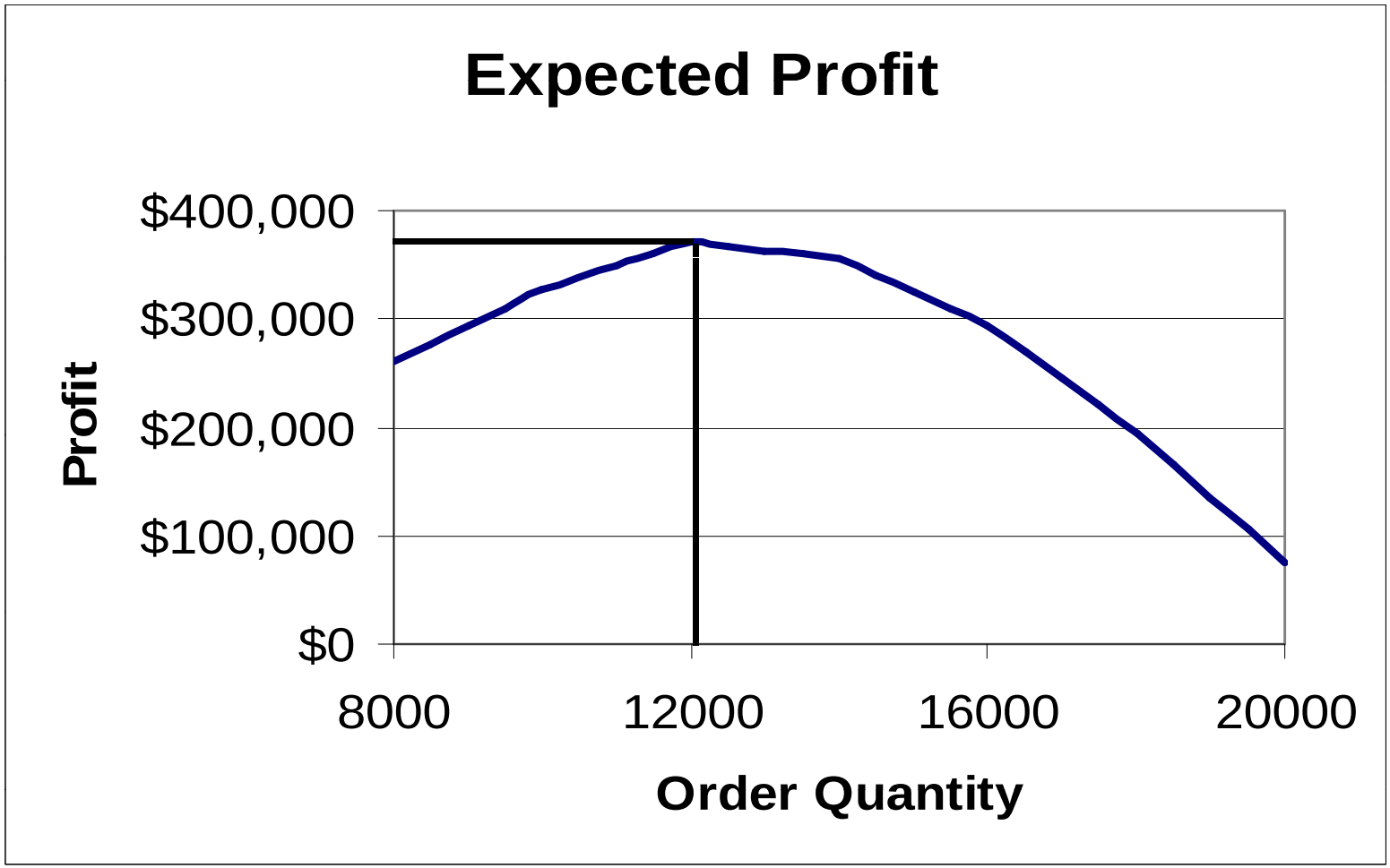

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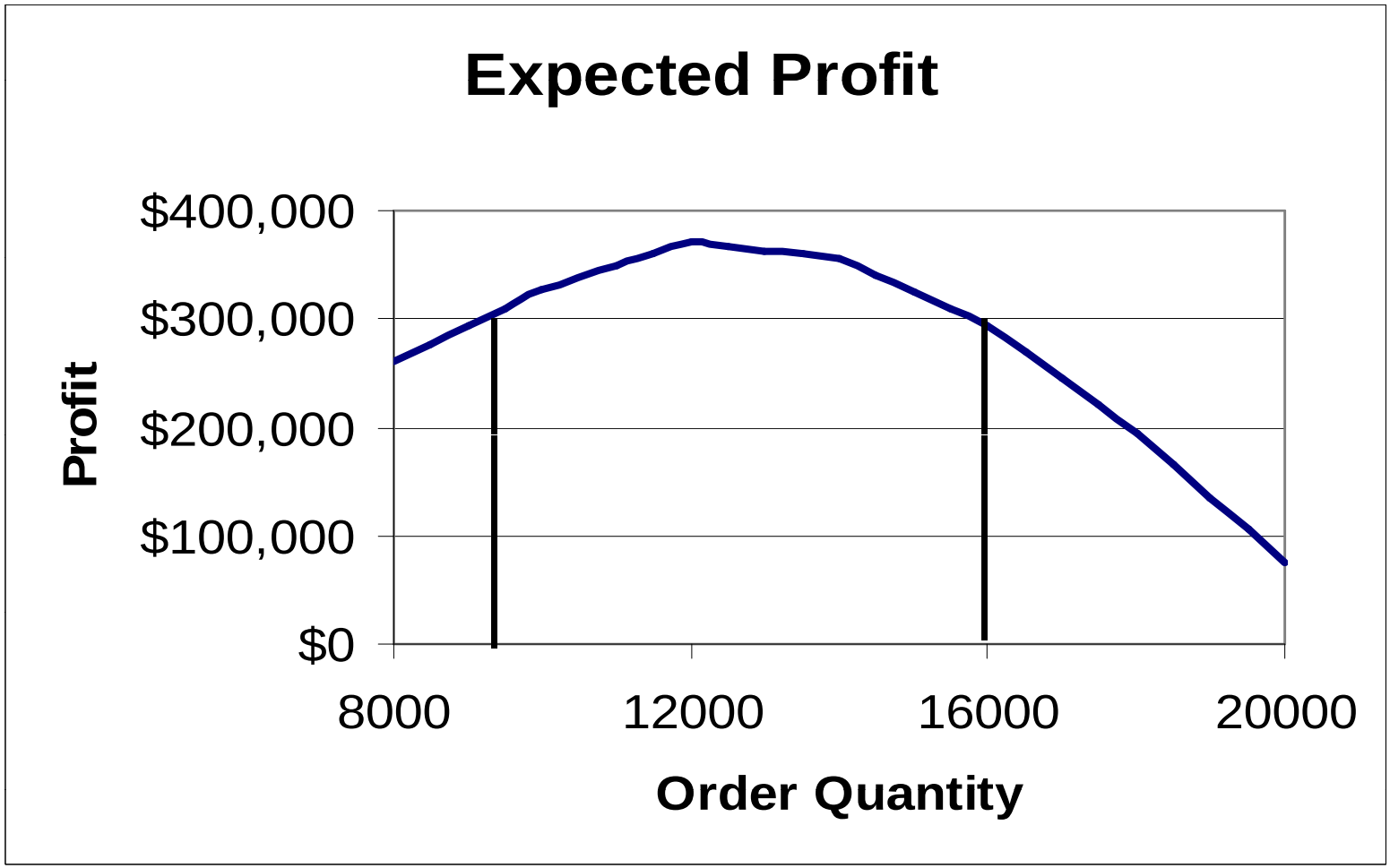
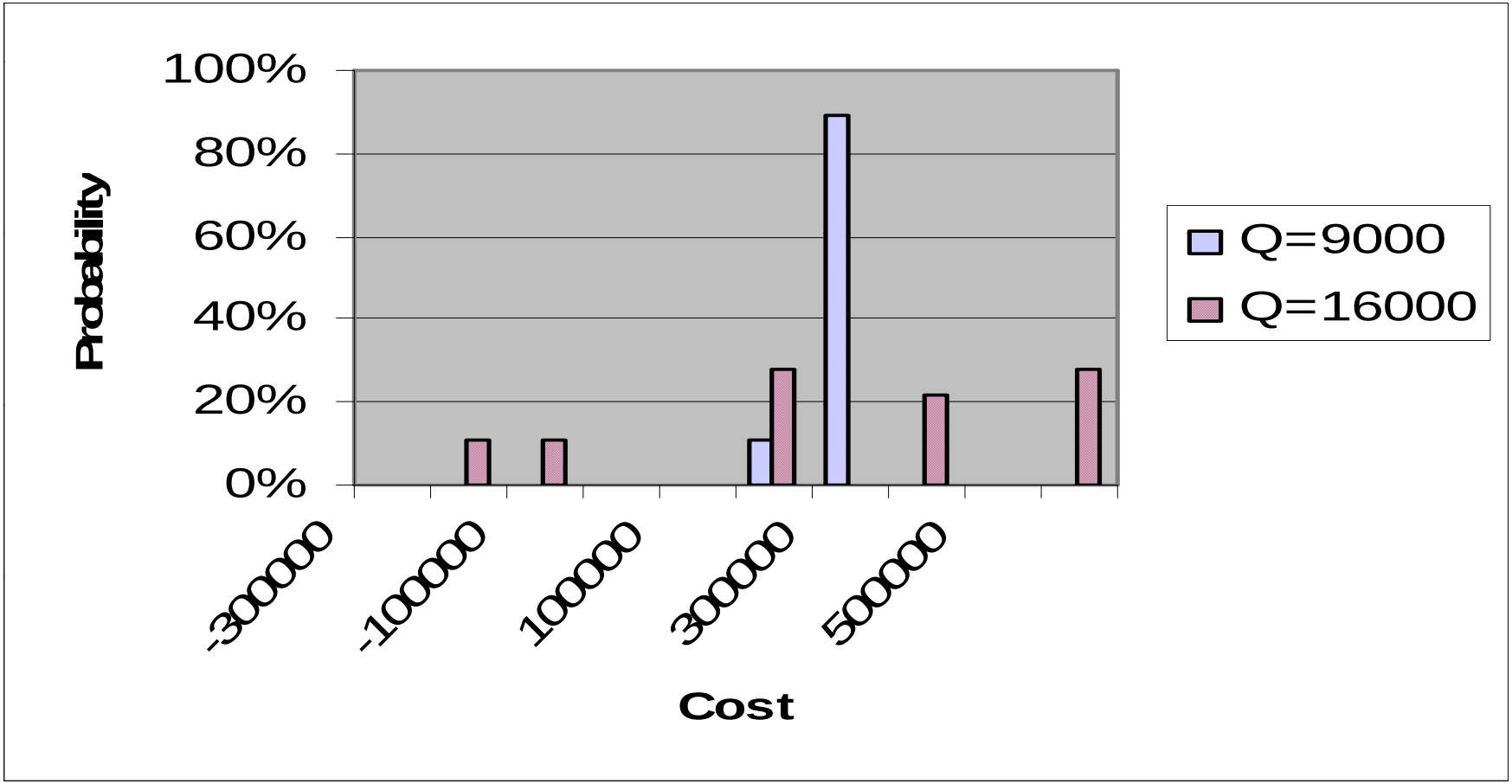


SnowTime: *Important Observations*

- Tradeoff between ordering enough to meet demand and ordering too much
- Several quantities have the same average profit
- Average profit does not tell the whole story

- Question: 9000 and 16000 units lead to about the same average profit, so which do we prefer?

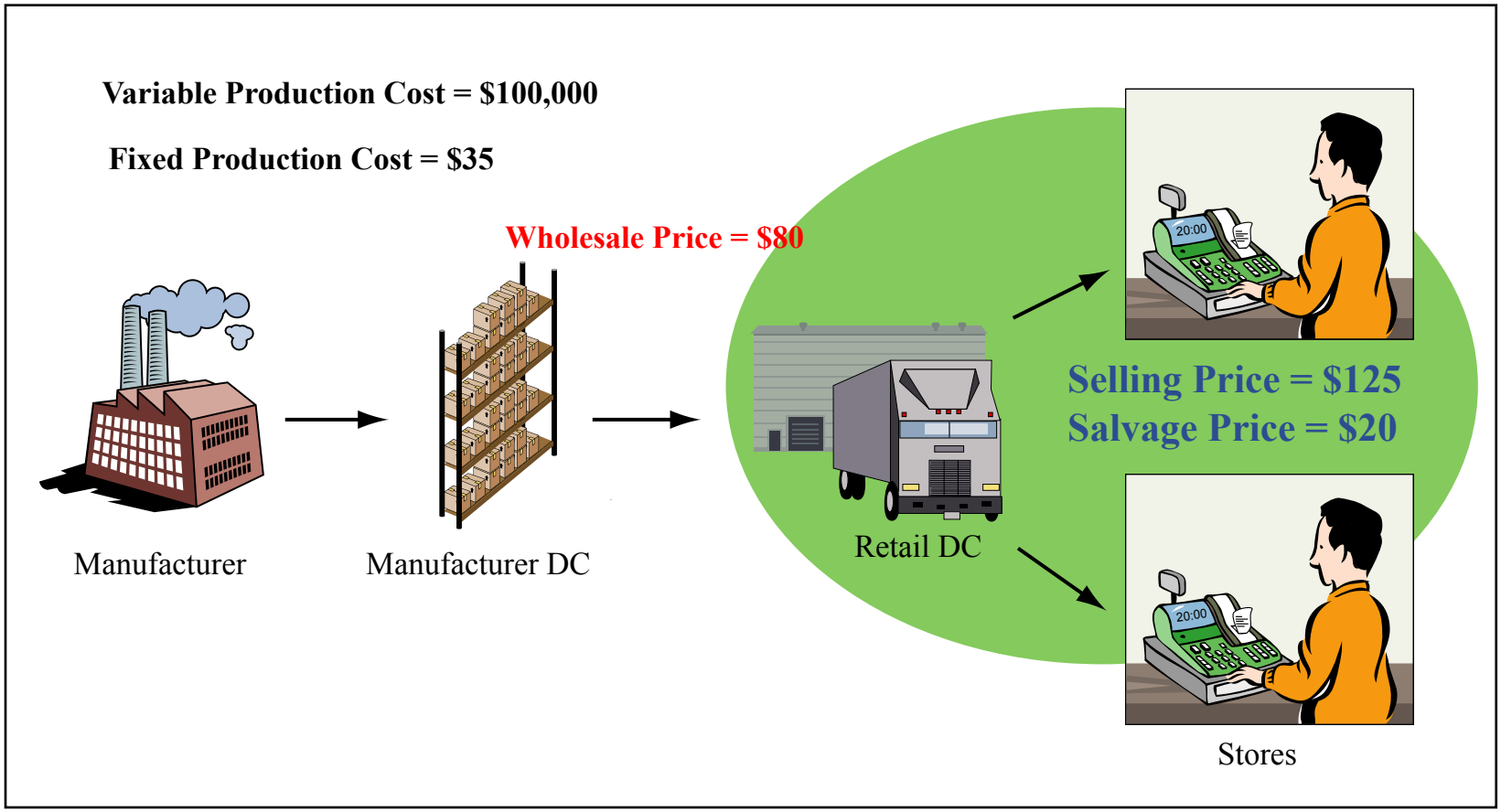
Probability of Outcomes



Key Insights from this Model

- The optimal order quantity is not necessarily equal to average forecast demand
- The optimal quantity depends on the relationship between marginal profit and marginal cost
- As order quantity increases, average profit first increases and then decreases
- As production quantity increases, risk increases. In other words, the probability of large gains ***and*** of large losses increases

