

Philosophy 244: #15— Identity and Descriptions

The last class ended with a paradox. It's a theorem of quantified modal logic with identity that $x=y \supset \Box x=y$. But this conditional seems false when applied to

The first PM General = the inventor of bifocals.

That is, it seems false that

The first PM General = the inventor of bifocals $\supset \Box$ (The FPMG = the IB)

There is perhaps a way of reading the consequent so that it comes out true:

Concerning the x and y who were FMPG and IB, necessarily $x = y$.

But it remains that there's another, and more natural reading, where it comes out false. What gives?

A key feature of *The first PMG = the inventor of bifocals* is that it's formulated with *definite descriptions* and not variables. Bertrand Russell way back when had already noticed problems with these. Take

The present king of France is bald.

bearing in mind that France is (and was) a republic. In the absence of a present king of France this statement is apt to seem false. But for the same reason

The present king of France is not bald.

seems also false. Isn't this a violation of the laws of logic (excluded middle, $P \vee \neg P$)? The answer is that it is if there's a (nonexistent) individual you're talking about here, but if not, well then maybe not. But this doesn't really get us very far, because the question remains, how does it manage *not* to violate excluded middle? How can *The KoF is bald* or *The KoF is not bald* be true if there's nothing for the singular term to refer to? Consider for that matter

The king of France (largest prime number, perfect cocktail,...) does not exist.

Russell thought it was nonsense to hear such a sentence as predicating of a nonexistent object "its" property of not existing. But how can a sentence *t is P*, where *t* is a term and *Q* a predicate, fail to predicate *Q*-ness of *t*?

This is where Russell had a new idea. If the term *t* is a variable or name, it can't. But if it's a definite description, something of the form *the φ*, it can. *The φ is ψ* means, and is true if and only if,

- (a) there's a least one φ
- (b) there's at most one φ
- (c) all φ s are ψ s.

Take *The first PMG invented bifocals*. It is true iff there's at least one first PMG, there's at most one first PMG, and all first PMGs invented bifocals. This makes clear intuitive sense and can even be formalized in LPC with identity.

The φ is $\psi = \exists x\varphi(x) \& \forall x\forall y(\varphi(y) \& \varphi(z) \supset y=z) \& \forall x(\varphi(x) \supset \psi(x))$

The way *The king of France is bald* and *The king of France isn't bald* manage to both be false is that the second isn't really the negation of the first. One says that there's a unique KoF and he's bald, the other that there's a unique KoF and he's not bald. The first conjunct fails in both cases. To negate *The KoF is bald* we'd have to say that one conjunct fails or the other: either there are no French kings, or there are at least two, or not all of them are bald. This is a possible reading of *The KoF is not bald*, just not the most natural one. The sentence can be translated into logic-ese either as the negation of *The KoF is bald* (in which case it's true) or as attributing the negation of baldness to the KoF (in which case it's false). The confusion arises because there are two ways of reading *not* in connection with a definite description:

External negation $\neg(\exists x\varphi(x) \& \forall y\forall y(\varphi(x)\&\varphi(y)\supset y=x) \& \forall x(\varphi(x)\supset\psi(x)))$

Internal negation $\exists x\varphi(x) \& \forall y\forall y(\varphi(x)\&\varphi(y) \supset y=x) \& \forall x(\varphi(x)\supset\neg\psi(x))$

This can be simplified we write the conjunction of (a) and (b) as $\exists!x\varphi(x)$ (E-shriek). Then the two readings are

External negation $\neg(\exists!x\varphi(x) \& \forall x(\varphi(x)\supset\psi(x)))$ and

Internal negation $\exists!x\varphi(x) \& \forall x(\varphi(x)\supset\neg\psi(x))$.

An alternative and more popular terminology focuses on the scope of the quantifier(s) rather than the operator. A quantifier takes wide scope if the operator lies within its scope; it takes narrow scope if it lies within the scope of the operator. External negation corresponds to the narrow scope reading and internal to the wide scope reading.

The contrast between wide and narrow scope readings takes on an even greater significance when it comes to identity statements. On Russell's account *The $\varphi = the \psi$* becomes

$\exists!x\varphi(x) \& \exists!x\psi(x) \& \forall x\forall y(\varphi(x)\&\psi(y)\supset x=y)$.

