3.185 Final Exam

Adam Powell

Monday December 15, 2003

Guidelines:

- This is a 3-hour final examination, with a closed book portion (Part A) and an open book portion (Part B).
- You will receive *both* Part A *and* Part B of the exam at the beginning of the three hours. You may (and are *strongly* advised to) look over the entire exam to decide how to partition your time. You *must*, however, work on the closed book portion first, and turn in your answers before you open any reference materials for the open book portion.
- You may use a calculator and the attached equation sheet in part A.
- You may use any non-human resources in part B (in the medical sense, not the philosophical sense), except for classes of devices which can communicate (*i.e.* no cell phones, wireless-networked PDAs, or laptops—with or without wireless networking).
- You may answer the questions within each part of the exam in any order you like.
- Start answering each question on a fresh sheet of paper. Read the questions carefully before attempting to answer them. *Please* write legibly.
- Write your name on *every* loose sheet of paper that you hand in.
- Graded exams and final grades will be available by December 23 (at the latest), but I may or may not be around to give them to you. If you wish to have these mailed to you, provide an address (U.S. Mail or interdepartmental) on the front cover of any of your exam answer booklets. You may also indicate that you wish to pick up your exam from my office during or after IAP. If you do not specify otherwise, I will send it to your term address if you live in a dormitory, or hold on to it otherwise. (Interdepartmental mail is notoriously unreliable to FSILGs—two years ago, a student with questions about his final didn't get it from interdepartmental mail until May!)
- I hope you do well. If everybody gets above a 90, I will give out a lot of As!

Sorry, no points for your names this time.

Part A: Closed Book

1. Time Scale and Fully-Dveloped Flow (18)

   (a) Write expressions for the timescale required to reach steady-state in diffusion, heat conduction and fluid flow (when limited by diffusive/viscous transport). (8)

   (b) Use the last of these to derive an expression for entrance length flow through a channel of thickness H, based on the distance traveled from the entrance at the average velocity over the steady-state timescale. (5)

   (c) The actual expression for the entrance length in channel flow is:

   $$L_e = \frac{H^2 u_{av}}{100\nu}.$$

   Describe why this entrance length is longer or shorter than the length you derived in part 1b. (5)

2. Biot and Nusselt Numbers (12)

   The Biot and Nusselt numbers are similar in form, but quite different in function. When comparing them here, please use either heat or mass transfer consistently throughout the problem.

   (a) Give expressions for the Biot and Nusselt numbers and show how those expressions differ. (4)

   (b) Describe the physical significance of the Biot and Nusselt numbers, illustrating that they are ratios of very different things. (8)

3. Floating Bubbles Out of Molten Glass (26)

   Entrapped bubbles are a major problem in glass processing. Here you will evaluate the possibility of removing bubbles by flotation.

   (a) Sketch the streamlines describing flow past a rising bubble in the bubble frame of reference. (4)

   (b) Based on a force balance, derive an expression for terminal velocity of spherical bubbles rising through a liquid in Stokes flow. (7)

   (c) Glass with viscosity $\mu = 1.0\frac{\text{kg}}{\text{m}\cdot\text{s}}$ and density $\rho = 3200\frac{\text{kg}}{\text{m}^3}$ sits in a container 0.1 m deep. What is the largest bubble size remaining in the glass after one minute? (7)

   (d) How much more time do you have to wait to cut the largest bubble diameter in half? (4)

   (e) Verify that Stokes flow applies for both cases. (4)

4. Estimating Fluid Velocity Using Heat Transfer (27)

A fluid is flowing below a solid plate at unknown velocity, but at known temperature T_∞. The top of the plate is at a known temperature T_0, and is losing heat by convective transport to the air above it: $q = h_{air}(T_0 - T_{air})$.

