# 3.185 Math Quiz

## Solutions

1. Write your name on the top of all answer booklets you turn in.

   There were many different answers, but I think everyone got it right.

2. Vector algebra

   For the two vectors: $\vec{A} = (1, 2, 3)$, $\vec{B} = (4, 5, 6)$

   (a) Their dot product $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z = 1 \times 4 + 2 \times 5 + 3 \times 6 = 32$.

   (b) Their cross product $\vec{A} \times \vec{B}$ is the determinant of the matrix:

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = (A_y B_z - A_z B_y)\hat{x} + (A_z B_x - A_x B_z)\hat{y} + (A_x B_y - B_x A_y)\hat{z},$$

$$\vec{A} \times \vec{B} = (12 - 15)x + (12 - 6)y + (5 - 8)\hat{z} = -3\hat{x} + 6\hat{y} - 3\hat{z}.$$

   (c) Their outer product $\vec{A}\vec{B}$ (also written $\vec{A} \otimes \vec{B}$) is:

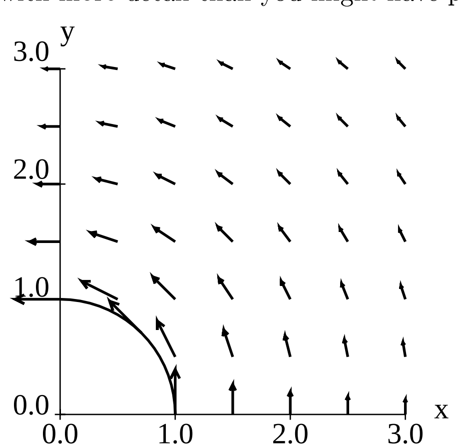
$$\vec{A} \otimes \vec{B} = \begin{pmatrix} A_x B_x & A_x B_y & A_x B_z \\ A_y B_x & A_y B_y & A_y B_z \\ A_z B_x & A_z B_y & A_z B_z \end{pmatrix} = \begin{pmatrix} 4 & 5 & 6 \\ 8 & 10 & 12 \\ 12 & 15 & 18 \end{pmatrix}$$

3. Vector calculus

   The velocity field was given as:

$$\vec{u} = \frac{1}{r}\hat{\theta} = \frac{x\hat{y} - y\hat{x}}{x^2 + y^2}$$

   (a) An example sketch (though with more detail than you might have provided):



(b) The theta velocity is: $u_\theta = 1/r$, so

$$\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(ru_\theta)\right) = \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{r}{r}\right)\right) = \frac{\partial}{\partial r}\left(\frac{1}{r}\cdot 0\right) = 0.$$

(c) The curl of this vector field, $\nabla \times \vec{u}$, is a scalar in two dimensions, and is equal to the $z$-component of the curl in three (which is the only non-zero component of the curl of a 2-D vector field):

$$\nabla \times \vec{u} = \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ u_x & u_y \end{vmatrix} = \frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y} = \frac{(x^2+y^2) - x \cdot 2x}{(x^2+y^2)^2} - \frac{-(x^2+y^2) + y \cdot 2y}{(x^2+y^2)^2}$$

$$\nabla \times \vec{u} = \frac{(y^2-x^2) - (y^2-x^2)}{(x^2+y^2)^2} = 0.$$

4. Error function derivatives

Calculate:

$$\frac{d}{dy}\mathrm{erf}(y^2).$$

The chain rule states:

$$(f(g(x)))' = g'(x)f'(g(x)).$$

Here we can use erf for $f$ and $g(y) = y^2$, so:

$$\frac{d}{dy}\mathrm{erf}(y^2) = 2y \cdot \mathrm{erf}'(y^2).$$

The fundamental theorem of calculus says that the derivative of an integral is the function itself, so that gives us erf′:

$$\frac{d}{dy}\mathrm{erf}(y^2) = 2y \cdot \frac{2}{\sqrt{\pi}} \exp\left(-(y^2)^2\right) = \frac{4y}{\sqrt{\pi}} \exp(-y^4).$$