3.46 PHOTONIC MATERIALS AND DEVICES
Lecture 4: Ray Optics, Electromagnetic Optics, Guided Wave Optics

Lecture | Notes

Light

<u>photon</u>

- exchanges energy with medium
  - Emission
  - absorption
  - scattering

<u>electromagnetic wave</u>

- nondissipative medium
  - Propagation
  - Interference
  - Diffraction

<u>ray optics</u>

- small λ approx.
  - Geometric optics

Photon

$E = h\nu$

$h = 6.626 \times 10^{-34}$ J·s

$\lambda = \dfrac{c}{\nu}$

mass = 0; charge = 0; spin = 1

Ray Optics

<u>"Optical" properties</u>

Complex index of refraction

$n_{complex} = n + iK$

n = refractive index
K = extinction coefficient

Complex dielectric function

$\varepsilon = \varepsilon_1 + i\varepsilon_2$

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Kramers-Kronig relations

Relate $\varepsilon_1(\omega)$ and $\varepsilon_2(\omega)$
α ≡ absorption coefficient

$$\alpha = \frac{2\omega K}{c}$$

Reflectivity (normal incidence)

$$R = \frac{(n-1)^2 + K}{(n+1)^2 + K}$$

- in transparent range of ω:

  $K \to 0;\ R \to \left(\frac{n-1}{n+1}\right)^2$

<u>Snell's Law</u>

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$$

<u>Total internal reflection</u>

$$\theta_1 > \theta_{ext} = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

Reflection (materials n_1, n_2)

$$R = \left(\frac{n-1}{n+1}\right)^2 \quad \text{normal incidence}$$

Diamond: $n \approx 2.4$
TiO_2: $n = 2.6$
$ZrSiO_4$: $n = 1.9$

Material	θ_c	n	R
Water	48.6°	1.33	0.02
Glass	41.8°	1.50	0.04
Crystal glass	31.8°	1.90	0.10
diamond	24.4°	2.42	0.17

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<u>Index matching</u>

$n_{(medium)} = n_{(material)} \Rightarrow$ no reflection

<u>Anti-reflection coating</u>

$$R = \frac{n_2^2 - n_1 n_3}{n_2^2 + n_1 n_3}$$

n_1 (air) ↓

n_2 (coating) t

n_3 (material) ↑

= 0 when $n_2 = \sqrt{n_1 n_3}$

Example

for solar cell: n_3 (silicon)

$n_2 t = \frac{\lambda}{4}$ quarter wave film

for glass:

$n_3 = 1.5$; air : $n_1 = 1.0 \Rightarrow n_2 = 1.22$

MgF_2 $n_2 = 1.384 \Rightarrow R = 0.12$

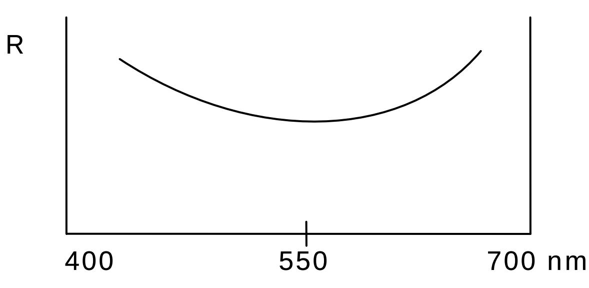
Example

AR coating for silicon

$n_{Si} = 3.5 \Rightarrow n_{AR} = 1.87$

$n_{SiO_2} = 1.51$

$\lambda = 550$ nm $\rightarrow t = 91$ nm



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Notes

Electromagnetic optics

Electromagnetic Field $\vec{E}(\vec{r},t)$, $\vec{H}(\vec{r},t)$

Maxwell's Equations

$$\vec{\nabla}\times\vec{H}=\varepsilon_0\frac{\partial\vec{E}}{\partial t} \qquad \vec{\nabla}\times\vec{E}=-\mu_0\frac{\partial\vec{H}}{\partial t}$$

$$\vec{\nabla}\cdot\vec{E}=0 \qquad \vec{\nabla}\cdot\vec{H}=0$$

Monochromatic EM Wave

$$\vec{E}(\vec{r},t)=\mathrm{Re}\left\{\vec{E}'(\vec{r})\exp(j\omega t)\right\}$$

