Unit 1: Real Numbers

Pset1 – Due September 17 (4 points each)

1. Prove Theorem I.11: If $ab = 0$ then $a = 0$ or $b = 0$
2. Prove Theorem I.25: If $a < c$ and $b < d$ then $a + b < c + d$.
3. Apostol page 43: 1j
4. Course Notes: A – Prove Theorem 6
5. Course Notes: A – Prove Theorem 12
6. Course Notes: A.10:6

Bonus: (Only to be attempted once other problems are completed) Let

$$A_n = \frac{x_1 + x_2 + \ldots + x_n}{n}, \quad G_n = (x_1 x_2 \ldots x_n)^{1/n}$$

represent the arithmetic and geometric mean, respectively, for a set of n positive real numbers.

- Prove that $G_n \leq A_n$ for $n = 2$.
- Use induction to show $G_n \leq A_n$ for any $n = 2^k$ where k is a positive integer.
- Now for any positive integer n, suppose $n < 2^m$ for some integer m. Using the set $\{x_1, x_2, \ldots, x_n, A_n, A_n, \ldots, A_n\}$ where the A_n appears $2^m - n$ times in the set, show that $G_n \leq A_n$.