Supply Contracts



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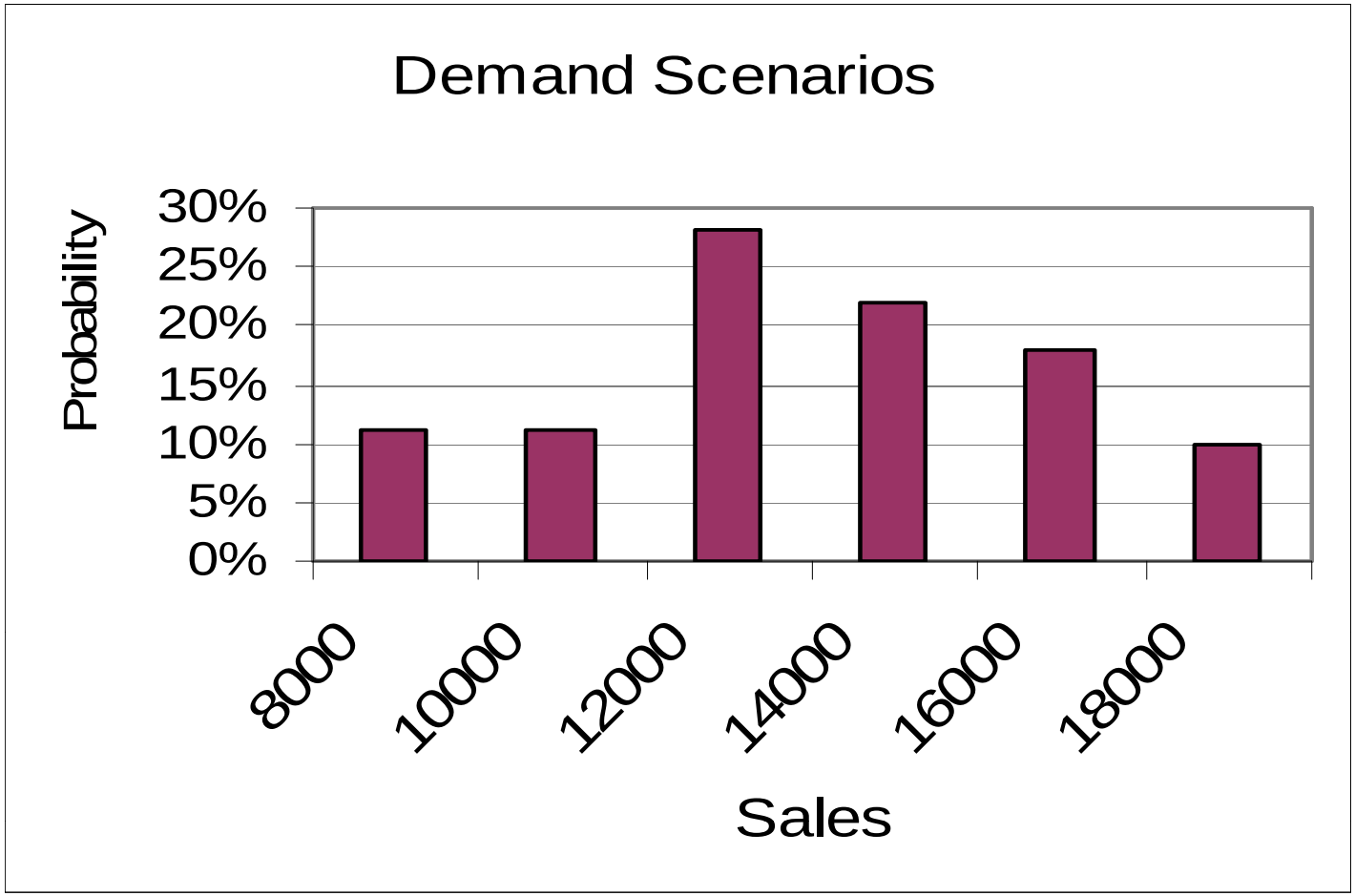
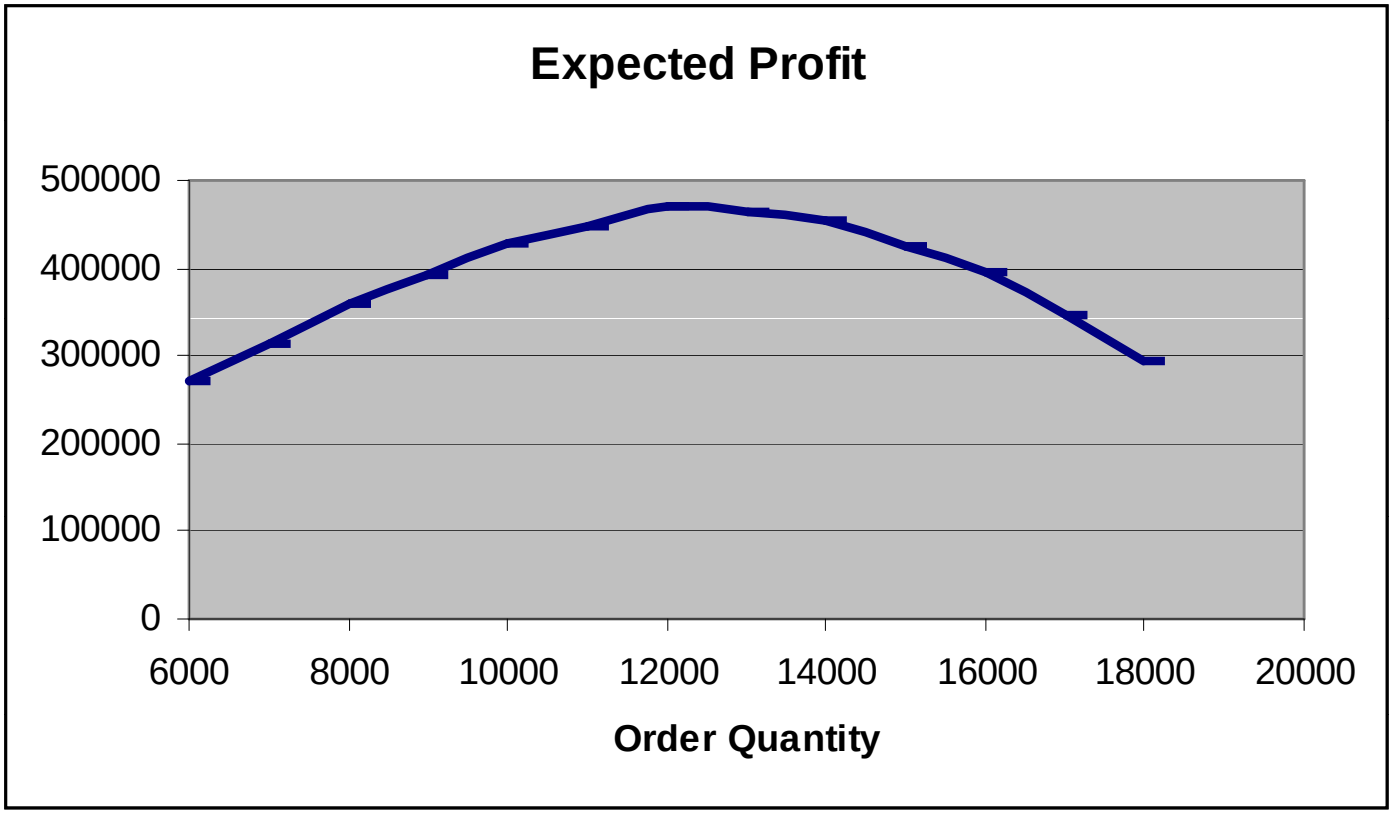
Demand Scenarios



Distributor Expected Profit



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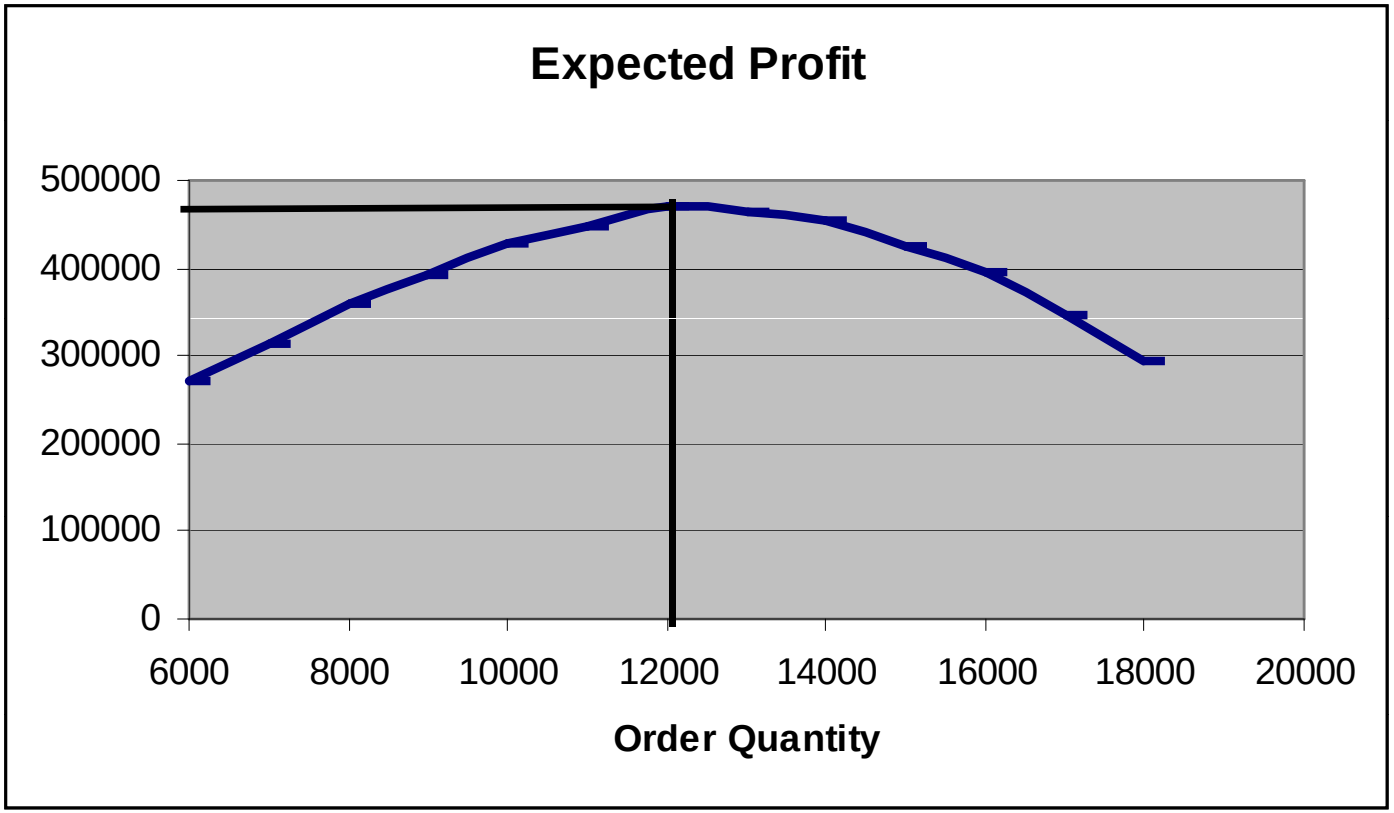
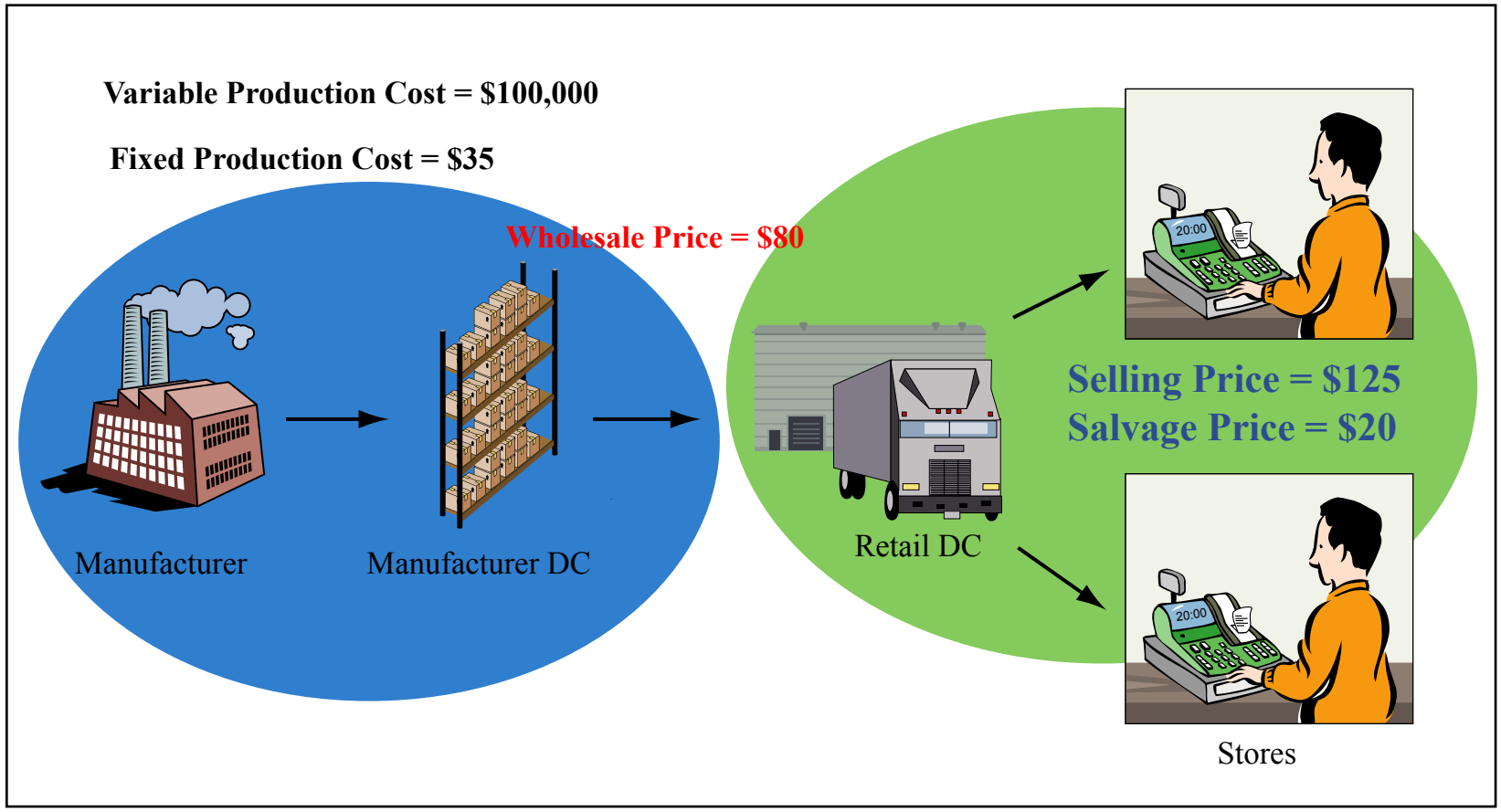


Supply Contracts (cont.)

