Engineering Economics:
Session 2

What is Value?

Review: Cash Flow Equivalence

Type	Notation	Formula	Excel
Single	Compound Amount (F/P,i,N)	$F = P(1+i)^N$	
	Present Worth (P/F,i,N)	$P = F/(1+i)^N$	
Uniform Series	Compound Amount (F/A, i, N)	$F = A\left(\frac{(1+i)^N - 1}{i}\right)$	
	Sinking Fund (A/F, i, N)	$A = F\left(\frac{i}{(1+i)^N - 1}\right)$	
	Present Worth (P/A, i, N)	$P = A\left(\frac{(1+i)^N - 1}{i(1+i)^N}\right)$	
	Capital Recovery (A/P, i, N)	$A = P\left(\frac{i(1+i)^N}{(1+i)^N - 1}\right)$	

Single Payment Example
Finding P given F

- **An investor can purchase land that will be worth $10k in 6 years**
- **If the investor's discount rate is 8%, what is the max they should pay today?**

$$P = F(P/F,i,N) = \frac{F}{(1+i)^N}$$

$$= \frac{\$10,000}{(1+0.08)^6}$$

$$= \$10,000 \cdot 0.6302$$

$$= \$6,300$$

Single Payment Example
Solving for i or N

- **What rate of return will you need to double your investment in 10 years?**

$$F = P(F/P,i,N) = P(1+i)^N$$

$$P(1+i)^{10} = 2P$$
$$(1+i)^{10} = 2$$
$$\ln(1+i)^{10} = \ln(2)$$
$$10 \bullet \ln(1+i) = \ln(2)$$
$$e^{\ln(1+i)} = e^{\frac{\ln(2)}{10}}$$
$$1+i = e^{\frac{\ln(2)}{10}}$$
$$i = e^{\frac{\ln(2)}{10}} - 1$$
$$i = 7.2\%$$

Single Payment Example
Solving for i or N

- **How many years must elapse for an investment to double at a rate of return of 6%?**

$$F = P(F/P,i,N) = P(1+i)^N$$

$$P(1+0.08)^N = 2P$$
$$(1.08)^N = 2$$
$$\ln(1.08)^N = \ln(2)$$
$$N \bullet \ln(1.08) = \ln(2)$$
$$N = \frac{\ln(2)}{\ln(1.08)}$$
$$N = 9$$

Discount Rate Approximation:
"Rule of 70 or 72"

- **To approximate effect of discounting: "Rule of 72" or "Rule of 70"**
  - **Number of years to double = 70 / Interest rate (in percent)**

$$P(1+i)^N = 2P$$
$$(1+i)^N = 2$$
$$\ln(1+i)^N = \ln(2)$$
$$N \bullet \ln(1+i) = \ln(2)$$
$$N = \frac{\ln(2)}{\ln(1+i)}$$

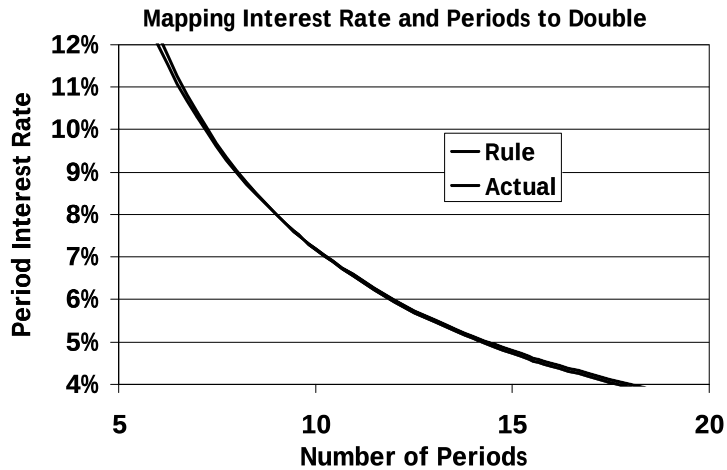
for small x

$$\ln(1+x) \approx x$$
$$\therefore$$
$$N \approx \frac{\ln(2)}{i} \approx \frac{0.69}{i}$$
$$N \approx \frac{70}{i\%}$$

