

3.185 Test 2

Advanced Heat Transfer, Fluid Dynamics

Solutions

1. Write your name on all of your answer booklets

Congratulations! Everyone got this right. If only that were true of #2...

2. Aaliyah was obviously: (b) A pop/R&B singer who lost her life at an early age in a plane crash. Nearly all of you were culturally savvy enough to know this.

The president of Bosnia-Herzegovina during its civil war was Alia Izetbegovic, correct pronunciation but wrong spelling; he passed about a month before the test. Latin for “others” could be *alia* or *alii*, wrong spelling and pronunciation (different accent). As for the Saudi peace movement... well, I wanted a fourth option, and pulled this one out of thin air.

One person actually got this wrong, but I think she was just testing to see whether I’d actually take off points; I didn’t fall for it.

3. Molecular beam epitaxy deposition rate

(a) What I was looking for here was:

- i. Use the Clausius-Clapeyron equation to calculate the vapor pressure from the source temperature.
- ii. Use the background pressure and temperature to estimate the mean free path of sublimating atoms in the chamber.
- iii. Calculate the Knudsen number, which is the ratio of mean free path to chamber size.
- iv. If the Knudsen number is greater than one, calculate the sublimation rate using the Langmuir equation.

Since I wrote on the board that you may assume “vacuum” behavior for this whole problem, steps 3(a)ii and 3(a)iii were not strictly necessary.

Also, given the original wording of the problem, another approach which answered the stated question was to calculate the sublimation flux using a heat balance. Assuming that the difference between flux in from the heat source (electron beam) called q_v , and heat flux conducted through the solid into the cooling water q_s , is all used to sublime the atoms, this gives:

$$q_v - q_s = \delta H_s \cdot J \Rightarrow J = \frac{q_v - q_s}{\Delta H_s}.$$

- (b) This implicitly assumed that electron beam power either went into sublimation, or into the cooling water, so the flux balance is again given by:

$$q_v - q_s = \Delta H_s \cdot J.$$

The net power is then the next flux times the area:

$$P_{beam} - P_{water} = \Delta H_s \cdot J \cdot A.$$

- (c) If sublimation flux distribution follows a cosine distribution (like radiating photons), and the sublimating atoms follow straight-line trajectories through the chamber (like photons), then guess what? We can use the radiation viewfactor to calculate the fraction of sublimating atoms which land on the substrate.

From here it's straightforward: $r_1 = 2\text{cm}$, $r_2 = 9\text{cm}$, $D = 30\text{cm}$, so $D/r_1 = 15$ (effectively ∞ on the graph) and $r_2/D = 0.3$, giving $F_{12} \simeq 0.1$.

The ratio of deposition flux at the substrate to sublimation flux from the source is this times the area ratio:

$$\frac{J_{subs}}{J_{source}} = \frac{A_1 F_{12}}{A_2}.$$

4. Freezing by radiation and convection

- (a) The thermal conductivity is given by the Wiedmann-Franz law:

$$k_{el} = L\sigma_{el}T = 2.45 \times 10^{-8} \frac{\text{W}\Omega}{\text{K}^2} \cdot 5 \times 10^5 (\Omega \cdot \text{m})^{-1} \cdot 1800\text{K} = 22 \frac{\text{W}}{\text{m} \cdot \text{K}}.$$

- (b) Total flux away from the top surface is radiative plus convective (T_s is top surface temperature):

$$q_{total} = q_{rad} + q_{conv} = \epsilon\alpha_{env}\sigma(T_s^4 - T_{env}^4) + h(T_s - T_{env}).$$

- (c) If the environment is much colder, then $T_s \gg T_{env}$ so $T_{env} \simeq 0$. If it is "black", then its absorbtivity is one. So the above expression simplifies to

$$q_{total} = \epsilon\sigma T_s^4 + hT_s = h_{total}T_s,$$

$$h_{total} = \epsilon\sigma T_s^3 + h = 200 + 100 \frac{\text{W}}{\text{m}^2 \cdot \text{K}} = 300 \frac{\text{W}}{\text{m}^2 \cdot \text{K}}.$$

- (d) Set the Biot number to 0.1 and solve for Y using h_{total} :

$$\text{Bi} = \frac{h_{total}Y}{k} = 0.1 \Rightarrow Y = \frac{0.1k}{h_{total}} = \frac{0.1 \cdot 22 \frac{\text{W}}{\text{m} \cdot \text{K}}}{300 \frac{\text{W}}{\text{m}^2 \cdot \text{K}}} = 0.0073\text{m}(7.3\text{mm}).$$

- (e) You're given an equation relating flux to solidification rate, so solve it for dY/dt :

$$q_L - q_S = \rho\Delta H_f \frac{dY}{dt} \Rightarrow \frac{dY}{dt} = \frac{q_L - q_S}{\rho\Delta H_f}.$$

Since liquid metal temperature is uniform, $q_L = 0$. At quasi-steady-state, the flux through the solid is equal to the flux leaving its top surface, which is $h_{total}T_s$, T_s being the surface temperature. Since the metal temperature is roughly uniform, $T_s \simeq T_m$ so we can use that:

$$\frac{dY}{dt} = \frac{h_{total}T_m}{\rho\Delta H_f} = \frac{300 \frac{\text{W}}{\text{m}^2 \cdot \text{K}} \cdot 1800\text{K}}{7500 \frac{\text{kg}}{\text{m}^3} \cdot 2.67 \times 10^5 \frac{\text{J}}{\text{kg}}} = 2.7 \times 10^{-4} \frac{\text{m}}{\text{s}}.$$