- Distributor optimal order quantity is 12,000 units
- Distributor expected profit is $470,000
- Manufacturer profit is $440,000
- Supply Chain Profit is $910,000
  - **IS there anything that the distributor and manufacturer can do to increase the profit of both?**

Supply Contracts



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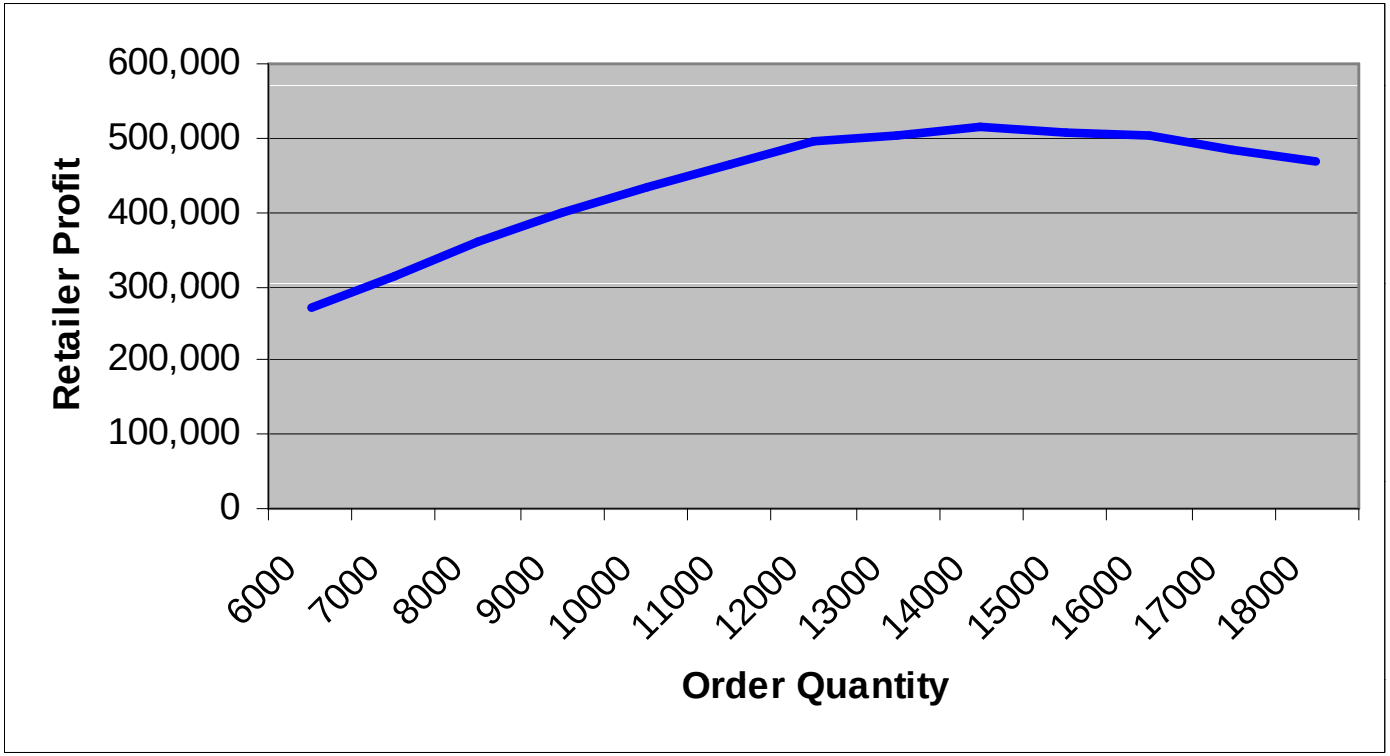
Retailer Profi.
(Buy Back=$55)



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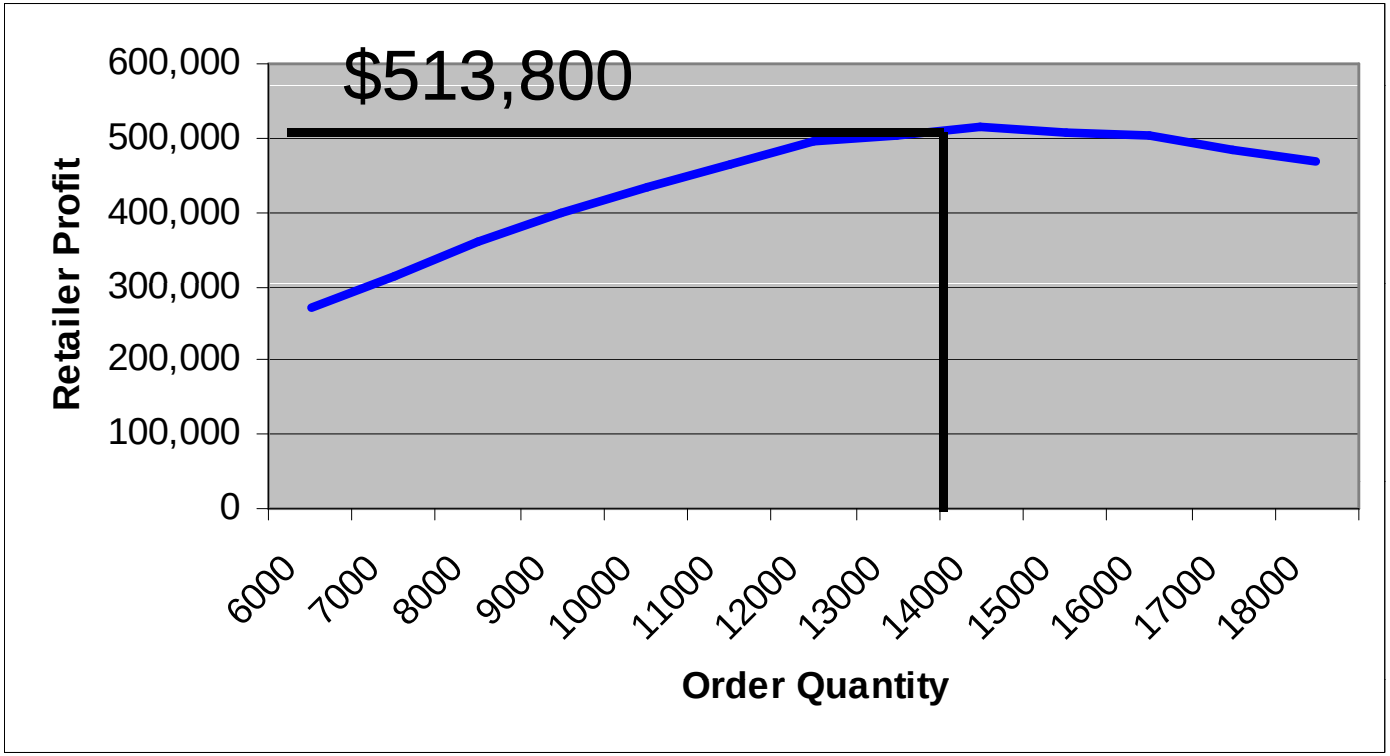

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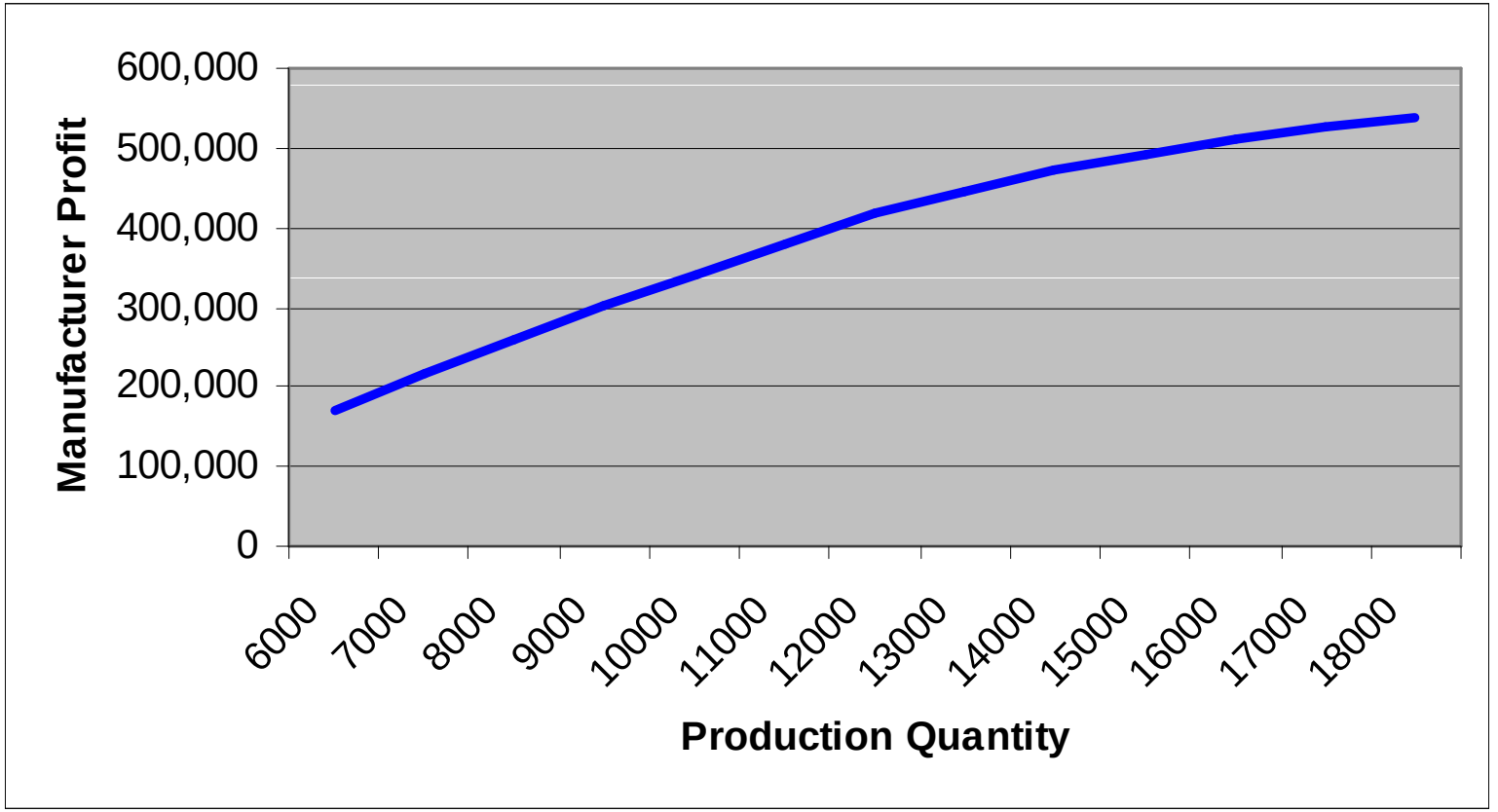

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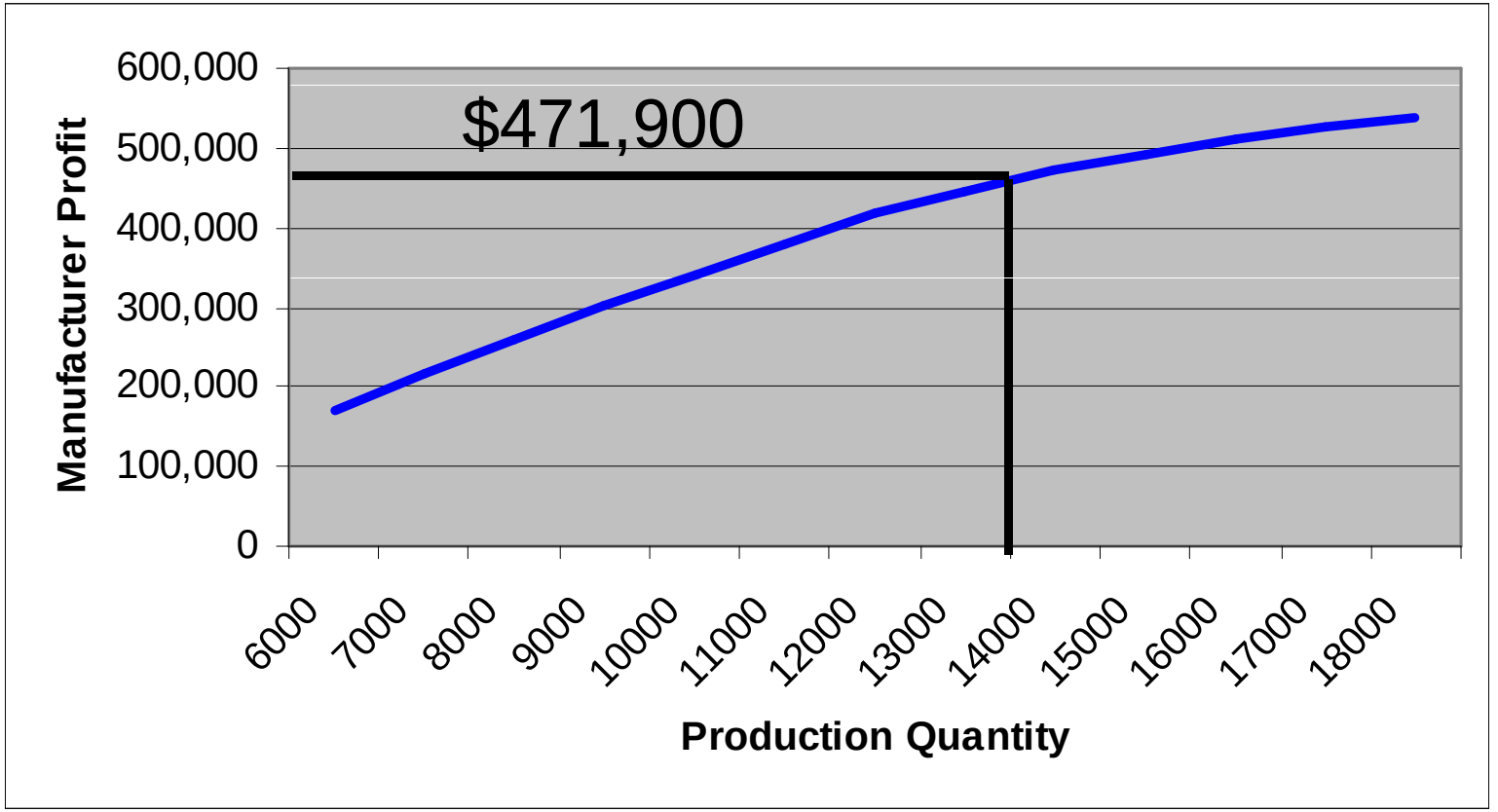

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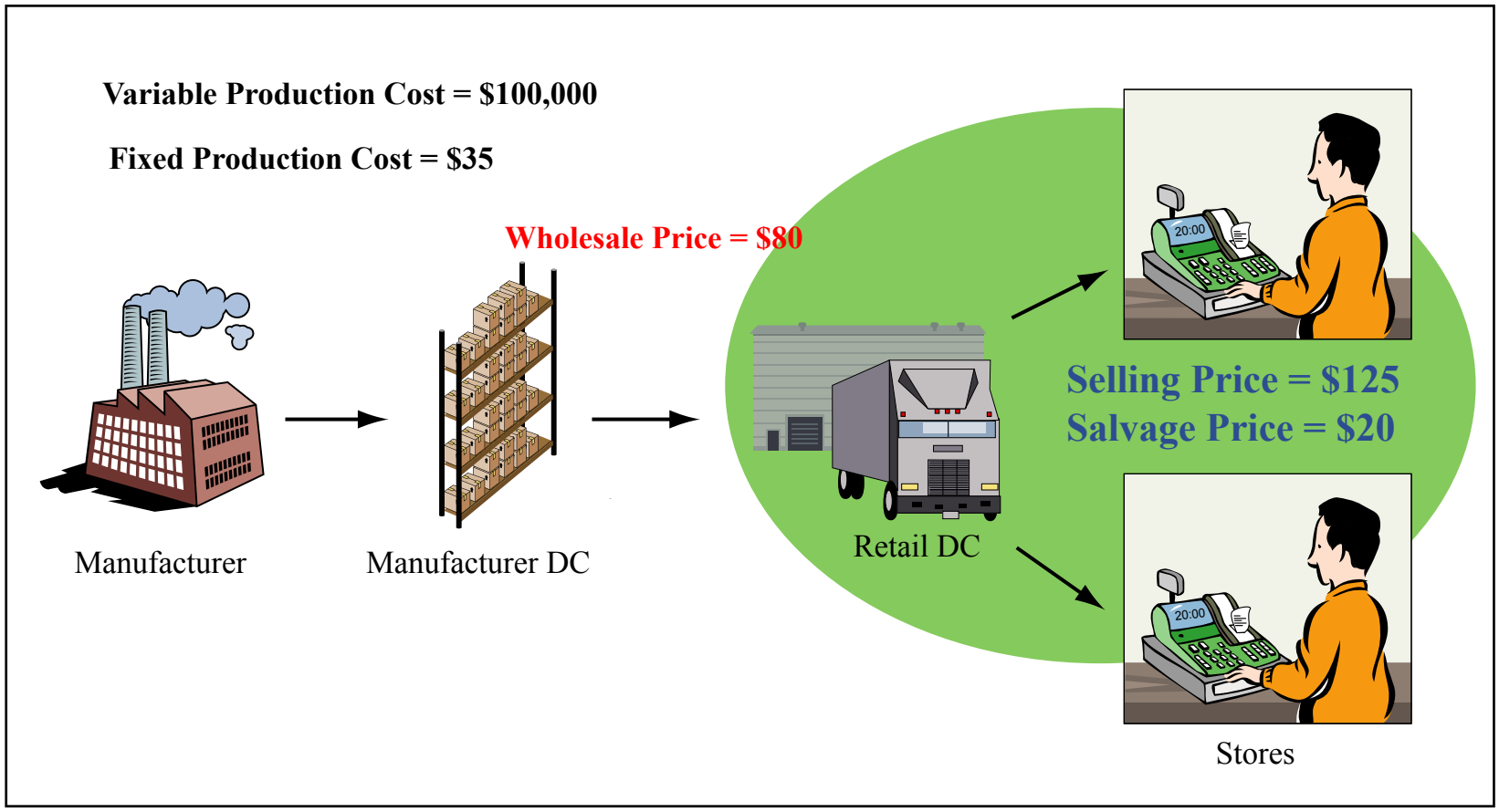

