Logic I - Session 13

Plan

- Damien on psets
- Quick summary of completeness
- Compactness
- Limitations of SL
- Intro to PL

Completeness

$\Gamma \nvdash \mathbf{P}$

↓

$\Gamma \cup \{\sim\mathbf{P}\}$ is C-SD

↓

$\Gamma \cup \{\sim\mathbf{P}\} \subseteq$ a MC-SD set Γ^*

↓

If <u>Γ^* is MC-SD</u> then <u>Γ^* is TF-C</u>

↓

$\Gamma \cup \{\sim\mathbf{P}\} \subseteq$ a TF-C set Γ^*

↓

$\Gamma \cup \{\sim\mathbf{P}\}$ is TF-C

↓

$\Gamma \nvDash \mathbf{P}$

Compactness

- A cool result of completeness:
  - **Compactness**: Γ is TF-C iff every finite subset of Γ is TF-C.
- So: a set Γ is TF-IC only if a finite subset of Γ is TF-IC.
- So, intuitively, there's no TF inconsistency that you need an infinite number of SL sentences to get!
- Let's prove compactness by proving each direction.

Compactness

- First, left-to-right:
  - If Γ is TF-C, then every finite subset of Γ is TF-C.
    - If there were a subset Γ- such that no TVA m.e.m. Γ- true, then there would be no TVA m.e.m. Γ true.
- Now, right-to-left:
  - If every finite subset of Γ is TF-C, then Γ is TF-C.
  - Equiv: **If Γ is TF-IC, then some finite subset Γ- of Γ is TF-IC.**

Compactness

- Assume **Γ is TF-IC**. Then there's no TVA that m.e.m. Γ true.
- So every TVA that m.e.m. Γ true makes some **R&~R** true.
- That is: $\Gamma \vDash$ **R&~R**.
- So by completeness, $\Gamma \vdash$ **R&~R**.
- But since every derivation is finite, there's a finite $\Gamma' \subseteq \Gamma$ such that $\Gamma' \vdash$ **R&~R**.
- So by soundness, $\Gamma' \vDash$ **R&~R**.
- But no TVA makes **R&~R** true.
- So no TVA makes Γ' true. I.e. Γ' is TF-IC.

- So if **Γ is TF-IC**, then **a finite subset of Γ is TF-IC.**

Limitations of SL

- We want our formal language and derivation system to help us prove that certain arguments are valid, that certain sets of sentences are inconsistent, etc...
- SL can't do that for some arguments and sentences.
- Everything in the house smells bad.
  Fido is in the house.
  So, Fido smells bad.
- Nothing has horns and also wings.
  Some animals at Neverland Ranch have horns.
  All chickens have wings.
  So not all animals at Neverland Ranch are chickens.