Scope ambiguities arise when we try to negate the identity.

Narrow Scope = External Negation $\neg(\exists!x\varphi(x) \& \exists!x\psi(x) \& \forall x\forall y(\varphi(x)\&\psi(y)\supset x=y))$.

Wide Scope = Internal Negation $\exists!x\varphi(x) \& \exists!x\psi(x) \& \forall x\forall y(\varphi(x)\&\psi(y)\supset\neg x=y)$.

As before, as long as the descriptions are uniquely satisfied these come out with the same truth value; so the contrast is somewhat trivial. It quickly stops being trivial, though, when we switch operators. Just as with \neg , there are two places for the \square to go relative to the quantifiers.

Narrow Scope = External Necessitation $\square(\exists!x\varphi(x) \& \exists!x\psi(x) \& \forall x\forall y(\varphi(x)\&\psi(y)\supset x=y))$.

Wide Scope = External Necessitation $\exists!x\varphi(x) \& \exists!x\psi(x) \& \forall x\forall y(\varphi(x)\&\psi(y)\supset\square x=y)$.

This is a distinction we've already bumped against. Why does *The FPMG = the IB* usually seem contingent but sometimes necessary? There are two ways to think of the matter. It's necessary is that its *internal* necessitation is true; it's contingent in that its *external* necessitation is false.

Russell was able to show that if $\exists!x\varphi(x)$ then the two readings agree in truth-value; in that case the contrast is only syntactic, not truth-conditional.

Internal/external is important in lots of quantified modal statements, whether they involve identity or not. Quine tries to convince us that there is no coherent property of being necessarily odd by pointing out that *Nine is necessarily odd* seems true while *The # of planets is necessarily odd* seems false. The latter though is ambiguous between an internal, wide scope reading and an external, narrow scope reading. And the wide scope reading is true. The falsity of the narrow scope disambiguation is irrelevant since it doesn't even purport to attribute a de re modal property to nine; rather it attributes de dicto necessity to *The # of planets is odd*

According to Russell, it follows from his analysis that definite descriptions aren't genuine singular terms. The appearance of a unitary term dissolves under analysis. Another way of saying the same thing is that a phrase like *the inventor of bifocals* receives not an explicit definition but a contextual one. An explicit definition would tell us what it stands for. The proposed contextual definition assigns no referent but explains the phrase's impact on the larger statements in which it is liable to occur. made by the entire sentences in which it occurs.

All that having been said, Russell did want a unified *notation* for embedded descriptions, and he chose the following: $\psi([\iota x]\varphi(x))$. This should be read as characterizing $[\iota x]\varphi(x)$ (*the first PMG*) as ψ (*an inventor*, say). Logically and linguistically people have come to take the iota notation to express a constituent after all: a kind of binary quantification. This is also a popular construal of *all*, *some*, *many*, and so on, which makes for a nice pattern:

- (every $x:\varphi(x)$) $\psi(x)$
- (many $x:\varphi(x)$) $\psi(x)$
- (the $x:\varphi(x)$) $\psi(x)$

The internal/external distinction for definite descriptions in modal contexts now becomes an analogue of the de re/de dicto ambiguities encountered earlier, as in, *All that glistens is not gold* (external negation, despite appearances) and *If you know something, it's got to be true* (external necessitation, sometimes fallaciously made internal).

- $\Box \forall x (Fx \supset Gx)$
- $\forall x (Fx \supset \Box Gx)$
- $\Box (\iota x:Fx) Gx$
- $(\iota x: Fx) \Box Gx$

Again, if the operator is \neg , scope ambiguities are truth-conditionally irrelevant if there's a unique φ . Nothing like that holds for \Box , so we have to be super-careful about the scope of definite descriptions in modal contexts.

Names and Function Symbols

Just saw how to add definite descriptions to the modal language; we treat them as newfangled quantifiers. The result though is that definite descriptions aren't referring phrases. And so we are still without any way of doing formally what we apparently do all the time in English, namely, refer to things by use of terms.

Not to say an internal reading is out of the question: if I know something, I can't be wrong.