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Retailer Profi.
(Wholesale Price $70, RS 15%)

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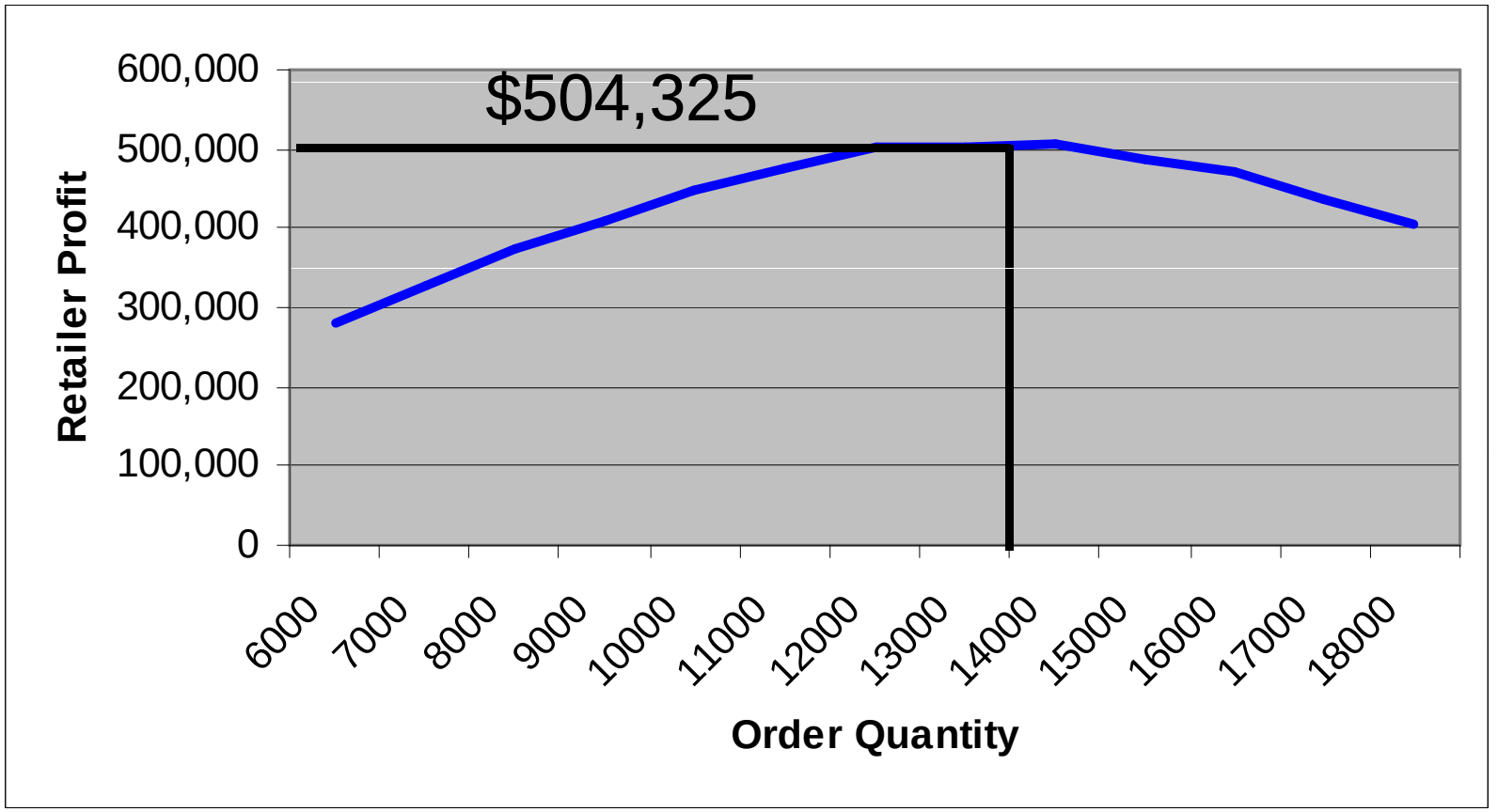

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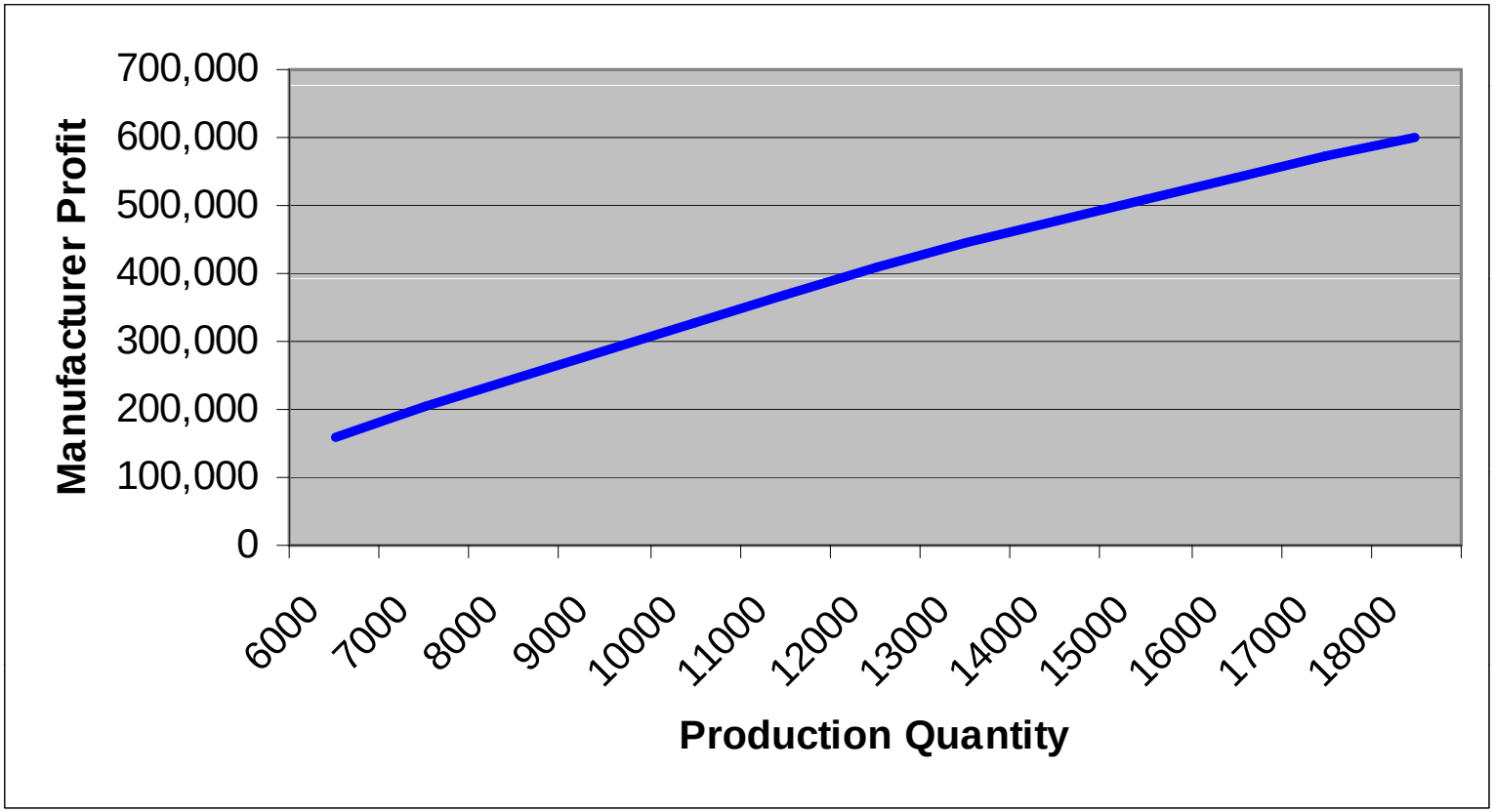

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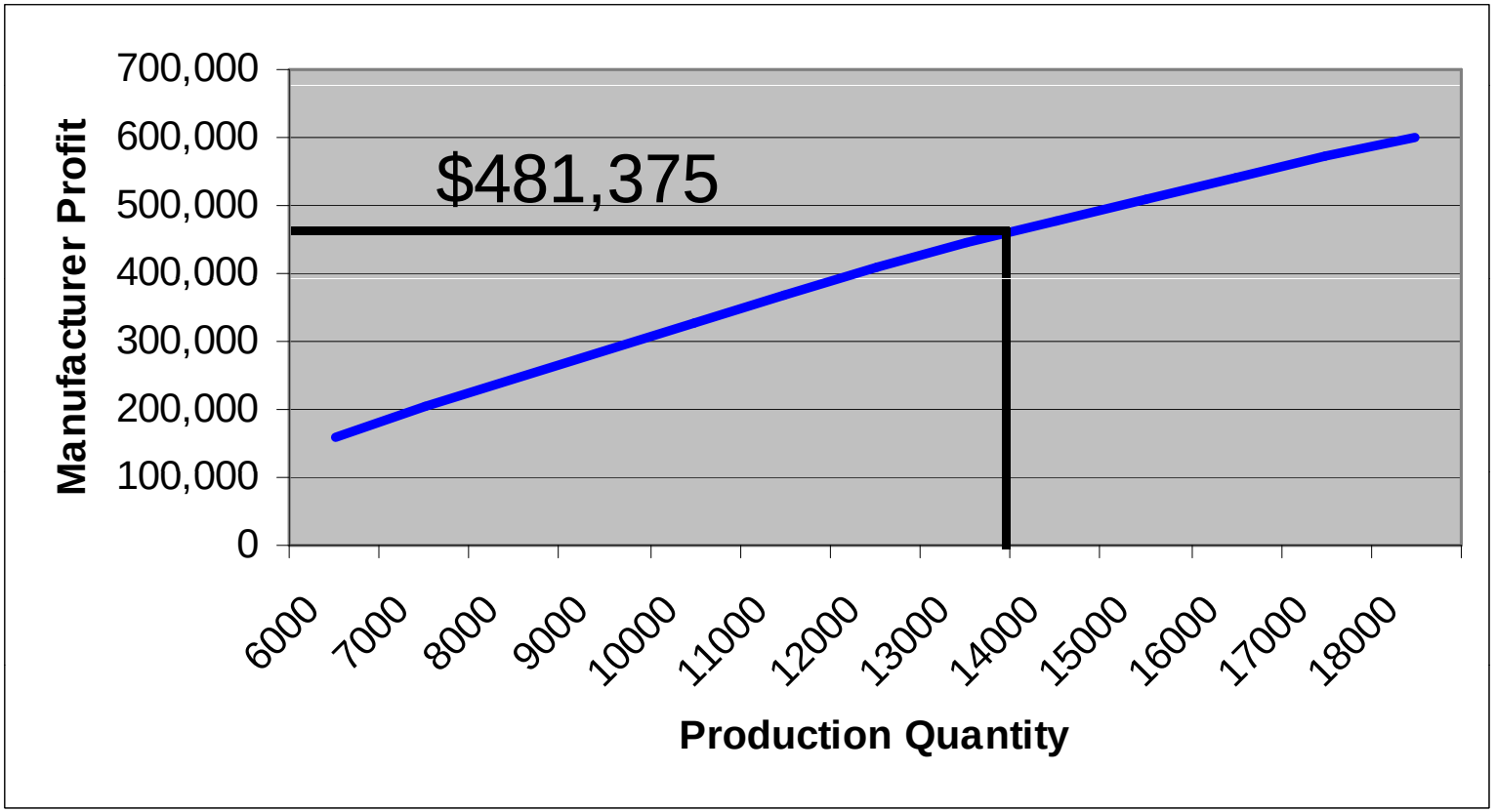
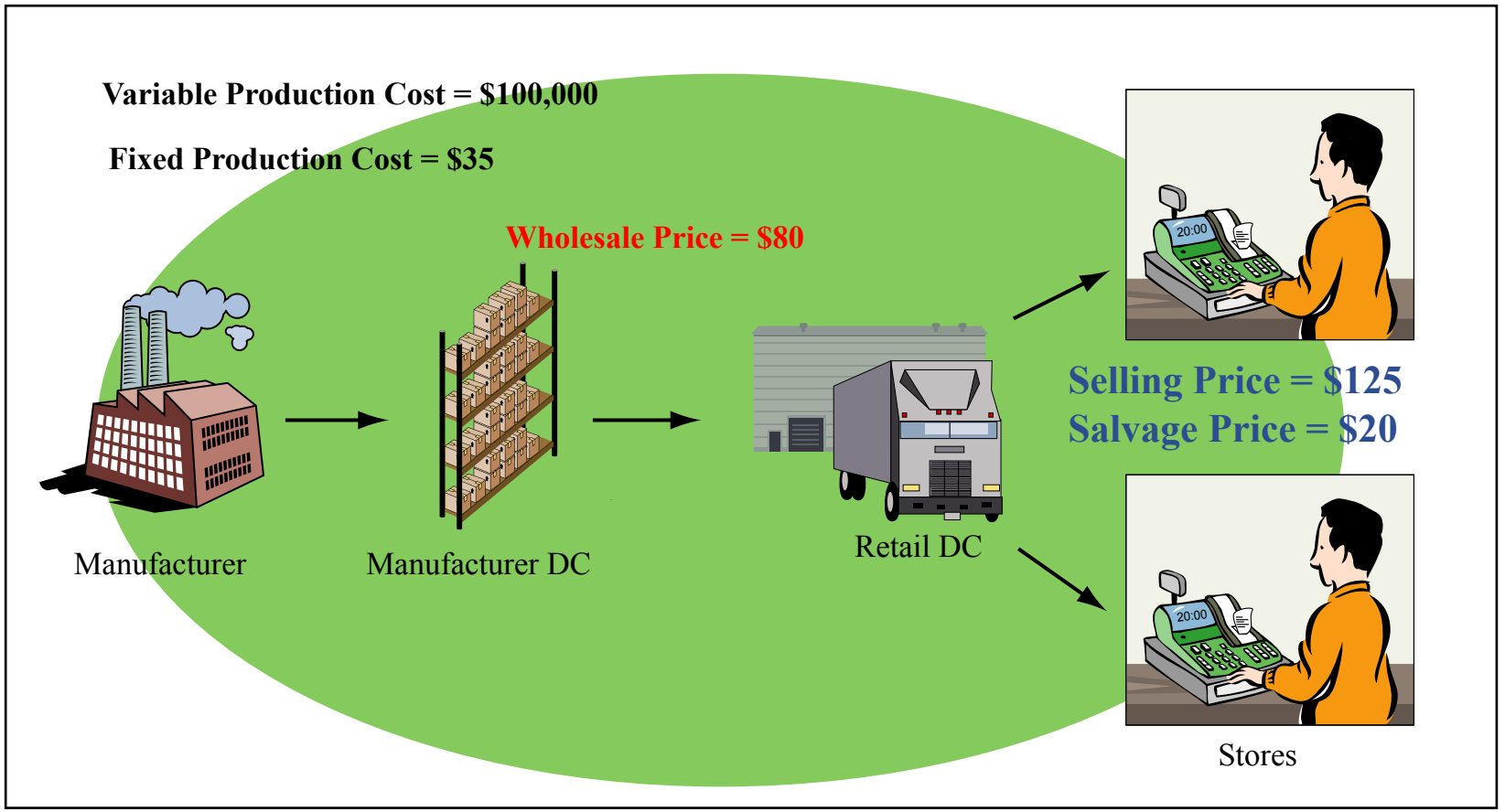