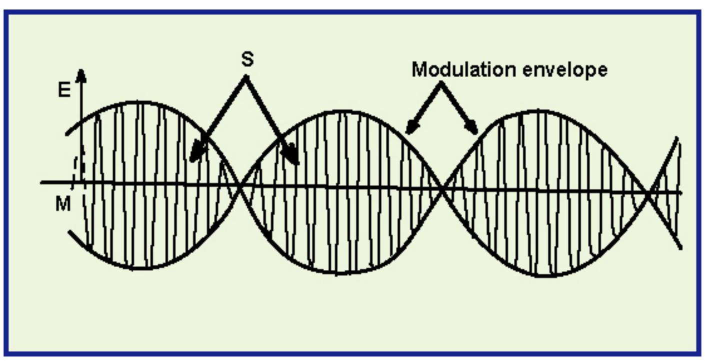
Each of the six scalar components of $\vec{E}$ & $\vec{H}$ must satisfy the Helmholtz Equation

$$\nabla^2 u+k^2 u=0$$

wave vector:

$$k = \frac{\omega}{c} = \omega(\varepsilon\mu_0)^{1/2} = nk_0 = \frac{n\omega}{c_0} = \frac{2\pi}{\lambda}$$

$c = \frac{\omega}{k}$: phase velocity; velocity $v_g=\frac{d\omega}{dk}$ = group



The carrier propagates with the phase velocity c. The slowly varying envelop propagates at the group velocity, v_g.

Transverse EM Plane Waves (TEM)

- $\vec{E}(\vec{r},t)$, $\vec{H}(\vec{r},t)$ are plane waves with wave vector $\vec{k}$
- $\vec{E}$, $\vec{H}$, $\vec{k}$ are mutually orthogonal

$$\vec{E}(\vec{r}) = E_0 e^{-j\vec{k}\vec{r}} \; , \; \vec{H}(\vec{r}) = H_0 e^{-j\vec{k}\vec{r}}$$

Phenomenology of Properties

Absorption

$$\chi = \chi' - i\chi'' \; ; \; \varepsilon = \varepsilon_0 (1 + \quad)$$

$$k = \omega(\varepsilon\mu_0)^{\frac{1}{2}} = (1 + \quad)^{\frac{1}{2}} k_0 = (1 + \chi' + i \quad '')^{\frac{1}{2}} k_0$$

$$= \beta - i\frac{1}{2}\alpha$$

$$U(x) = Ae^{-ikx} = Ae^{\frac{-\alpha x}{2}} e^{-i\beta x}$$

$$I(x) \propto |U(x)|^2 \propto e^{-\alpha x}$$

Resonant atoms in host medium

$$n(\nu) \approx n_0 + \frac{\chi'(\nu)}{2n_0} \; , \; \alpha(\nu) \approx -\left(\frac{2\pi\nu}{n_0 \quad _0}\right)\chi''(\nu)$$

Fiber materials for transmission

- Electronic polarizability not important for IR fibers
- Heavy atom → weaker bond
  → long λ_0

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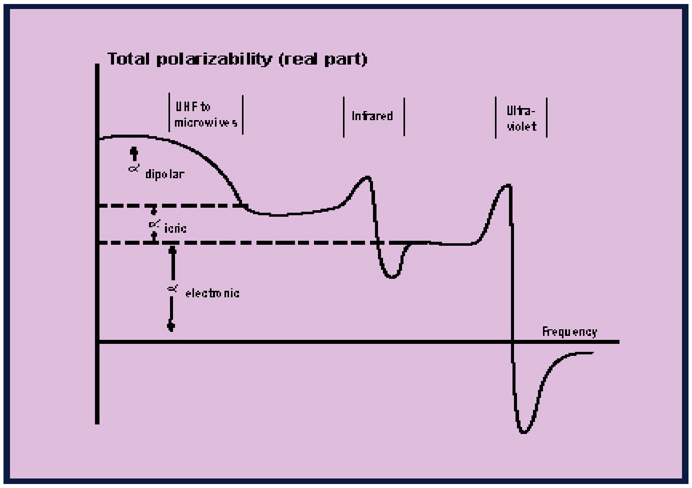
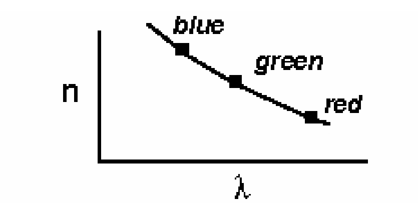
Notes

