

Subject 24.241. Logic I. Homework due Thursday, October 27

In doing the derivations, you may use any of the theorems and derived rules listed at the end of the section "Substitution of Equivalents" on the Stellar site.

I. Please derive each of the following sentences from the empty set of premisses:

- a) $((P \rightarrow (Q \rightarrow (R \rightarrow S))) \rightarrow (R \rightarrow (P \rightarrow (Q \rightarrow S))))$
- b) $((P \rightarrow (Q \rightarrow R)) \rightarrow ((P \rightarrow Q) \rightarrow (P \rightarrow R)))$
- c) $((P \rightarrow Q) \vee (\neg P \rightarrow R))$
- d) $((P \rightarrow Q) \vee (Q \rightarrow R))$
- e) $((P \rightarrow R) \wedge (Q \rightarrow R)) \rightarrow ((P \vee Q) \rightarrow R)$

2. Translate each of the following arguments into the language of the sentential calculus, then derive the translated conclusion from the translated premisses:

- a) If the top card is an ace, then, unless the second card is also an ace, you'll win \$50.
If the top card and the second card are both aces, you'll win \$100.
Therefore, if the top card is an ace, you'll win either \$50 or \$100.
- b) If God is perfectly good, then there is evil in the world only if God cannot prevent it.
If there is evil in the world and God cannot prevent it, then God is not omnipotent.
There is evil in the world.
Therefore, God is not both omnipotent and perfectly good.

3. Suppose that Γ is a set of SC sentences with the following two properties:

- a) Every sentence derivable from Γ is an element of Γ .
- b) For each sentence, either the sentence or its negation is in Γ .

Show that, unless Γ is the set of all sentences, Γ is a complete theory.