Supply Contracts

Strategy	Retailer	Manufacturer	Total
Sequential Optimization	470,700	440,000	910,700
Buyback	513,800	471,900	985,700
Revenue Sharing	504,325	481,375	985,700

Supply Contracts



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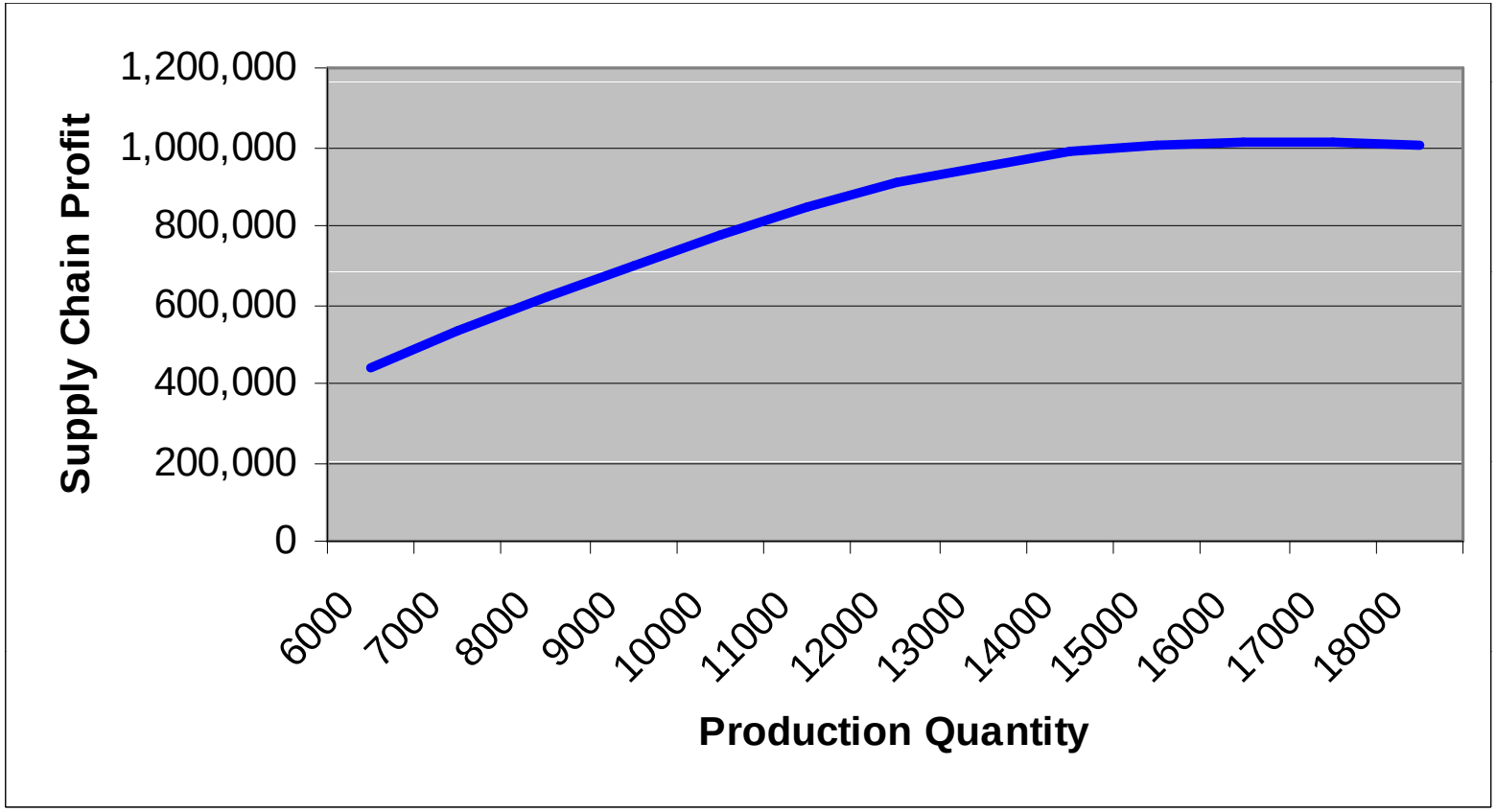
Supply Chain Profit



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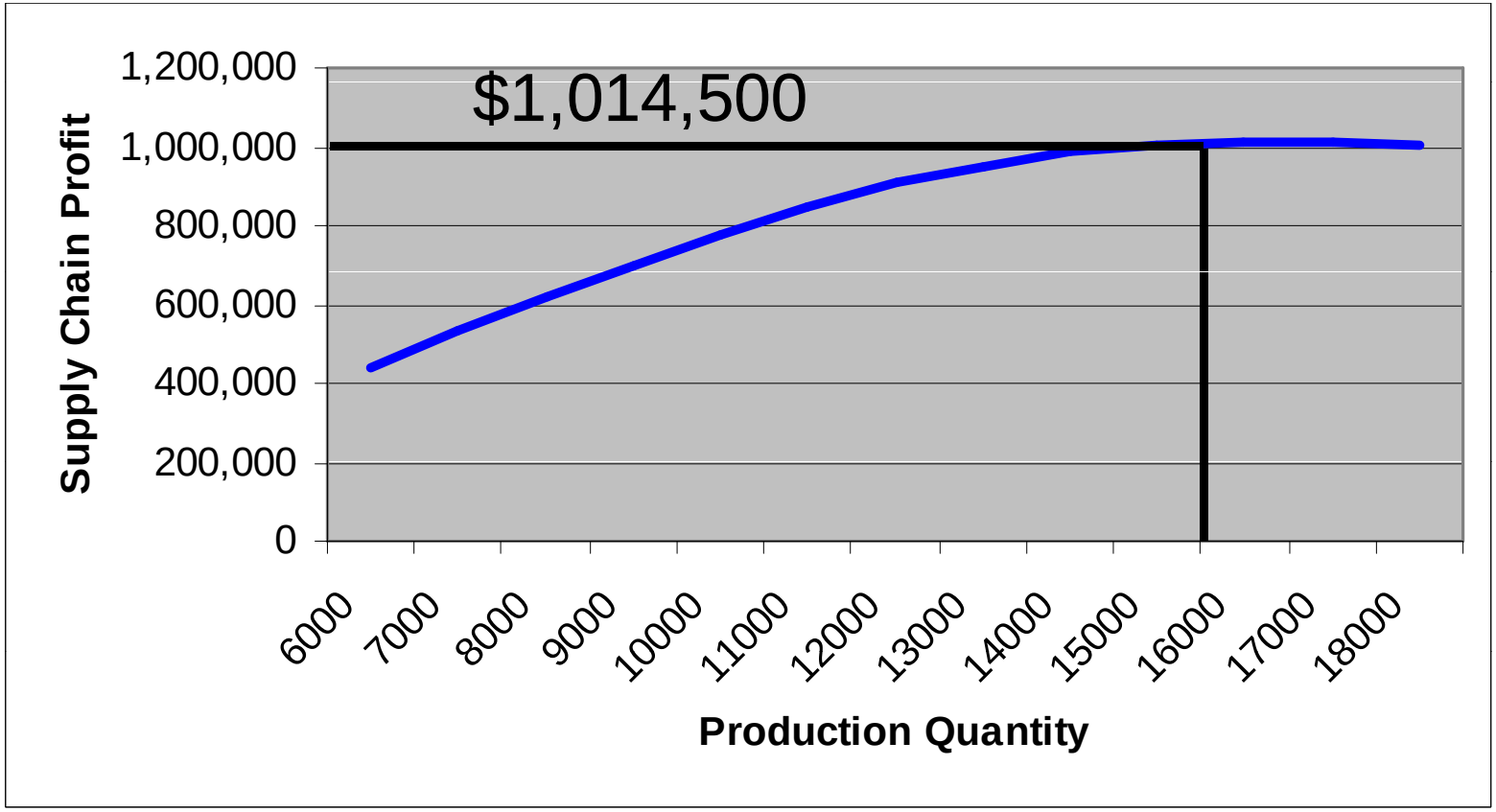

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Global Optimization			1,014,500

Supply Contracts: Key Insights

- Effective supply contracts allow supply chain partners to replace **sequential optimization** by **global optimization**
- Buy Back and Revenue Sharing contracts achieve this objective through **risk sharing**
- **No one has an incentive to deviate from the contract terms**

Supply Contracts: Case Study

- Example: Demand for a movie newly released video cassette typically starts high and decreases rapidly
  - Peak demand last about 10 weeks
- Blockbuster purchases a copy from a studio for $65 and rent for $3
  - Hence, retailer must rent the tape at least 22 times before earning profit
- Retailers cannot justify purchasing enough to cover the peak demand
  - In 1998, 20% of surveyed customers reported that they could not rent the movie they wanted

Supply Contracts: Case Study

- Starting in 1998 Blockbuster entered a revenue sharing agreement with the major studios
  - Studio charges $8 per copy

    Blockbuster pays 30 45% of its rental income
- Even if Blockbuster keeps only half of the rental income, the breakeven point is 6 rental per copy
- The impact of revenue sharing on Blockbuster was dramatic
  - Rentals increased by 75% in test markets
  - Market share increased from 25% to 31% (The 2nd largest retailer, Hollywood Entertainment Corp has 5% market share)

What are the drawbacks of RS?

- Administrative Cost
  - Lawsuit brought by three independent video retailers who complained that they had been excluded from receiving the benefits of revenue sharing was dismissed (June 2002)
  - The Walt Disney Company has sued Blockbuster accusing them of cheating its video unit of approximately $120 million under a four year revenue sharing agreement (January 2003)
- Impact on sales effort
  - Retailers have incentive to push products with higher profit margins
  - Automotive industry: automobile sales depends on retail effort

What are the drawbacks of RS?

- Retailer may carry substitute or complementary products from other suppliers
  - One supplier offers revenue sharing while the other does not
    - Substitute products: retail will push the product with high margin
    - Complementary products: retailer may discount the product offered under revenue sharing to motivate sales of the other product

SnowTime Costs: Initial Inventory

- Production cost per unit (C): $80
- Selling price per unit (S): $125
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- Q is production quantity, D demand

- Profit =
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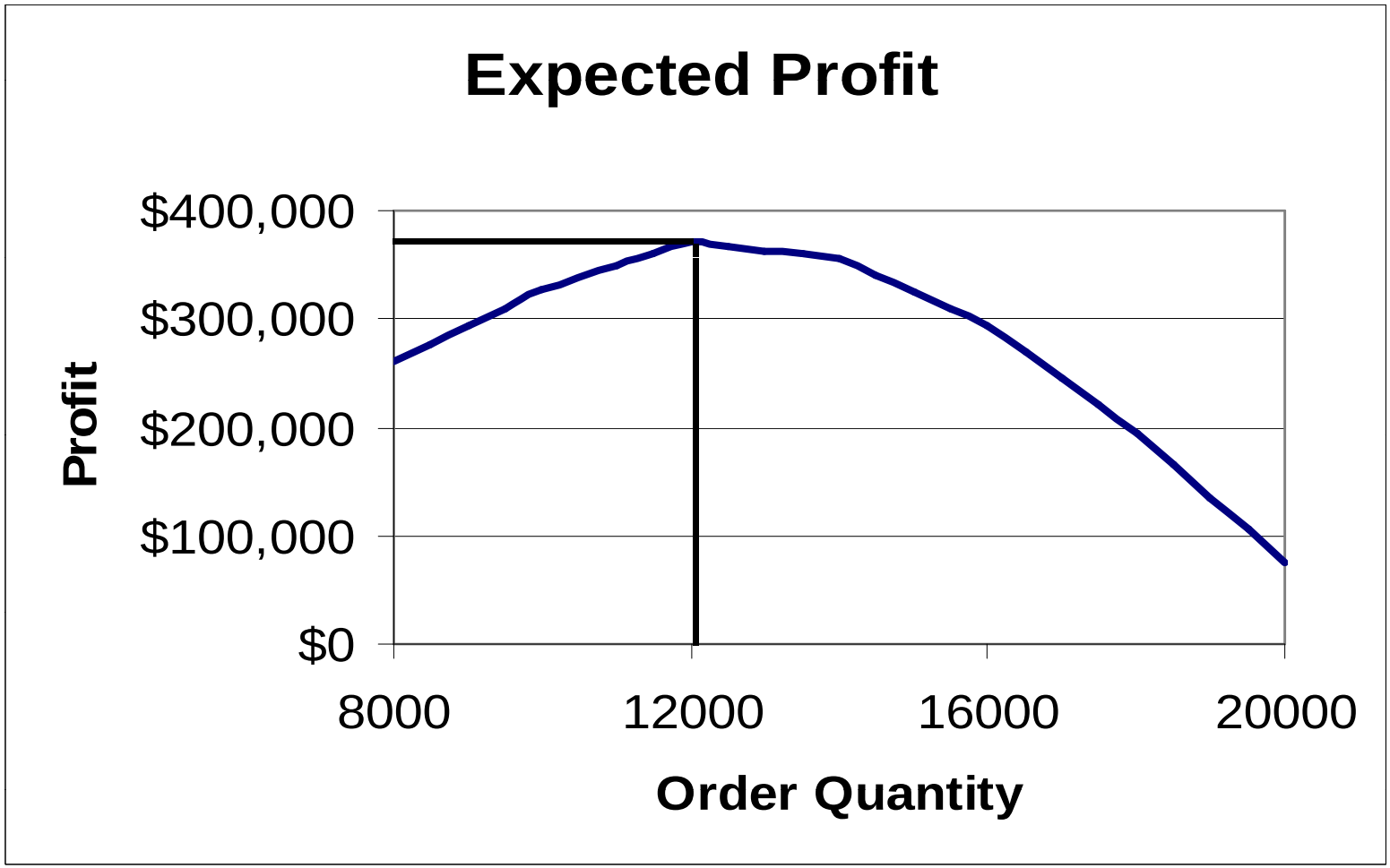
SnowTime Expected Profit

