24.222 Decisions, Games, and Rational Choice
Spring 2008

Problem set number one

These questions involve making some intuitive judgments about probabilities in certain kinds of situations as well as the computation of some probabilities as a function of judgments about others. They also involve reflection on what one is doing when one makes such intuitive judgments, and on the status of the theoretical assumptions presupposed in the computations. All of the problems are adapted from examples discussed in Richard Jeffrey, *Subjective Probability: The Real Thing* (Cambridge, University Press, 2004).

1. Suppose you were to shake a thumbtack and let it land on a hard floor. Can you assign a probability to the proposition that it lands point up? If you had to bet on whether it landed point up or with the point on the floor, what do you think fair odds would be? (Suppose you choose the odds, and I choose which side of the bet to take. What odds would you propose?) No fair experimenting

2. There are three coins in a bag: one two-headed, one two-tailed, and one fair. You shake the bag, and let one fall onto the table, and it shows a head. What should your probability be that it is the two-headed coin? (a) 1/2, since of the three equally probable possibilities, you have eliminated just one (that the coin is two-tailed. (b) 2/3, since one has also eliminated the possibility that the coin is fair, but landed tails up. Which argument, if either, is right?

3. Suppose you are told that Pat is a typical middle class Cambridge homeowner. Let A be the proposition that before the end of 2008, Pat will have installed a burglar alarm in her home. Let B be the proposition that Pat's home will have been burglarized before the end of 2008. Two questions: (1) Which of the two probabilities (representing your degrees of belief), pr(A/B) or pr(A/~B) is higher? (2) Which of the two probabilities, pr(B/A) or pr(B/~A) is higher? Kahneman, Slovic and Tversky reported that when asked a similar question in an experiment, "a large majority of subjects (132 of 161) stated that pr(A/B) > pr(A/~B) and that pr(B/A) < pr(B/~A), contrary to the laws of probability."

   First, prove that it is right that this is contrary to the laws of probability. That is, prove that pr(A/B) > pr(A/~B) if and only if pr(B/A) > pr(B/~A). Second, how do you answer the questions? What do you think explains the data?

4. You are suffering from a disease that, according to your manifest symptoms, is either disease A or disease B. A happens to be 19 times as common as B, and this demographic statistic is the only evidence that is relevant (without further investigation) to determining which of the two you have. The two diseases are both fatal if untreated, and there is a treatment available for both, but unfortunately, it is dangerous - in fact fatal - to combine the two treatments. There is, however, a test which your doctor has ordered, which yields a definitive diagnosis, and the diagnosis is correct, both with cases of A and cases of B, 90% of the time. (That is, when the patient has A, the test result is right in 90% of the cases, and when the patient has B, the test result is right in 90% of the cases.) The test

reports that you are suffering from disease B. Which treatment should you take, and why?

5,   John Venn, a 19th century probability theorist (and the inventor of Venn diagrams), made the following point about testimony: "There are, broadly speaking, two different ways in which we may suppose testimony to be given. It may, in the first place, take the form of a reply to . . . a question. . . . framed to be answered by yes or no. . . . On the other hand, the testimony may take the form of a more original statement or piece of information. Instead of saying 'Did A happen?' we may ask 'What happened?' Here if the witness speaks the truth he must be supposed, as before, to have but one way of doing so. . . . But if he errs, he has many ways of going wrong." The point was illustrated with the following example of two contrasting cases:

(a) In an urn with 1000 balls, one is green and the rest are red. A ball is drawn at random, and seen only by a slightly color blind witness, who reports that the ball was green. Given that the witness's reliability in distinguishing red from green is .9 (that is, pr(he says X/X) = .9, where X = Red, and where X = Green.), what is your probability that the witness was right on this occasion?

(b) Again, the urn has 1000 balls, but this time they are numbered, 1 to 1000. Again, the witness's reliability is .9, in the sense that pr(he says X/X) = .9, for each X from 1 to 1000. A ball is drawn, and the witness reports that it was numbered "25" Assume that if he is wrong, he is as likely to choose one wrong number as any other. What is the probability that he is right, in this case?

After answering the question for the two examples, comment on whether the examples illustrate the phenomenon that Venn was describing. (Consider, for example, a variation of case (b) in which, rather than asking the witness what number he saw, you ask him whether or not the number he saw was 25, and he answers "yes".)

Please think about these five questions, and give me your thoughts about them by **Monday, February 25**.