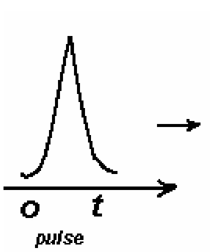


Frequency dependence of the several contributions to polarizability.

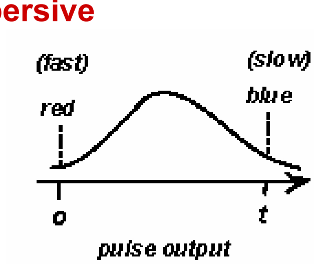
Dispersion $\equiv \frac{dn}{d\lambda}$



normal dispersion



Normal dispersive medium →



group index $n_g = n - \lambda_0 \frac{dn}{d\lambda_0}$

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Notes

group velocity

$$v_g = \frac{c_0}{n_g} = c_0\left(n - \lambda_0 \frac{dn}{d\lambda_0}\right)^{-1}$$

Dispersion coefficient

$$D_\lambda = \frac{d}{d\lambda}\left(\frac{1}{v_g}\right) = -\frac{\lambda_0}{c_0}\frac{d^2 n}{d{\lambda_0}^2}$$

$$D_\lambda = \frac{\text{temporal spread}}{\text{length} \cdot \text{spectral width}} = \frac{\text{ps}}{\text{km} \cdot \text{nm}}$$

$$|D_\lambda| \sigma_\lambda = \frac{\text{seconds of pulse broadening}}{\text{distance travel}}$$

σ_λ : spectral width

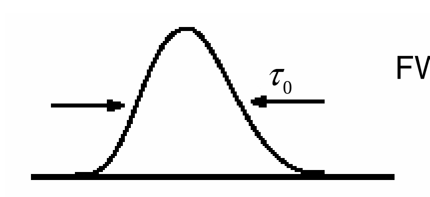
<u>pulse delay</u>: $\tau_d = \frac{z}{v}$

<u>pulse spreading</u>: $D_\nu = \frac{d}{d\nu}\left(\frac{1}{v_g}\right)$

$\sigma_\tau = |D_\nu| \sigma_\nu z$ temporal width

<u>Gaussian pulse</u>

$$A(0,t) = \exp -\left(\frac{t^2}{{\tau_0}^2}\right)$$



$\text{FWHM} = \tau_0$

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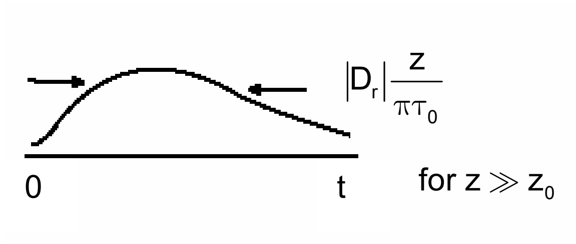
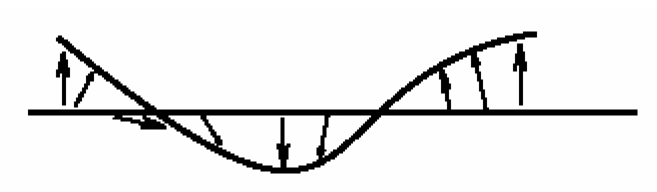
$$\tau_2 = \tau_0 \left[1 + \left(\frac{z}{z_0} \right)^2 \right]^{\frac{1}{2}}$$



Polarization

The time course of direction of $\vec{E}\left(\vec{r},t\right)$



Helical rotation of <u>circular polarization</u>

1. <u>Plane Polarization</u>

   $\vec{E}$ at fixed direction of $\vec{k}$

   $\vec{E}(z,t) = a_y \vec{y} e^{i(kz-\omega t)}$; $\omega = kc$

   <u>monochromatic light</u>

   $$\vec{E}\left(\vec{r},t\right) = \mathrm{Re}\left\{ \vec{A} \exp\left[i2\pi \left(t - \frac{z}{c} \right) \right] \right\}$$