For this problem, you may assume:

- Everything is steady-state.
- The boundary layers in the fluid below the plate are much thinner than the length, and flow in them remains laminar.
- The fluid Prandtl number is about 10.
- The solid thermal conductivity is large enough relative to heat transfer coefficients above and below that the plate temperature is a roughly uniform $T = T_s$ across the bottom.
- $T_\infty > T_s > T_0 > T_{air}$.

(a) Sketch the velocity and thermal boundary layers in the fluid beneath the plate. (5)

(b) Outline the steps, including functional relationships (*e.g.* friction factor=f(Re)) and the few equations which you are required to memorize (*e.g.* Nu_x=...), required to calculate the average heat transfer coefficient from the fluid to the plate based on its free-stream velocity U_∞, plate length, and fluid properties. (8)

(c) If the solid thermal conductivity is large enough to satisfy the fourth assumption above, is the Biot number likely to be large or small? What other temperature in the system will most likely be closest to T_s? (5)

(d) Outline the steps required to estimate the free stream velocity U_∞ in the fluid below the plate, based on T_0, T_{air}, T_∞, h_{air}, plate length, and properties of the fluid below the plate. (9)

5. Mass Transfer Boundary Layers (17)

(a) Sketch the development of laminar and turbulent velocity boundary layers in flow past a flat plate. Your sketch need not be to scale. (4)

(b) If the flat plate is slowly dissolving into the fluid with a Prandtl number of 1000, what will be the approximate ratio of velocity boundary layer thickness to concentration boundary layer thickness δ_u/δ_C in the laminar region? (4)

(c) Write an expression for the turbulent diffusivity D_t as a function of turbulent viscosity μ_t. What is the turbulent Prandtl number? (4)

(d) At a high local Reynolds number (say $\text{Re}_x = 10^9$), sketch a graph of effective diffusivity (turbulent+molecular) vs. y (distance from the plate). Include on your sketch the transitions from your sketch in part 5a. (5)

Helpful equations for Part A

- Bubble friction factor in Stokes flow:

$$f = \frac{16}{\mathrm{Re}}.$$

- Buoyancy force on a bubble (V is bubble volume):

$$F_b = \rho_{fl} g V.$$

- Boundary layer thickness ratio for $\mathrm{Pr} < 0.1$:

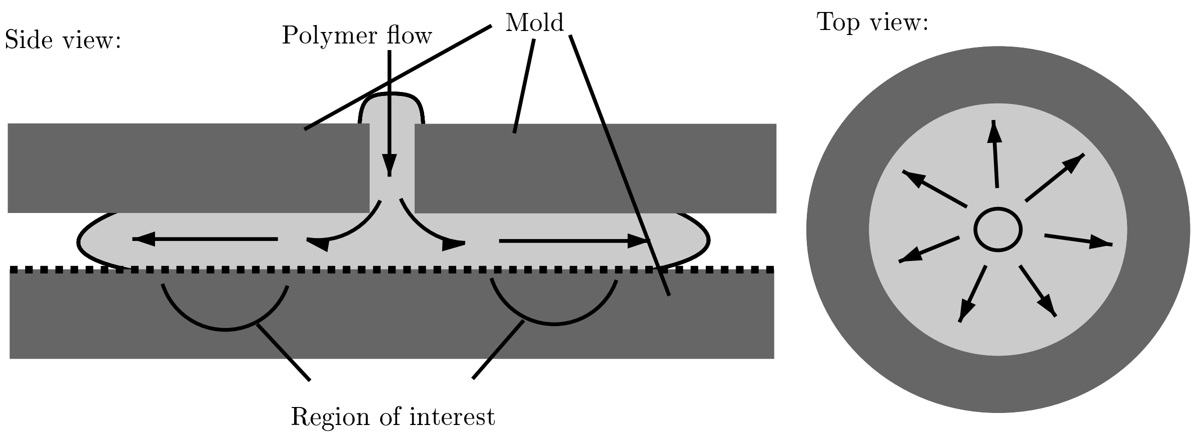
$$\frac{\delta_C}{\delta_u} = \frac{0.72}{\sqrt{\mathrm{Pr}}}.$$

