Engineering Risk Benefit Analysis

1.155, 2.943, 3.577, 6.938, 10.816, 13.621, 16.862, 22.82, ESD.72, ESD.721

RPRA 4. Availability

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Definitions

Unavailability: $q(t) = \Pr[\text{down at } t]$

Availability: $a(t) \equiv 1 - q(t) = \Pr[\text{up at } t]$

$$q(t) + a(t) = 1$$

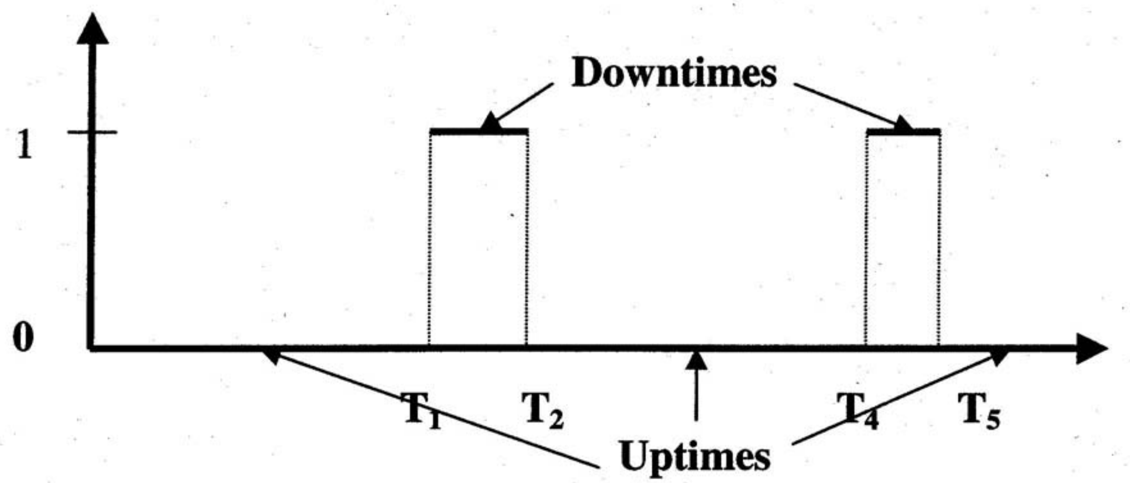
Unattended components

q(t) = F(t) = Pr[T < t]

Example: 2-out-of-3 system of exponential components

$$Q_s(t) = F_s(t) = 3(1 - e^{-\lambda t})^2 - 2(1 - e^{-\lambda t})^3$$

Continuously monitored repairable components



Continuously monitored repairable components (2)

Average unavailability:

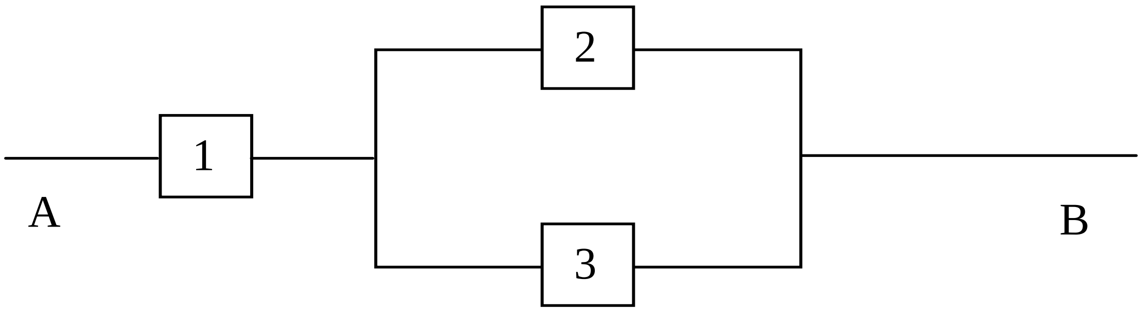
$$q = \frac{MTTR}{MTTF + MTTR}$$

For the exponential failure distribution:

$$MTTF = \frac{1}{\lambda} \qquad \Rightarrow$$

$$q = \frac{\tau}{\tau + \frac{1}{\lambda}} = \frac{\lambda\tau}{1 + \lambda\tau}, \quad \text{Note:} \quad q \cong \lambda\tau \quad \text{for} \quad \lambda\tau < 0.1$$

Example



i	MTTF (hrs)	MTTR (hrs)
1	800	8
2	600	15
3	600	15

Example (2)

What is the reliability of the system for one month assuming that no repair is available?

Step 1: System Logic

Minimal path sets: $\{Y_1, Y_2\}$ $\{Y_1, Y_3\}$

Structure function for success:

$$Y_S = 1 - (1 - Y_1 Y_2)(1 - Y_1 Y_3) \Rightarrow$$

$$Y_S = Y_1(Y_2 + Y_3 - Y_2 Y_3)$$

Example (3)

Step 2: Reliability of the system in terms of component reliabilities:

$$R_S = R_1(R_2 + R_3 - R_2R_3)$$

Component reliabilities:

$$R_1 = \exp(-720/800) = 0.407$$

$$R_2 = R_3 = \exp(-720/600) = 0.301$$

Therefore, $R_S = 0.208$

Example (4)

What is the availability of the system assuming that the repair process starts immediately upon detection of failure?

Structure function for success

$$Y_S = Y_1(Y_2 + Y_3 - Y_2 Y_3)$$

Availability of the system in terms of component availabilities:

$$A_S = a_1(a_2 + a_3 - a_2 a_3)$$

Example (5)

Component availabilities:

$$a_1 = 1 - (8/800) = 0.990$$

$$a_2 = a_3 = 1 - (15/600) = 0.975.$$

Therefore, $A_S = 0.989.$

Note

Minimal cut sets: X_1 and X_2X_3

***Structure function for failure*:**

$$X_S = 1 - (1 - X_1)(1 - X_2X_3)$$

$$X_S = X_1 + X_2X_3 - X_1X_2X_3$$

Note (2)

The unreliability of the system for one month is:

$$F_S = F_1 + F_2F_3 - F_1F_2F_3$$

where: $F_1 = 1 - \exp(-720/800) = 0.593$

$F_2 = F_3 = 1 - \exp(-720/600) = 0.699.$

Thus,

$F_S = 0.792 = 1 - 0.208 = 1 - R_S$

PROBABILITY OF SYSTEM FAILURE OR SUCCESS

1. **Determine the structure function.**

2. **Express system (un)reliability or (un)availability as a function of component (un)reliabilities or (un)availabilities.**

3. **Determine component (un)reliabilities or (un)availabilities.**