Discount Rate Approximation:
"Rule of 70 or 72"

- **To approximate effect of discounting: "Rule of 72" or "Rule of 70"**
  - **Number of years to double = 70 / Interest rate (in percent)**
- **Examples**
  - **When would $1000 invested at 10% double?**
    Rule → 7.2 years Actual → 7.273
  - **What is the value of $1000 in 8 years, at 9%?**
    Rule → $2,000 Actual → $1,993

Discount Rate Approximation:
“Rule of 70 or 72”



Review:
Finite Series of Equal Payments

a) Future Value (F)

$$= \sum_{i}^{N} A(1+r)^{i}$$

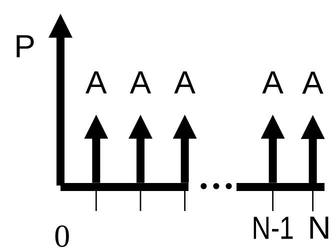
$$= A\,\frac{[(1+r)^{N}-1]}{r}$$

b) Payment (A)

$$= P \times r\,\frac{[(1+r)^{N}]}{[(1+r)^{N}-1]}$$

$$= P\ (crf)$$

crf = Capital Recovery Factor



Using the Compound Amount Factor: Finding F, Given i, A, N

- **Suppose:**
  - **You put $3k into savings for 10 years (@end of ea. yr)**
  - **Your savings account earns 7%**
  - **What is your account worth after 10years?**

$$\begin{aligned} F &= A(F/A,i,N) \\ &= A\left(\frac{(1+i)^N-1}{i}\right) \\ &= \$300\left(\frac{(1.07)^{10}-1}{0.07}\right) \\ &= \$300(13.82) \\ &= \$4,145 \end{aligned}$$

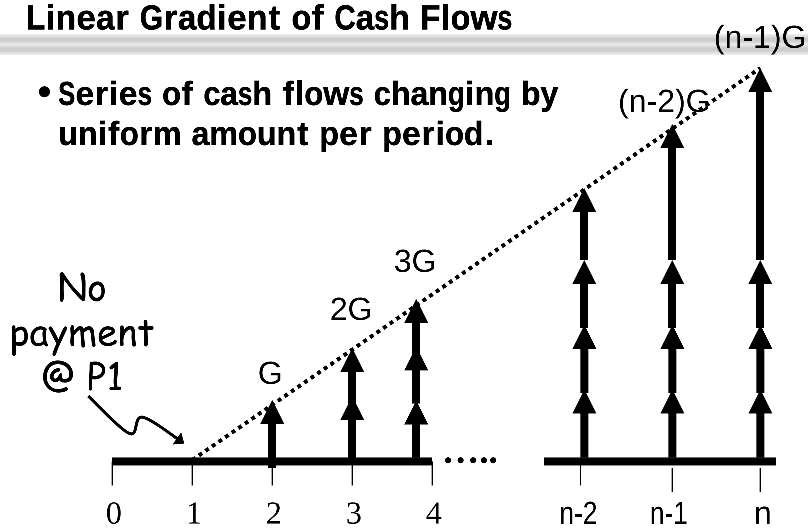
Using the Capital Recovery Factor: Finding A, given P

- **Suppose:**
  - **Your firm purchases lab equipment for $250k**
  - **The loan's interest rate is 8%**
  - **What payment will repay the loan?**

$$\begin{aligned} A &= P(A/P,i,N) \\ &= P\left(\frac{i(1+i)^N}{(1+i)^N-1}\right) \\ &= \$250,000\left(\frac{0.08(1.08)^6}{(1.08)^6-1}\right) \\ &= \$250,000(0.2163) \\ &= \$54,075 \end{aligned}$$