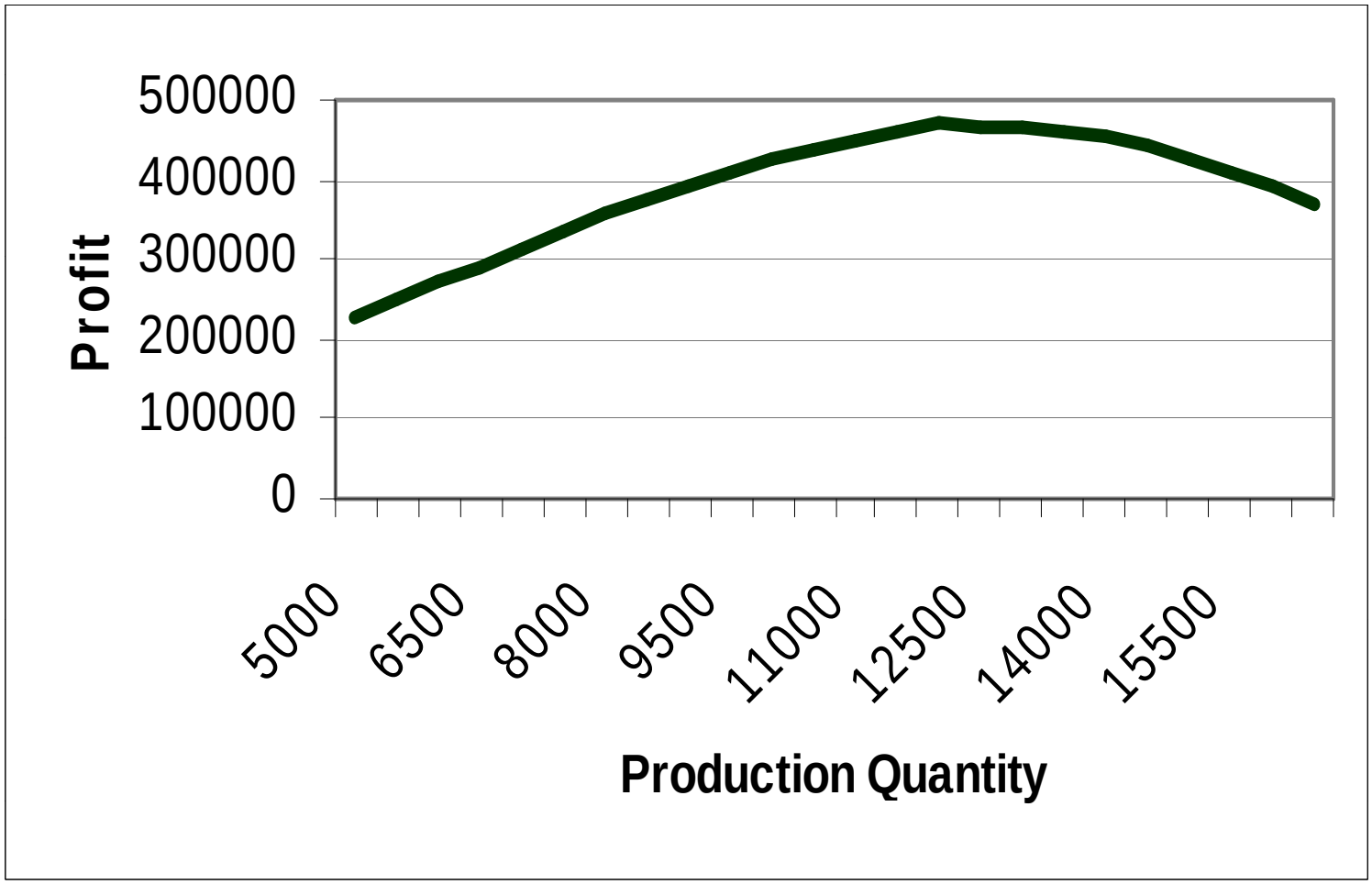


Initial Inventory

- Suppose that one of the jacket designs is a model produced last year.
- Some inventory is left from last year
- Assume the same demand pattern as before
- If only old inventory is sold, no setup cost

- **Question:** If there are 7000 units remaining, what should SnowTime do? What should they do if there are 10,000 remaining?

Initial Inventory and Profit



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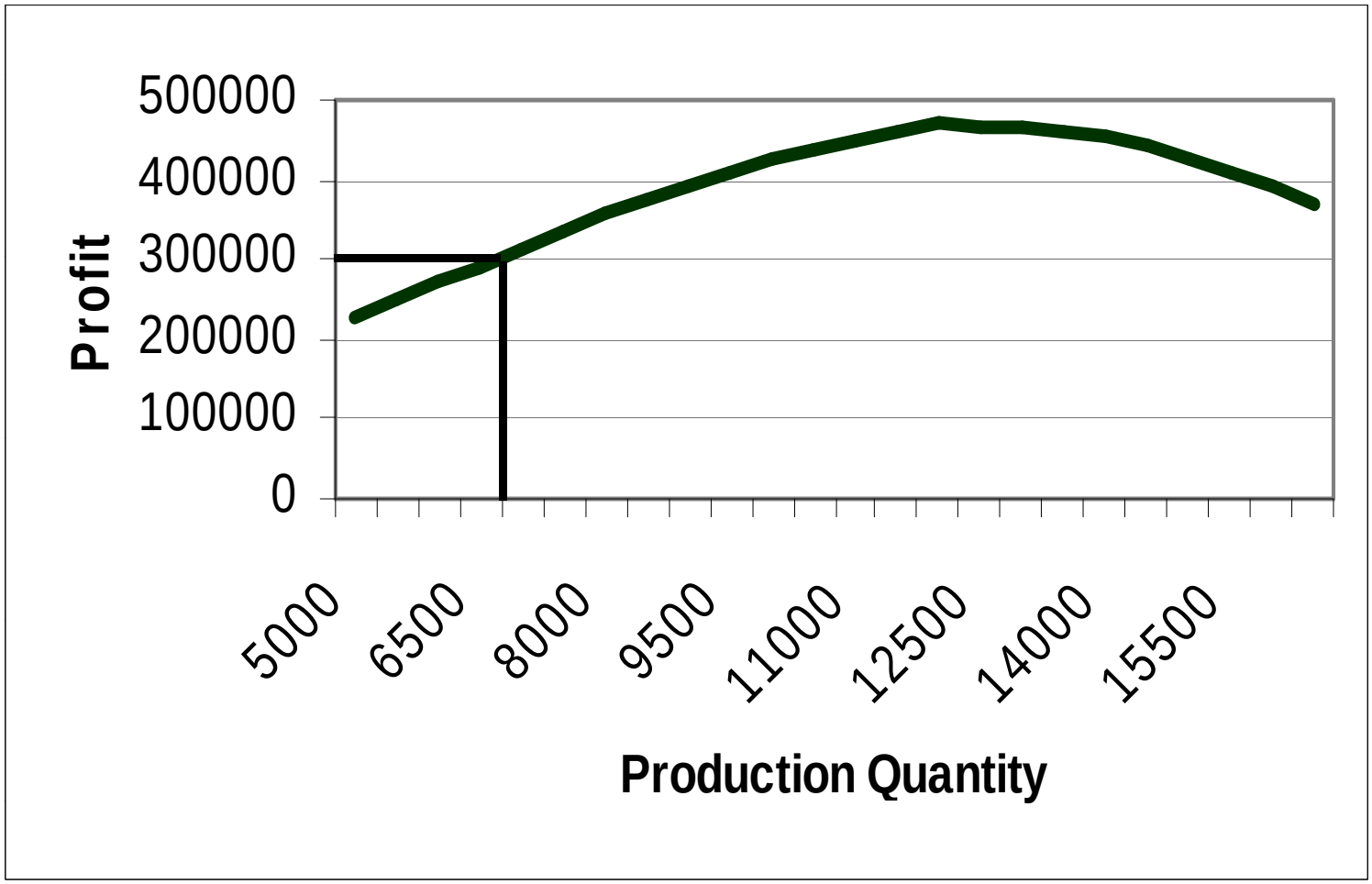

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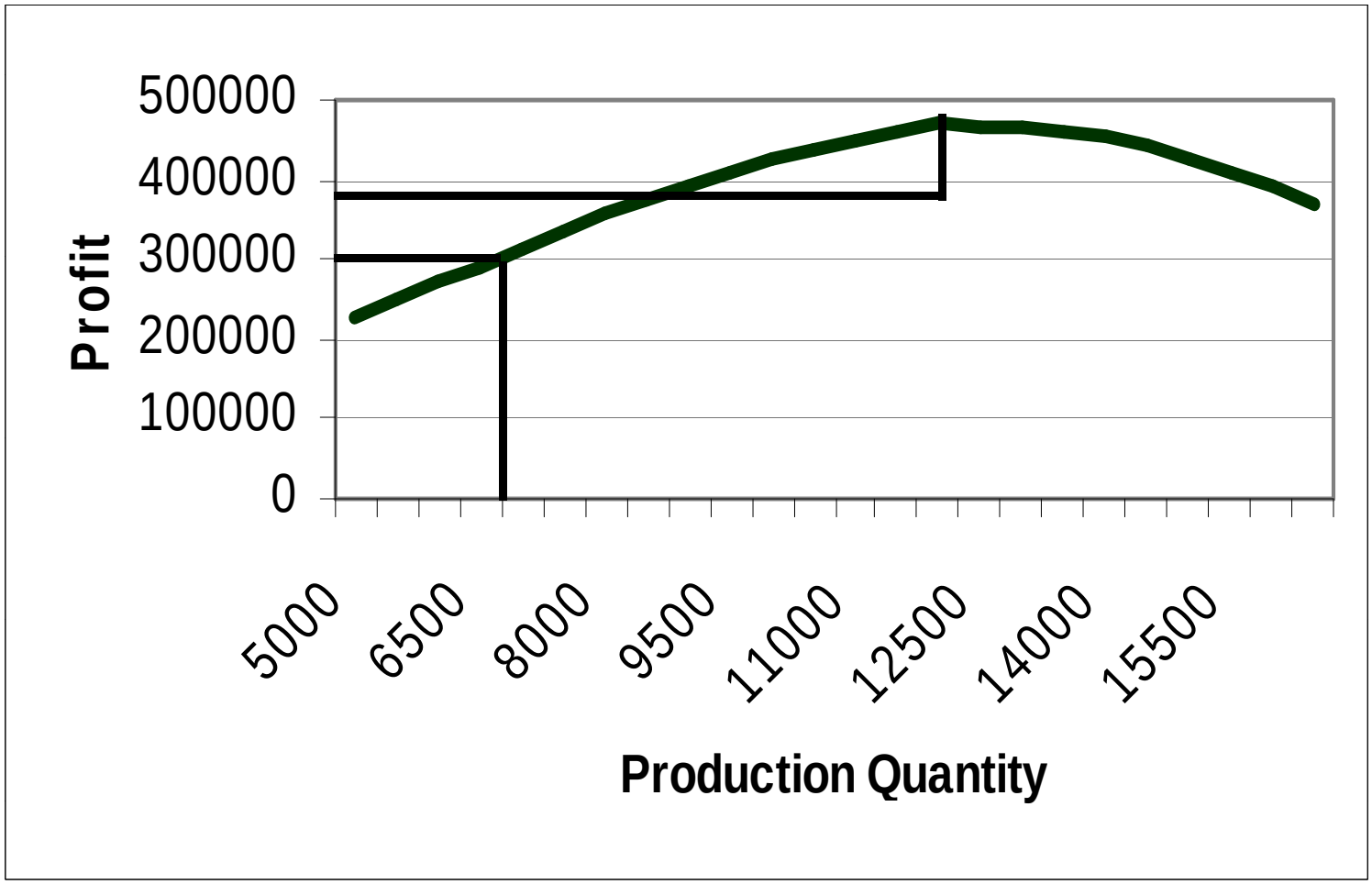

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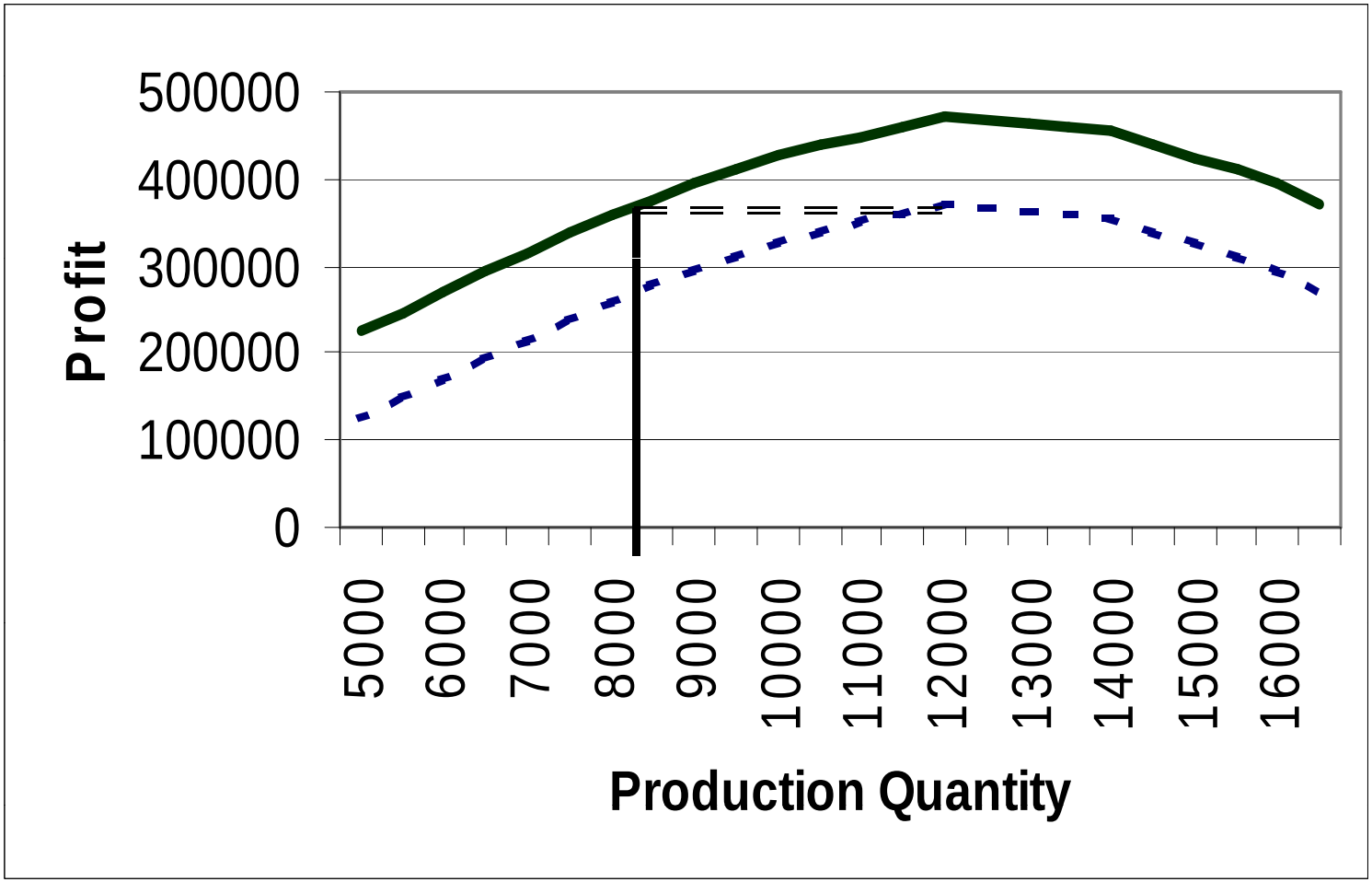


(s, S) *Policies*

- For some starting inventory levels, it is better to not start production
- If we start, we always produce to the same level
- Thus, we use an *(s,S)* policy. If the inventory level is below *s*, we produce up to *S*.
- *s* is the reorder point, and *S* is the order-up-to level
- The difference between the two levels is driven by the fixed costs associated with ordering, transportation, or manufacturing

