

### Unit 3: Limits and Continuity - Week 1

#### Pset 3

Due September 30 (4 points each)

- (1) page 83:21
- (2) page 94:10
- (3) For step functions  $s(x), t(x)$  defined on  $[a, b]$  prove the Cauchy-Schwarz inequality:

$$\left( \int_a^b s(x)t(x)dx \right)^2 \leq \int_a^b s(x)^2 dx \cdot \int_a^b t(x)^2 dx.$$

Show that equality holds iff  $s(x) = ct(x)$  where  $c \in \mathbb{R}$ .

- (4) Bonus: Let  $B = \{x \in [0, 1] | x = m/2^n \text{ for some } m, n \in \mathbb{Z}\}$  Prove that the function

$$f(x) = \begin{cases} 1 & : x \in B \\ 0 & : x \notin B \end{cases}$$

is not integrable on  $[0, 1]$  by our definition of integrability.

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