Here's how to do it, starting with individual constants or names. Names a, b, \dots can occur in exactly same nonquantificational positions as variables. The difference is that each one is interpreted as standing for a "fixed" object $o = V(a)$, as opposed to a "variable" object given by the variable assignment μ . Using "term" to cover both individual variables and individual constants, we can say that if φ is an n -place predicate and $t_1 \dots t_n$ are terms, then $\varphi(t_1 \dots t_n)$ is a wff.

$$[Vt] \ V_{\mu}(\varphi(t_1 \dots t_n), w) = 1 \text{ iff } \langle V_{\mu}(t_1) \dots V_{\mu}(t_n), w \rangle \in V(\varphi).$$

Notice that V assigns a name a the same object in every world, or more exactly, a name stands for an object period; it's $o = V(a)$, not $o = V(a, w)$.

An idea advocated by Quine is that names are theoretically dispensable. For each would-be name a we could have a one-place predicate φ_a such that $\langle o, w \rangle \in V(\varphi)$ iff $o = V(a)$. The role of a would then be played by $(\iota x: \varphi_a x)$. Instead of $\psi(a)$ we'd say $(\iota x: \varphi_a x) \psi(x)$. If we like Russell's Theory of Descriptions, it can now be used to eliminate even the iota-phrase.

Here's a problem that may have occurred to you. Names lack scope. $\Box \psi(a)$ is unambiguous. Definite descriptions having scope, how can a description hope to take the place of a name? Suppose we want to necessitate $(\iota x: \varphi_a x) \psi(x)$. Is the result to be $\Box (\iota x: \varphi_a x) \psi(x)$ or $(\iota x: \varphi_a x) \Box \psi(x)$? As you might expect and hope, it turns out to make no difference; the truth value is the same in either case.

What about function symbols? An example of a binary function symbol is $+$ in arithmetic. It takes two terms t and t' and forms with them an additional term, viz. $t+t'$. Generally speaking if θ is an n -place function symbol and $t_1 \dots t_n$ are terms, then $(\theta t_1 \dots t_n)$ is a term. V maps θ to a function $V(\theta)$ from n -tuples of domain elements to domain elements. The evaluation rule is

$$[V\theta] \ V_{\mu}(\theta(t_1 \dots t_n)) = V_{\mu}(\theta)(V_{\mu}(t_1) \dots V_{\mu}(t_n)).$$

Functional terms are dispensable too, it turns out! The work of θ could be done by an $n+1$ place predicate φ_{θ} , where $\varphi_{\theta}(x_1 \dots x_n y)$ means that $y =$ the result of applying θ to $x_1 \dots x_n$. The role of $\theta(t_1 \dots t_n)$ would be played by $(\iota y: \varphi_{\theta}(t_1 \dots t_n y))$. Note the absence of any world-variable in $[V\theta]$. Functional terms, like names, stand for the same object in every possible world; they are *rigid*. Similarly it is always the same value of y that makes $V_{\mu}(\varphi_{\theta}(x_1 \dots x_n y), w) = 1$. Hence again the truth-value of $\Box (\iota y: \varphi_{\theta}(t_1 \dots t_n y)) \psi(y)$ always agrees with that of $(\iota y: \varphi_{\theta}(t_1 \dots t_n y)) \Box \psi(y)$.

Don't rejoice yet, however. All of this is because functional terms have been interpreted here as rigid. A lot of English expressions with a functional look to them are not functional no longer. Take *oldest-child*(Pat) or *combined-weight*(Jupiter, Mars). These yield a different value in different worlds for a fixed input.

If we want to accommodate English functional terms, we need to be looking at non-rigid, world-relative function symbols. While we're at it we may as well look into the possibility of non-rigid *names*. Otherwise we will have to regard it as a necessary truth that water = H₂O. Can a necessary truth really call for empirical investigation? Next time: contingent identity and intensional objects.

This should remind u of Russell's result about negation. Scope differences vis a vis negation collapse if the definite description is uniquely satisfied. Scope differences vis a vis necessitation collapse if the definite description is uniquely satisfied in every world, and by the same object. (This is rigidity.) Since there is no need for "nominal" definite descriptions to play a scope indicator role there is no danger of confusion if we write $(\iota x \varphi_a x)$ wherever we would have written a .

What's an example of a ternary function symbol? Can it be decomposed into binaries?

Functional definite descriptions playing no scope-indicator role, we may write $(\iota y: \varphi_{\theta}(t_1 \dots t_n y))$ wherever we would have written $\theta(t_1 \dots t_n)$.

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