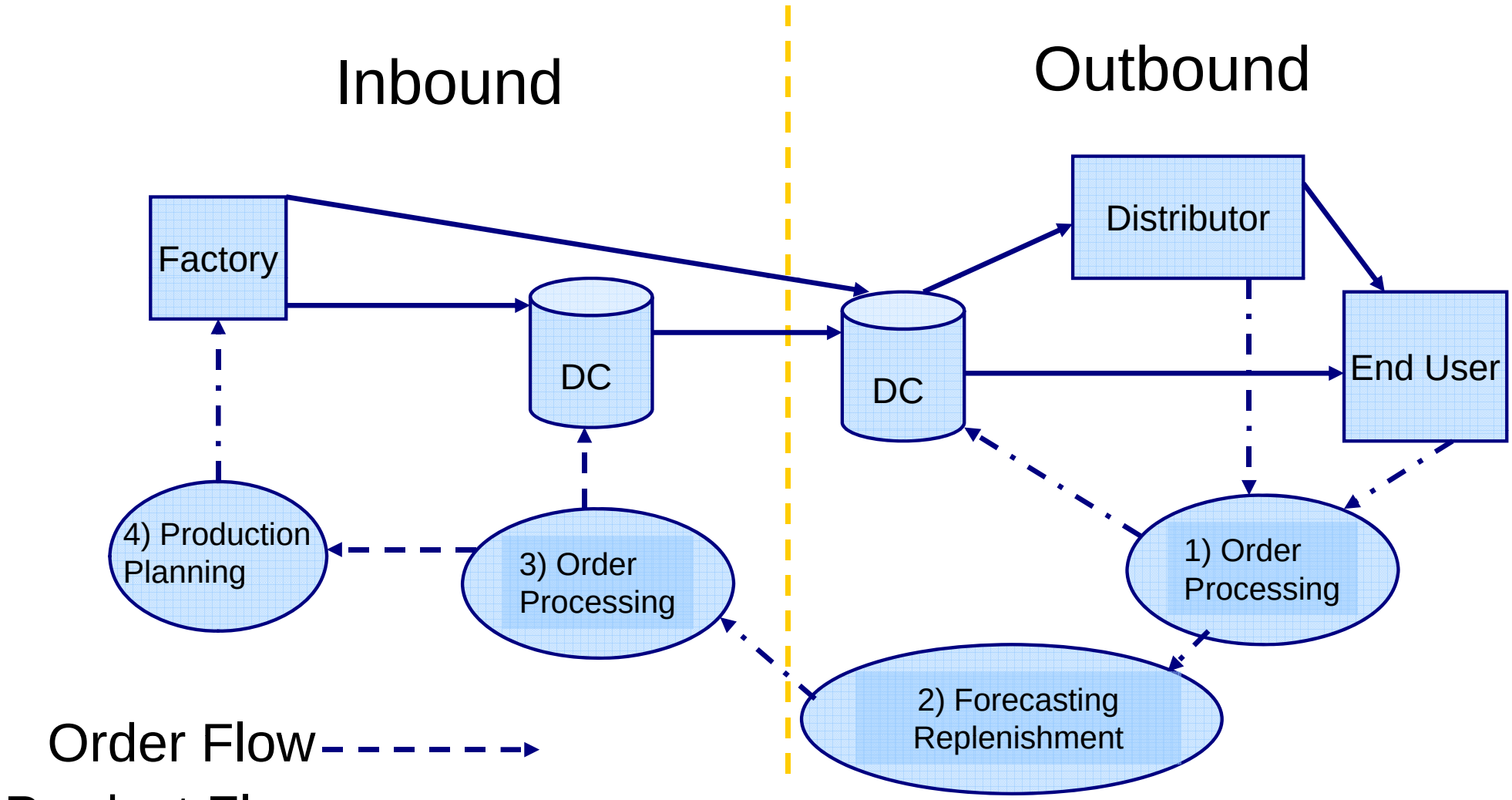
A Multi-Period Inventory Model

- Often, there are multiple reorder opportunities
- Consider a central distribution facility which orders from a manufacturer and delivers to retailers. The distributor periodically places orders to replenish its inventory

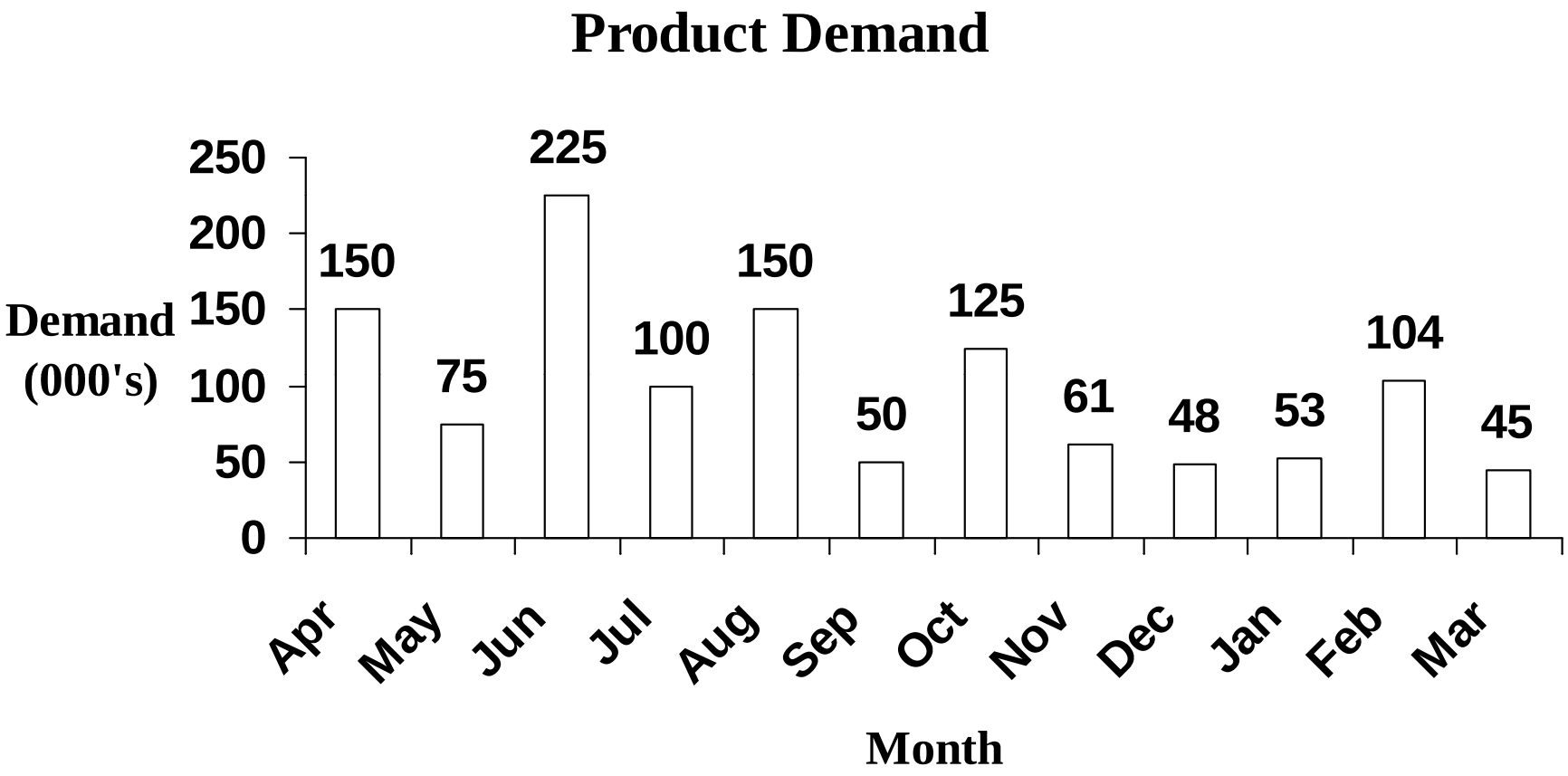
Case Study: Electronic Component Distributor

- Electronic Component Distributor
- Parent company HQ in Japan with world-wide manufacturing
- All products manufactured by parent company
- One central warehouse in U.S.

Case Study: The Supply Chain



Demand Variability: Example 1



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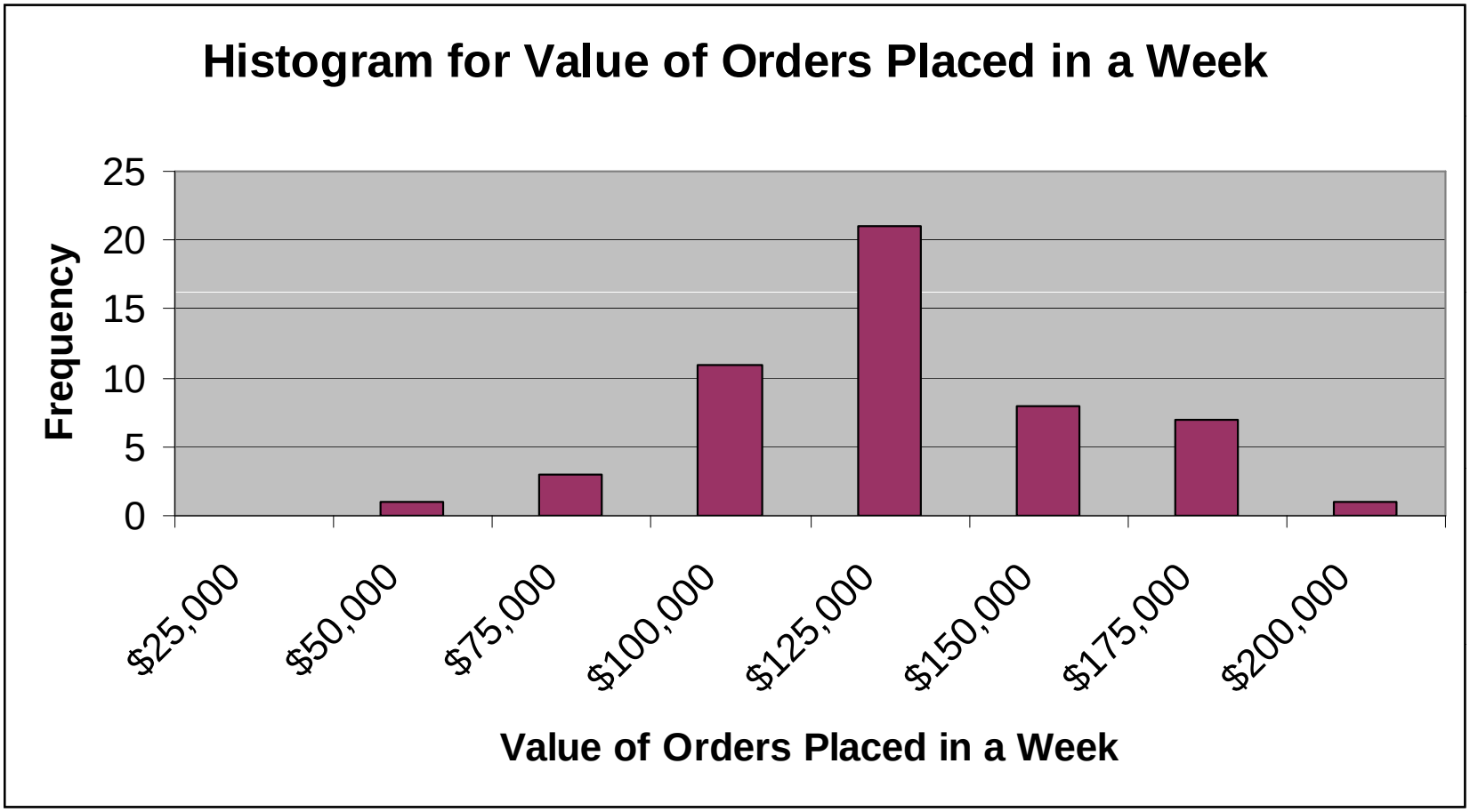
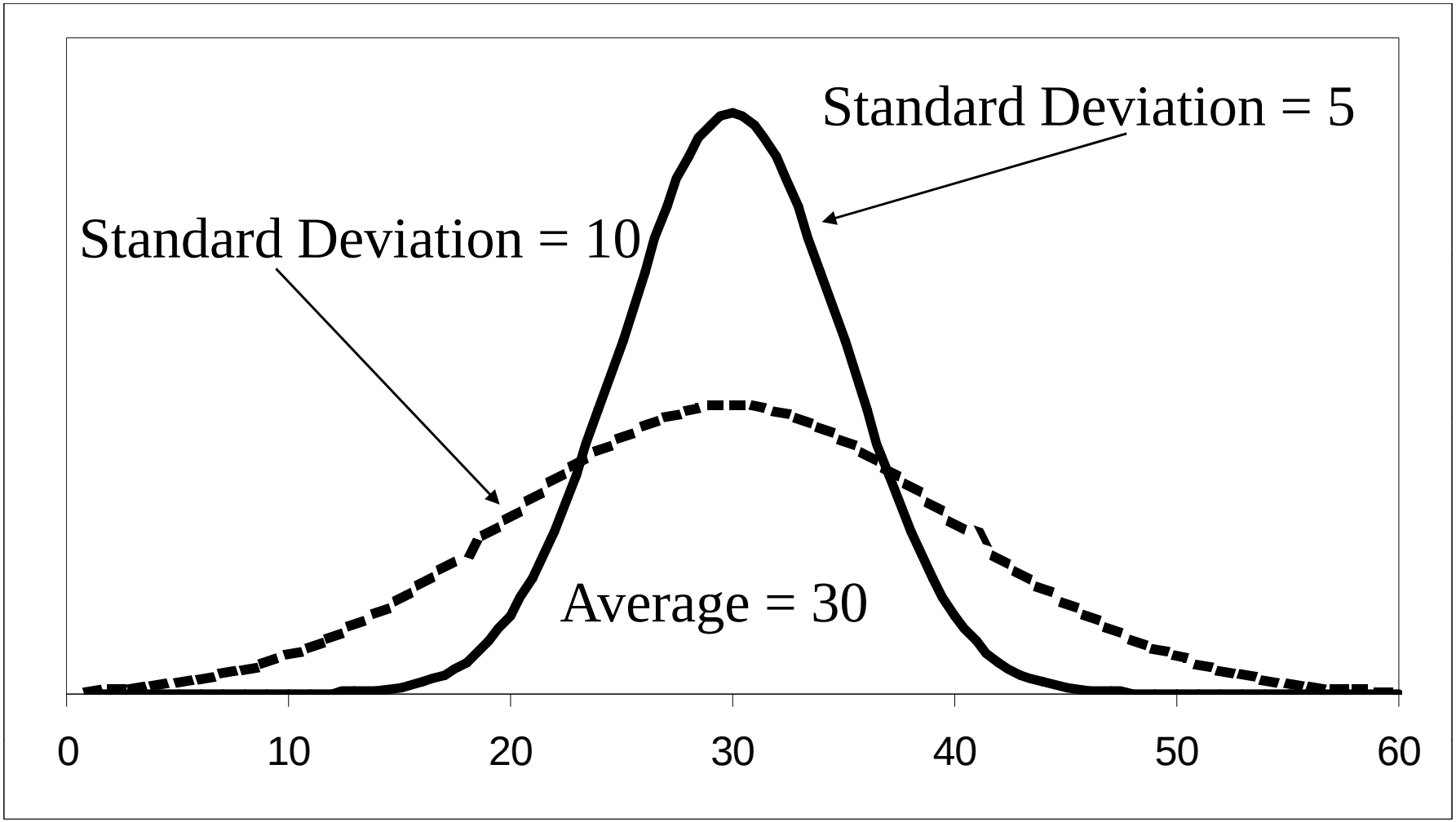