Other Special Cases: Linear Gradient of Cash Flows

- **Series of cash flows changing by uniform amount per period.**



Deriving Equivalence for a Linear Gradient of Payments

$$P = 0 + \frac{G}{(1+i)^2} + \frac{2G}{(1+i)^3} + \ldots + \frac{(N-1)G}{(1+i)^N}$$

$$P = \sum_{n}^{N} \frac{(n-1)G}{(1+i)^n}$$

Let $x = 1/(1+i)$

$$P = 0 + ax^2 + 2ax^3 + \ldots + (N-1)ax^N$$

$$P = ax(0 + x + 2x^2 + \ldots + (N-1)x^{N-1})$$

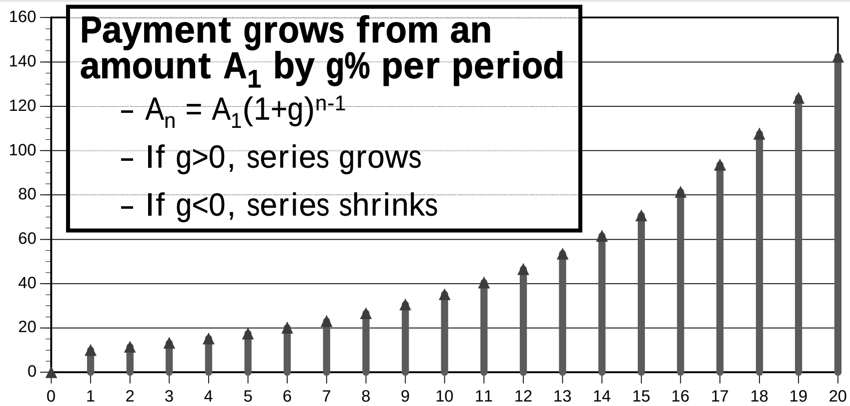
$$0 + x + 2x^2 + \ldots + (N-1)x^{N-1} = x\left[\frac{1 - Nx^{N-1} + (N-1)x^N}{(1-x)^2}\right]$$

$$P = G\left(\frac{(1+i)^N - iN - 1}{i^2(1+i)^N}\right)$$

Other Special Cases:
Geometric Series

Payment grows from an amount A_1 by g% per period

- $A_n = A_1(1+g)^{n-1}$
- If g>0, series grows
- If g<0, series shrinks



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Department of Materials Science & Engineering

Geometric Gradient of Payments

$$P = 0 + \frac{G}{(1+i)^2} + \frac{2G}{(1+i)^3} + \dots + \frac{(N-1)G}{(1+i)^N}$$

$$P = \sum_{n=1}^{N} A_1 \frac{(1+g)^{n-1}}{(1+i)^n}$$

$$P = \begin{cases} A_1\left(\dfrac{1-(1+g)^N(1+i)^{-N}}{i-g}\right) & ,\text{if } i \neq g \\ \dfrac{NA_1}{(1+i)} & ,\text{if } i = g \end{cases}$$

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Example Problem: Geometric Series

- Facility has aging cooling system which currently runs 70% of the time the plant is open
  - Pump will only last 5 more years. As it deteriorates, the pump run time is expected to increase 7% per year
- New cooling system would only run 50% of the time
- Assumptions
  - Either pump uses 250 kWh, Electricity cost $0.05/KWh
  - Plant runs 250 days per year, 24 hours per day
  - Firm's discount rate is 12%
- What is the value of replacing the pump?

Example Problem: Geometric Series

- **Current pump power cost =** 70% x 250kWh x $0.05/kWh x 250 days x 24 hrs/day = $52,500
- New pump power Cost = $37,500

$$P_{Old} = \$52,500\left(\frac{1-(1.07)^{5}(1.12)^{-5}}{0.12-0.07}\right)$$

$$= \$214,360$$

$$P_{New} = \$37,500(P/A, 12\%, 5)$$

$$P_{New} = \$37,500(3.605)$$

$$= \$135,200$$

$$Value = P_{Old} - P_{New} = \$79,160$$