Limits and Continuity - Week 2

Pset 4
Due October 8

(1) Page 138: 17, 18, 21 (1,1,and 2 points respectively)
(2) Let $A(x) = \int_{-2}^{x} f(t)dt$ where $f(t) = -1$ if $t < 0$ and $f(t) = 1$ if $t \geq 0$. Graph $y = A(x)$ for $x \in [-2, 2]$. Using ϵ, δ, show that $\lim_{x\to 0} A(x)$ exists and find its value. (You may want to draw yourself a picture of $|A(x) - A(0)|$ by considering the appropriate regions on a $t-y$ coordinate plane that contains the graph of $y = f(t)$. This will help you see geometrically how to write δ in terms of ϵ.)
(3) Notes F.2:2
(4) Suppose that g, h are two continuous functions on $[a, b]$. Suppose there exists $c \in (a, b)$ such that $g(c) = h(c)$. Define $f(x)$ such that $f(x) = g(x)$ for $x < c$ and $f(x) = h(x)$ for $x \geq c$. Prove that f is continuous on $[a, b]$.
(5) Let $f(x) = \sin(1/x)$ for $x \in \mathbb{R}, x \neq 0$. Show that for any $a \in \mathbb{R}$, the function $g(x)$ defined by

$$g(x) = \begin{cases} f(x) & : x \neq 0 \\ a & : x = 0 \end{cases}$$

is not continuous at $x = 0$.
(6) page 145:5

Bonus: Let f be a bounded function that is integrable on $[a, b]$. Prove that there exists $c \in \mathbb{R}$ with $a \leq c \leq b$ such that $\int_a^b f(x)dx = 2\int_a^c f(x)dx$.