   ν = frequency of photons

   z = direction of propagation

   c = phase velocity

Amplitude has $\vec{x}$ and $\vec{y}$ component:

$$\vec{A} = A_x \vec{x} + A_y \vec{y}$$

Lecture	Notes

$$\vec{E}(z,t) = E_x \vec{x} + E_y \vec{y}$$

$$\downarrow$$

$$a_x \cos\left[2\pi\nu\left(t - \frac{z}{c}\right) + \phi_x\right]$$

$\Rightarrow$ at fixed z, $\vec{E}$ rotates periodically in x-y plane

2. <u>General Solution</u>: elliptical polarization

$$\frac{E_x^2}{a_x^2} + \frac{E_y^2}{a_y^2} - 2\cos\phi \frac{E_x E_y}{a_x a_y} = \sin^2\phi$$

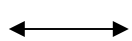
Matrix Representation

Matrix representation is a simplified way to perform first order calculations where small angles can be assumed. It can be used for order of magnitude calculations to obtain general values for a broad range of optical devices.

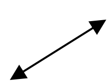
$$E = \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} \qquad \begin{matrix} A_x = a_x \;\; ^{i\phi x} \\ A_y = a_y e^{i\phi y} \end{matrix}$$

"Jones" vector: $\vec{J} = \begin{bmatrix} A_x \\ A_y \end{bmatrix} =$ operator on $\vec{E}$

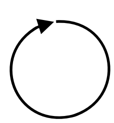
$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ linear poliarized in $\vec{x}$



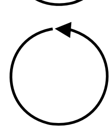
$\begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$ linear poliarized at θ to $\vec{x}$



$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ i \end{bmatrix}$ right circular



$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -i \end{bmatrix}$ left corner



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Linear polarization ≡ Σ (right + left circular)

$$\begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} = \frac{1}{\sqrt{2}} e^{-i\theta} + \frac{1}{\sqrt{2}} e^{i\theta}$$

Jones Transformation Matrix

$$\vec{J}_2 = \vec{T}\,\vec{\ }_1$$

$$\begin{pmatrix} A_{2x} \\ A_{2y} \end{pmatrix} = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} \begin{pmatrix} A_{1x} \\ A_{1y} \end{pmatrix}$$

<u>Linear Polarizer</u>

T = $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ (polarizes wave in x-direction)

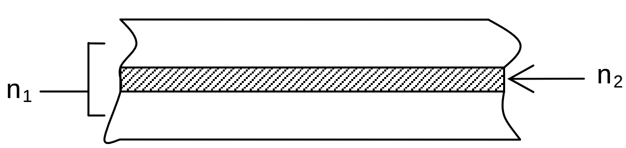
$A_{1x}, A_{1y} \rightarrow A_{1x}, 0$

$$\vec{E}_{out} = \vec{T}\,\vec{\ }_{in}$$

Guided Wave Optics – Introduction

- Free space
- Guided by <u>confinement</u> in <u>high refractive</u> index medium

<u>Optical wave guide</u> $n_2 > n_1$



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Notes

Planar Mirrors

TEM plane waves

$\lambda = \frac{\lambda_0}{n}$

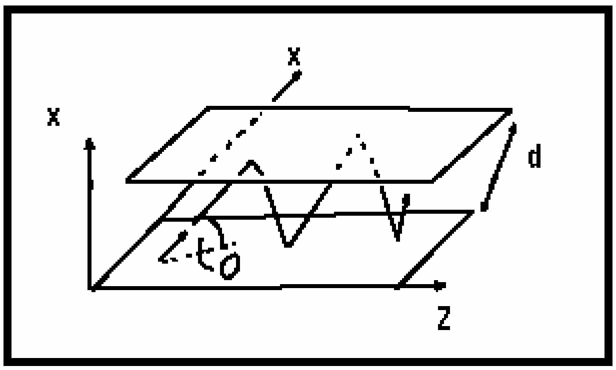
k = nk_0

$k = nk_0$

$c = \frac{c_0}{n}$

polarized in x-direction

$\vec{k}$ in y-z plane at θ to z-axis



1. $\vec{E} \,||\,$ mirror plane

2. each reflection → $\Delta\phi = \pi$ with $\vec{A}, |\vec{k}|$ unchanged

3. self-consistency: after two reflections, wave reproduces itself ≡ eigenmode of wave