- Boundary layer thickness ratio for $\mathrm{Pr} > 5$:

$$\frac{\delta_C}{\delta_u} = \frac{0.975}{\sqrt[3]{\mathrm{Pr}}}.$$

Part B: Open Book

1. Shear Stress in CD Injection Molding (24)



On Test 2, you simplified the Navier-Stokes equations describing liquid polymer flow into a CD mold. The resulting equations (should have) looked like:

$$\frac{1}{r}\frac{\partial}{\partial r}(r u_r) = 0.$$

$$\rho u_r \frac{\partial u_r}{\partial r} = -\frac{\partial p}{\partial r} + \mu \frac{\partial^2 u_r}{\partial z^2}.$$

$$0 = -\frac{1}{r}\frac{\partial p}{\partial \theta},$$

$$0 = -\frac{\partial p}{\partial z} + F_z,$$

where the r-derivative in the viscous term of the r-momentum equation is removed based on the mass conservation equation. Here you will simplify further and solve the equations.

Data:

- Polymer flow rate: $Q = 5 \times 10^{-6} \frac{\text{m}^3}{\text{s}}$.
- CD mold thickness: $\delta = 1\text{mm}$.
- Polycarbonate viscosity at injection temperature: $\mu = 200 \frac{\text{kg}}{\text{m}\cdot\text{s}}$
- Polycarbonate density at injection temperature: $\rho = 1200 \frac{\text{kg}}{\text{m}^3}$.

(a) Solve the mass conservation equation (the top one) to give a general equation for u_r as a function of r. (5)

(b) Calculate the average velocity in the r-direction at $r = 1\text{cm}$, and calculate the Reynolds number there. What additional assumption can you make, and which term in the r-momentum equation (the second one) does it let you cancel? (6)

(c) Based on the momentum equations with $F_z = -\rho g$, the pressure field is described by:

$$p = f(r) - \rho g z.$$

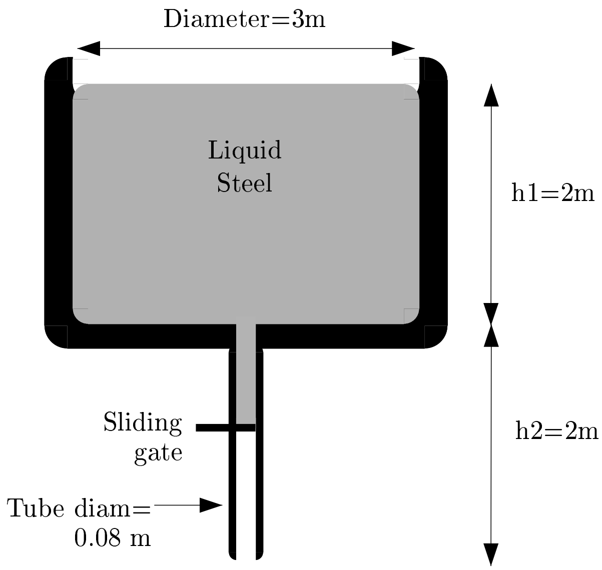
Using this, find a general solution to the r-momentum equation to give u_r as a function of z. (4)

(d) Reconcile your solutions to parts 1a and 1c to give an expression for u_r as a function of r and z. (4)

(e) Calculate the shear stress τ_{zr} against the top or bottom face of the mold at $r = 2\text{cm}$. (5)

2. Ladle Metallurgy I: Scaleup of a Lab Design (31)

A ladle is a large cylindrical vessel for chemically treating liquid steel (*e.g.* vacuum degassing, argon/oxygen bubbling, etc.) which looks something like the 100-ton ladle in the diagram below:



In a university laboratory, experimentation on this scale is completely unpractical, so researchers build a $\frac{1}{100}$ scale miniature ladle to test new technologies. Their model ladle has a liquid steel height of 2 cm and diameter of 3 cm, the tube draining it is 2 cm long and 0.8 mm in diameter, etc.

