

Subject 24.241. Logic I. Homework due Thursday, Sept. 22.

For these problems, use the version of the sentential calculus in which the atomic sentences are English capital letters.

1. Classify each of the following five expressions as either an atomic sentence, a negation, a conjunction, a disjunction, a conditional, a biconditional, or not an SC sentence. For each of the expressions that is an SC sentence, give a structure tree:

- a)  $\neg(P \vee (Q \wedge \neg R))$
- b)  $(\neg P \vee (Q \wedge \neg R))$
- c)  $((\neg P \vee Q) \wedge \neg R)$
- d)  $\neg \neg (\neg \neg \phi \leftrightarrow \neg \neg \neg \neg \psi)$
- e)  $((A \rightarrow B) \rightarrow C) \rightarrow ((A \rightarrow C) \rightarrow C)$

2. Give an example of each of the following:

- a) a disjunction both of whose disjuncts are atomic
- b) a disjunction both of whose disjuncts are conjunctions both of whose conjuncts are disjunctions.
- c) a conditional whose antecedent is a biconditional and whose consequent is a negated biconditional.

3. Describe an algorithm that, given an SC sentence as input, determines which of the six categories (atomic, negation, conjunction, disjunction, conditional, or biconditional) it falls into. Explain why your algorithm gives the correct classification of each of the SC sentences from problem 1.

4. A *substitution* is an operation that replaces each atomic sentence by an SC sentence, making sure that the same replacement is made for all instances of a given atomic sentence. In other words, a substitution is a function  $s$  associating SC sentences with SC sentences that meets the following conditions:

$$\begin{aligned} s(\phi \vee \psi) &= (s(\phi) \vee s(\psi)) \\ s(\phi \wedge \psi) &= (s(\phi) \wedge s(\psi)) \\ s(\phi \rightarrow \psi) &= (s(\phi) \rightarrow s(\psi)) \\ s(\phi \leftrightarrow \psi) &= (s(\phi) \leftrightarrow s(\psi)) \\ s(\neg \phi) &= \neg s(\phi) \end{aligned}$$

Let  $s$  be a substitution with  $s("P") = "(P \vee Q)"$ ,  $s("Q") = "((P \vee Q) \rightarrow R)"$ , and  $s(\phi) = \phi$  for  $\phi$  an atomic sentence other than " $P$ " or " $Q$ ."

- a) What is  $s("(P \vee R)")$ ?
- b) What is  $s("(P \vee Q)")$ ?
- c) What is  $s(s("(P \vee R")))$ ?
- d) Give two distinct SC sentences  $\phi$  and  $\psi$  with  $s(\phi) = s(\psi)$ .