   ⇒ "bounce angles" θ are discrete (quantized)

   $m\lambda = 2d\sin\theta_m$

$\vec{E}_m(y,z) = U_m(y)\exp(-i\beta_m z)$

$\beta = k_z = k\cos\theta$ propagation constant

= β_m (quantized)

= $k\cos\theta_m$

$U_m(y)$ = transverse distribution

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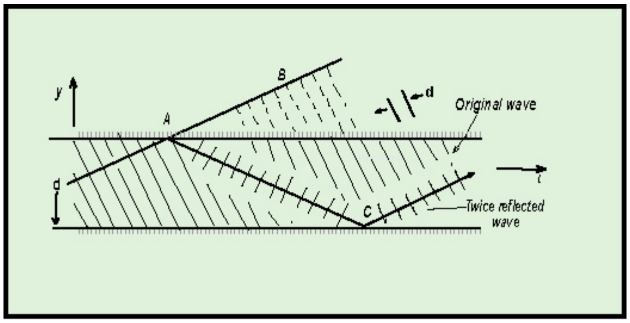
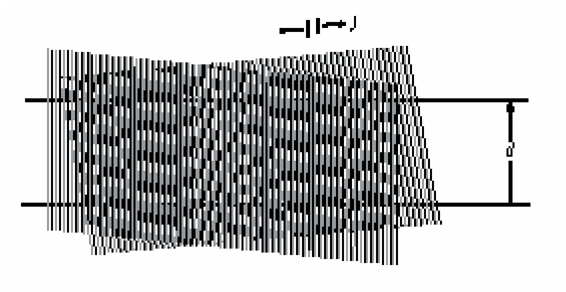


(a) Condition of self-consistency: as a wave reflects twice it duplicates itself



(b) At angles for which self-consistency is satisfied, the two waves interfere and create a wave that does not change with t.

<u>Optical power</u> $\propto |E|^2 \propto a_m^2$

<u>Number of Modes</u> M

$$M \geq \frac{2d}{\lambda}$$

M ↑ with d

$\lambda_{max} = 2d$: cut off λ

$\nu_{min} = \frac{c}{2d}$: cut off ν

d ≤ λ ≤ 2d <u>single mode</u>

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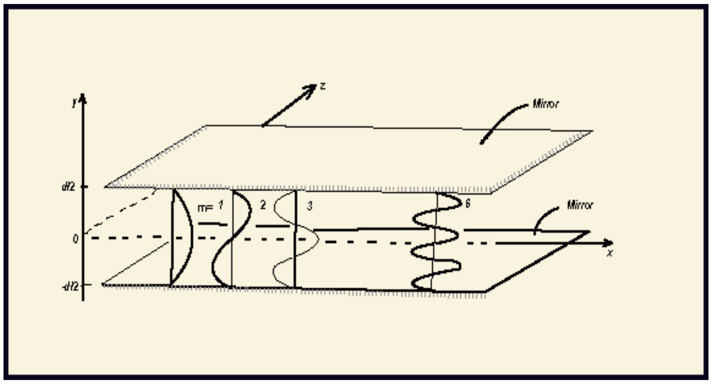

Field distributions of the modes of a planar-mirror waveguide

Group velocity of pulse

$$v_g = \frac{d\omega}{d\beta}$$

$$\beta_m^2 = \left(\frac{\omega}{c}\right)^2 - \frac{m^2\pi^2}{d^2} \quad \text{dispersion relation}$$

$$v_{mode} = \frac{d\omega}{d\beta_m} = c^2 \frac{\beta_m}{\omega}$$

$$= c^2 \frac{k \cos \theta_m}{\omega} = c \cdot \cos \theta_m$$

- longer zigzag path → slower group velocity
- different modes → different v_g → different transverse u(y) as wave propagates.

Notes