- (f) We have resistances in series, due to conduction through the solid and evaporation/convection from the surface. So the heat flux in the solid goes like:

$$q_s = \frac{T_m - T_{env}}{\frac{Y}{k_s} + \frac{1}{h_{total}}}.$$

The trouble is, h_{total} is a function of the surface temperature T_s , which is somewhere between T_{env} and T_m . We can relate T_s to the conductive flux, which is the same as the flux q above:

$$q_s = k_s \frac{T_m - T_s}{Y} \Rightarrow T_s = T_m - \frac{qY}{k_s}.$$

We plug this into the h_{total} expression from part 4c:

$$h_{total} = \epsilon\sigma T_s^3 + h = \epsilon\sigma \left(T_m - \frac{qY}{k_s} \right)^3 + h,$$

then plug that into the heat flux:

$$q_s = \frac{T_m - T_{env}}{\frac{Y}{k_s} + \frac{1}{\epsilon\sigma \left(T_m - \frac{q_s Y}{k_s} \right)^3 + h}}.$$

Then we need to solve this for q_s (probably numerically), and use that in the solidification rate equation from part 4e.

5. Injection molding a CD

(a) Assumptions which could be made here included:

- Newtonian behavior with uniform viscosity (since that was given).
- Incompressible flow with uniform density; this and the previous assumption permitted use of the simpler closed-form Newtonian Navier-Stokes equations.
- Laminar flow (with small thickness and high viscosity this is reasonable).
- Steady-state (given); time derivatives are all zero.
- Fully-developed (given as short entrance length); flow is purely radial, $u_\theta = u_z = 0$. But velocity derivatives in the flow direction (the r -direction) are *not* zero, as conservation of mass requires that there be faster flow near the center and slower flow near the outside.
- Edge effects and axisymmetric are the same thing here; velocity derivatives in the θ direction are zero.
- Gravity acts in the negative z -direction; $F_r = F_\theta = 0$, $F_z = -\rho g$.

(b) Since $u_\theta = u_z = 0$ (because flow is fully-developed), the mass conservation equation reduces to:

$$\frac{1}{r} \frac{\partial}{\partial r} (ru_r) = 0.$$

Steady-state, fully-developed, axisymmetric, and $F_r = 0$ reduce the r -momentum equation to:

$$\rho u_r \frac{\partial u_r}{\partial r} = -\frac{\partial p}{\partial r} + \mu \left[\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial}{\partial r} (ru_r) \right) + \frac{\partial^2 u_r}{\partial z^2} \right].$$

With a bit more sophistication, one might realize that the r -derivative in the viscosity term is just a derivative of what we found to be zero in the mass equation, so that too could be eliminated.

The θ -momentum equation is almost completely eliminated:

$$0 = -\frac{1}{r} \frac{\partial p}{\partial \theta},$$

so there are no pressure gradients in the θ -direction, and the z -momentum equation only has pressure gradients due to weight of the polymer:

$$0 = -\frac{\partial p}{\partial z} + F_z \left(= -\frac{\partial p}{\partial z} - \rho g \right).$$

6. Flow rate and pressure in a tube

(a) To calculate pressure drop ΔP required to pump at a given flow rate Q :

- Calculate average velocity from flow rate: $u_{av} = Q/A_{xs} = Q/\pi R^2$.
- Calculate Reynolds number using average velocity: $Re = \rho u_{av} d/\eta$.

- If $Re < 2100$, one can use the laminar friction factor: $f = 16/Re$, or just use the Hagen-Poiseuille equation and be done with it. Otherwise, one must use a graph to determine the friction factor.
- Shear stress is friction factor times kinetic energy density: $\tau = fK = f \cdot \frac{1}{2}\rho u_{av}^2$.
- Drag force is shear stress times tube inside surface area: $F_d = \tau A = fKA = f \cdot \frac{1}{2}\rho u_{av}^2 \cdot 2\pi RL$.
- Pressure drop is drag force per unit cross-section area: $\Delta P = \frac{F_d}{A_{xs}} = \frac{F_d}{\pi R^2}$.

Combining the last three steps gives:

$$\Delta P = \frac{f \cdot \frac{1}{2}\rho u_{av}^2 \cdot 2\pi RL}{\pi R^2} = \frac{f\rho u_{av}^2 L}{R} \Rightarrow \frac{\Delta P}{L} = \frac{f\rho u_{av}^2}{R}.$$

- (b) Since the tube radius is 0.5 cm, the cross section area is $0.25\pi\text{cm}^2$, making things easier. Also, for the laminar cases we bypass last three steps above:

$$Q = \frac{\pi\Delta PR^4}{8\mu L} \Rightarrow \frac{\Delta P}{L} = \frac{8\mu Q}{\pi R^4} = \frac{8\mu u_{av}}{R^2}.$$

Flow rate, cm^3/s :	0.25π	2.5π	25π
Average velocity, cm/s :	1	10	100
Reynolds number:	100	1000	10,000
Friction factor:	0.16	0.016	0.0072
Pressure drop $\Delta P/L$, $\text{g}/(\text{cm}^2 \cdot \text{s}^2)$:	0.32	3.2	144

- (c) The third case is turbulent, and the nearly five-fold jump in the relative pressure drop (relative to what the laminar flow calculation would give) is pretty dramatic!