Data:

- Molten steel viscosity: $\mu = 5 \times 10^{-3} \frac{\mathrm{kg}}{\mathrm{m \cdot s}}$.
- Molten steel density: $\rho = 7000 \frac{\mathrm{kg}}{\mathrm{m^3}}$.

(a) Derive an expression for flow rate in laminar fully-developed flow through a vertical tube driven by gravity *and* a pressure gradient. (6)

(b) Use this expression from part 2a, and the assumption that pressure at the tube entrance is $p_{atm} + \rho g h_1$, to calculate the flow rate through the tube of the miniature laboratory apparatus (with the gate open). (5)

(c) Verify that flow in part 2b is laminar and fully-developed, satisfying the assumptions in part 2a. (f you didn't get an answer to part 2b, describe how you would verify these assumptions.) (5)

So the researchers do the experiment and measure the flow rate, and it agrees well with your results from part 2b. They team up with a steel company to scale the design to full scale. They think they can use the same equation to estimate flow rate. But because you've taken 3.185, you know better than that...

(d) Calculate the velocity and flow rate through the tube in the full-scale ladle full of liquid steel using a methodology more appropriate for this larger size, even if not exactly accurate. (5)

(e) Estimate the velocity boundary layer thickness at the exit of the tube. How accurate is the methodology you used in part 2d? If not accurate, does it overestimate or underestimate the flow rate? (6)

(f) Are these misguided researchers overestimating or underestimating the flow rate by using the laminar fully-developed flow expression? (4)

3. Ladle Metallurgy II: Natural Convection (20)

   Unless the liquid metal is stirred, natural convection plays a dominant role in fluid flow within the ladle (whether the gate is open or closed), because metal loses heat to the surroundings through the somewhat cooler ladle. Use the full-scale dimensions of the ladle for this problem.

   Data:

   - Liquid steel thermal conductivity: $k = 15\frac{\mathrm{W}}{\mathrm{m \cdot K}}$.
   - Liquid steel heat capacity: $c_p = 700\frac{\mathrm{J}}{\mathrm{kg \cdot K}}$.
   - Liquid steel thermal expansion coefficient: $\beta = 10^{-5}\mathrm{K}^{-1}$.
   - Temperature difference between steel bulk and ladle inner wall: $\Delta T = 100K$.

   (a) Sketch the natural convection boundary layers and direction of flow in the ladle. (5)

   (b) Calculate the Grashof and Prandtl numbers. Is flow likely to be laminar or turbulent? (5)

   (c) Estimate the average heat transfer coefficient from the liquid metal to the ladle walls. How does this compare with other heat transfer coefficients with which you are familiar, (*e.g.* hot solid metal in air or water on test 1)? (5)

   (d) As it turns out, the boundary layer thickness times circumference times average velocity total a flow rate comparable to that calculated in part 2d. What does this say about the temperature of the first metal drained from the ladle: will it be hotter or colder than the average temperature? (5)

4. The Complete Idiot's Guide to ... (25)

   Choose a process for making material (or a material whose performance in service depends on transport) with which you are familiar, either by experience at an internship/UROP or through studies in classes, and write a "Complete Idiot's Guide" to one aspect of that process consisting of three elements:

   - A very brief description of the process or performance characteristics.
   - *Either* a step-by-step guide telling how to calculate an important parameter of the process using 3.185 concepts (flow rate, heat transfer coefficient, etc.);
   - *Or* if calculation is too challenging, the (abbreviated) dimensional analysis and a step-by-step guide to physical modeling (*e.g.* wind tunnel, water modeling, etc.) required to estimate the parameter.
   - A closing sentence or two telling how calculation of this parameter is important to the design, operation or control of the process or material performance.

   (5 pts writing, 5 pts process description, 10 pts step-by-step guide, 5 pts closing/relevance to process.)