Reminder: *The Normal Distribution*



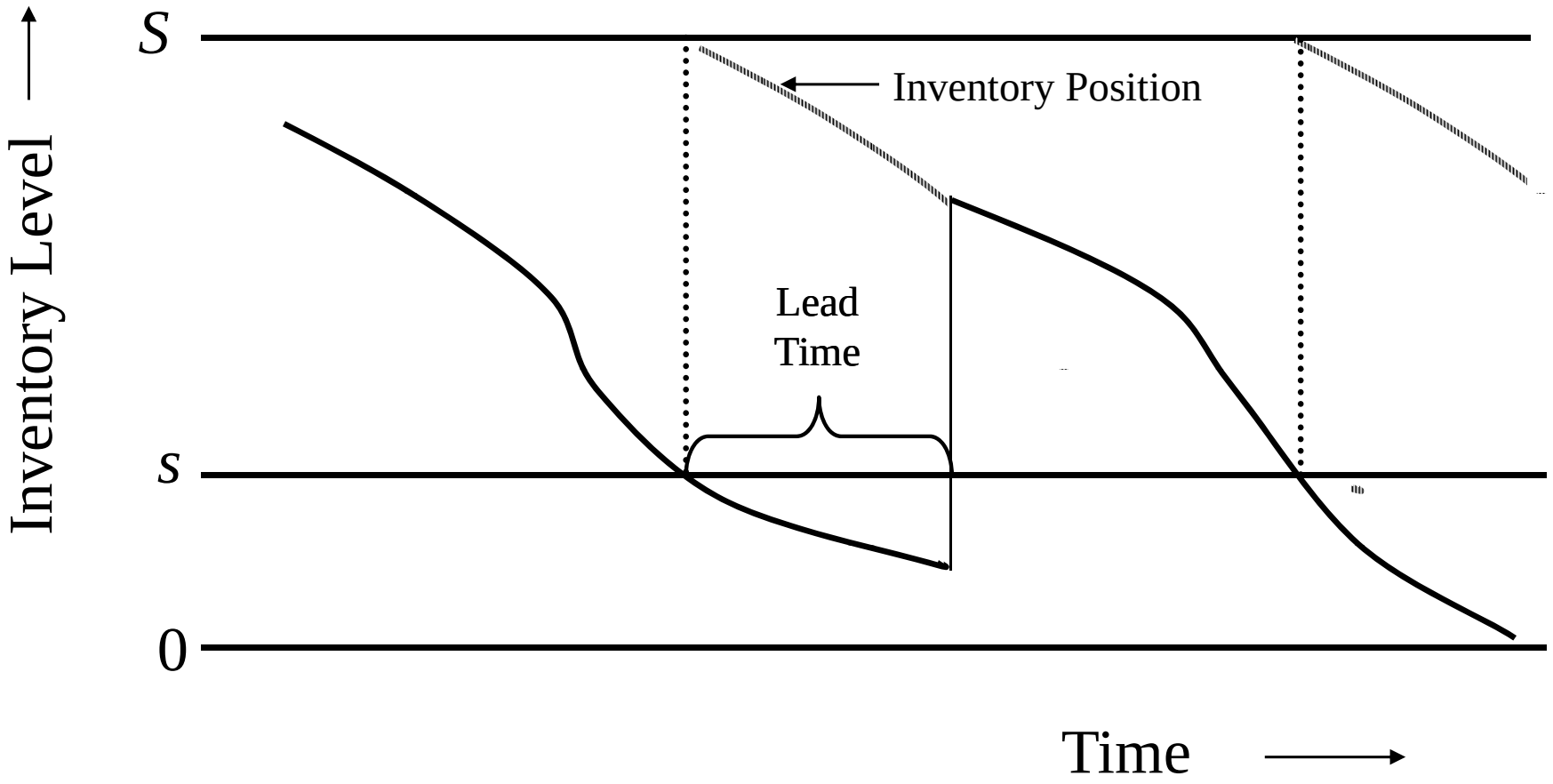
The DC holds inventory to:

- Satisfy demand during lead time
- Protect against demand uncertainty
- Balance fixed costs and holding costs

The Multi-Period Inventory Model

- Normally distributed random demand
- Fixed order cost plus a cost proportional to amount ordered.
- Inventory cost is charged per item per unit time
- If an order arrives and there is no inventory, the order is lost
- The distributor has a required service level. This is expressed as the the likelihood that the distributor will not stock out during lead time.
- Intuitively, how will this effect our policy?

A View of (s, S) Policy



The (s,S) Policy

- (s, S) Policy: Whenever the inventory position drops below a certain level, s, we *order to raise the inventory position to level S.*
- The reorder point is a function of:
  - The Lead Time
  - Average demand
  - Demand variability
  - Service level

Notation

- AVG = average daily demand
- STD = standard deviation of daily demand
- LT = replenishment lead time in days
- h = holding cost of one unit for one day
- SL = service level (for example, 95%). This implies that the probability of stocking out is 100%-SL (for example, 5%)
- Also, the **Inventory Position** at any time is the actual inventory plus items already ordered, but not yet delivered.

Analysis

- The reorder point has two components:
  - To account for average demand during lead time:

    $$\mathbf{LT} \times \mathbf{AVG}$$

  - To account for deviations from average (we call this **safety stock**)

    $$z \times \mathbf{STD} \times \sqrt{\mathbf{LT}}$$

    where z is chosen from statistical tables to ensure that the probability of stockouts during leadtime is 100%-SL.

Example

- The distributor has historically observed weekly demand of:

  AVG = 44.6 STD = 32.1

  Replenishment lead time is 2 weeks, and desired service level SL = 97%
- Average demand during lead time is:

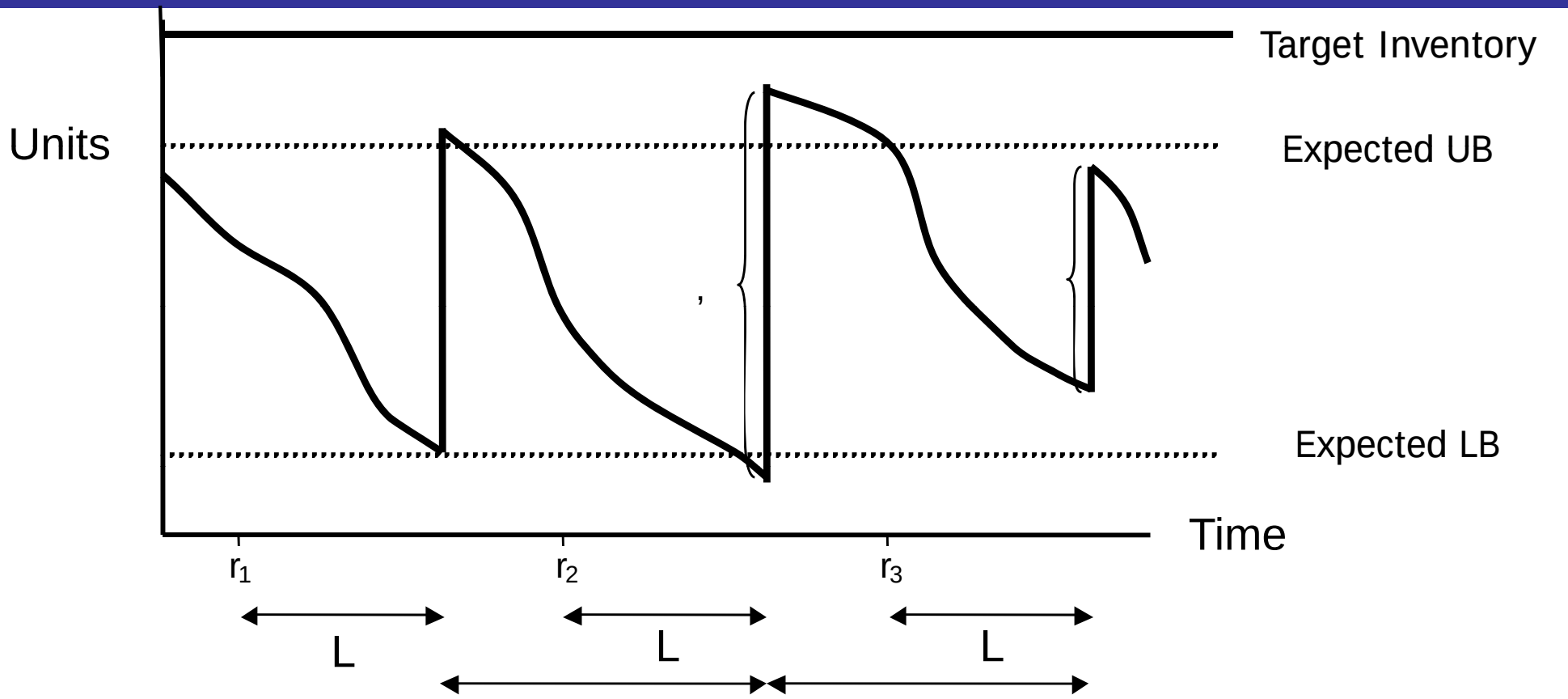
  $44.6 \times 2 = 89.2$
- Safety Stock is:

  $1.88 \times 32.1 \times \sqrt{2} = 85.3$
- Reorder point is thus 175, or about 3.9 weeks of supply at warehouse and in the pipeline

Fixed Order Schedule

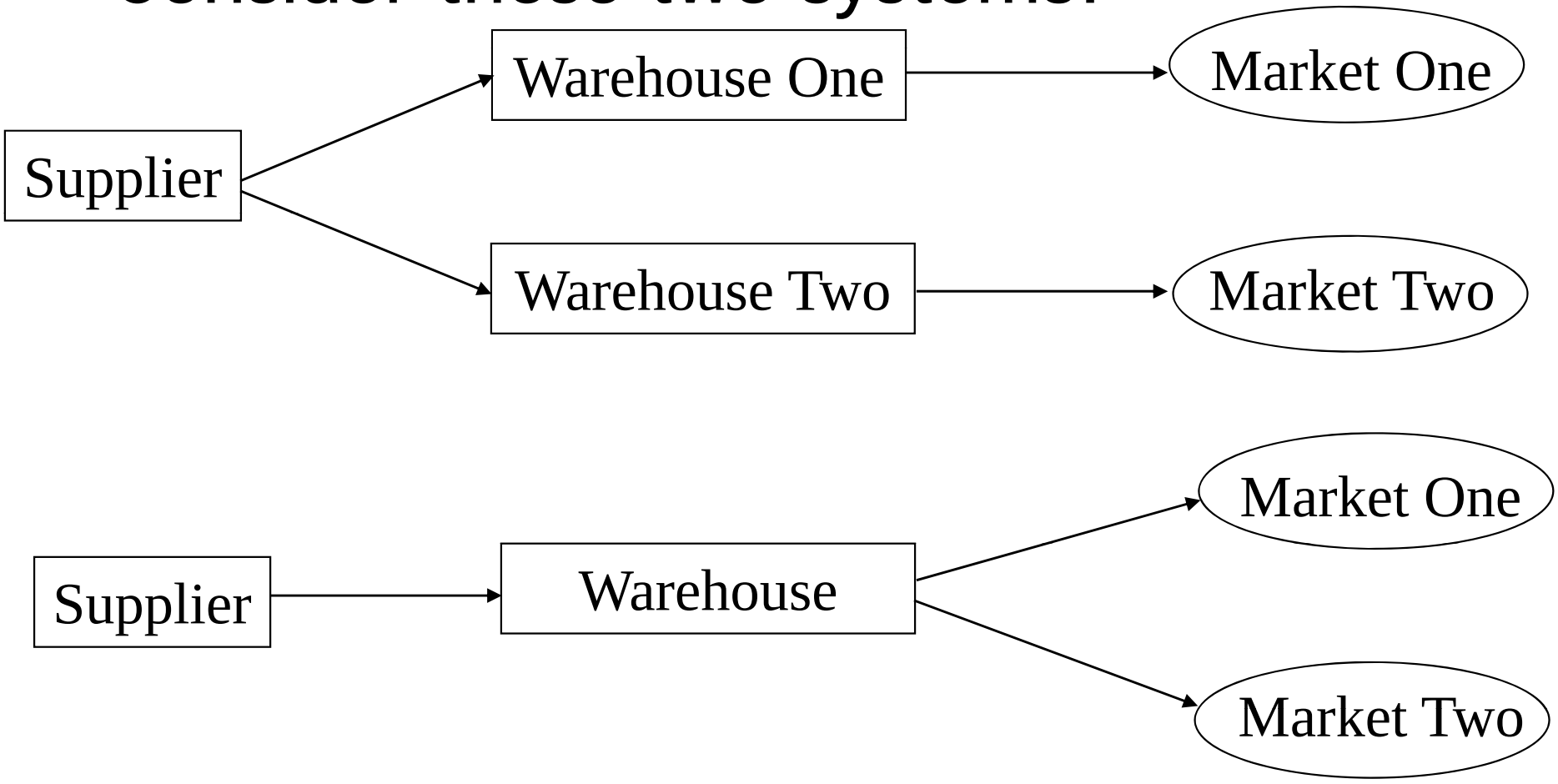
- Suppose the distributor places orders every month
- What policy should the distributor use?
- What about the fixed cost?

Base-Stock Policy



Risk Pooling

- Consider these two systems:



Risk Pooling

- For the same service level, which system will require more inventory? Why?
- For the same total inventory level , which system will have better service? Why?
- What are the factors that affect these answers?

Risk Pooling Example

- Compare the two systems:
  - two products
  - maintain 97% service level
  - $60 order cost
  - $.27 weekly holding cost
  - $1.05 transportation cost per unit in decentralized system, $1.10 in centralized system
  - 1 week lead time

Risk Pooling Example

Week	1	2	3	4	5	6	7	8
Prod A, Market 1	33	45	37	38	55	30	18	58
Prod A, Market 2	46	35	41	40	26	48	18	55
Prod B, Market 1	0	2	3	0	0	1	3	0
Product B, Market 2	2	4	0	0	3	1	0	0

Risk Pooling Example

Warehouse	Product	AVG	STD	CV
Market 1	A	39.3	13.2	.34
Market 2	A	38.6	12.0	.31
Market 1	B	1.125	1.36	1.21
Market 2	B	1.25	1.58	1.26

Risk Pooling Example

Warehouse	Product	AVG	STD	CV	s	S	Avg. Inven.	% Dec.
Market 1	A	39.3	13.2	.34	65	158	91	
Market 2	A	38.6	12.0	.31	62	154	88	
Market 1	B	1.125	1.36	1.21	4	26	15	
Market 2	B	1.25	1.58	1.26	5	27	15	
Cent.	A	77.9	20.7	.27	118	226	132	26%
Cent	B	2.375	1.9	.81	6	37	20	33%

Risk Pooling: *Important Observations*

- Centralizing inventory control reduces both safety stock and average inventory level for the same service level.
- This works best for
  - High coefficient of variation, which reduces required safety stock.
  - Negatively correlated demand. Why?
- What other kinds of risk pooling will we see?

To Centralize or not to Centralize

- What is the effect on:
  - Safety stock?

    Service level?
  - Overhead?
  - Lead time?
  - Transportation Costs?

Inventory Management: Best Practice

- Periodic inventory review policy (59%)
- Tight management of usage rates, lead times and safety stock (46%)
- ABC approach (37%)
- Reduced safety stock levels (34%)
- Shift more inventory, or inventory ownership, to suppliers (31%)
- Quantitative approaches (33%)

Changes In Inventory Turnover

- Inventory turns increased by 30% from 1995 to 1998
- Inventory turns increased by 27% from 1998 to 2000
- Overall the increase is from 8.0 turns per year to over 13 per year over a five year period ending in year 2000.

Inventory Turnover Ratio

Industry	Upper Quartile	Median	Lower Quartile
Dairy Products	34.4	19.3	9.2
Electronic Component	9.8	5.7	3.7
Electronic Computers	9.4	5.3	3.5
Books: publishing	9.8	2.4	1.3
Household audio & video equipment	6.2	3.4	2.3
Household electrical appliances	8.0	5.0	3.8
Industrial chemical	10.3	6.6	4.4

Factors that Drive Reduction in Inventory

- Top management emphasis on inventory reduction (19%)
- Number of SKUs in the warehouse (10%)
- Improved forecasting (7%)
- Use of sophisticated inventory management software (6%)
- Coordination among supply chain members (6%)
- Others

Factors that will Drive Inventory Turns Change by 2000

- Better software for inventory management (16.2%)
- Reduced lead time (15%)
- Improved forecasting (10.7%)
- Application of SCM principals (9.6%)
- More attention to inventory management (6.6%)
- Reduction in SKU (5.1%)
- Others

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