Derivatives

Pset 4 (4 pts each)
Due October 15

(1) Notes H.8:2
(2) Notes H.9:6,7
(3) Page 155:8
(4) We define a set $A \subset \mathbb{R}$ to be dense in $\mathbb{R}$ if every open interval of $\mathbb{R}$ contains at least one element of A. Let A be a dense subset of $\mathbb{R}$. Let $f(x)$ be a continuous function such that $f(x) = 0$ for all $x \in A$. Prove that $f(x) = 0$ for all $x \in \mathbb{R}$.
(5) Let $f(x)$ be a continuous function on $[0, 1]$ and consider $w \in \mathbb{R}$. Show that there exists $z \in [0, 1]$ such that the distance between $(w, 0)$ and the curve $y = f(x)$ is minimized by $(z, f(z))$. (Hint: Notice I'm not telling you to find the value for z, just to show it exists. If you can figure out the right function to use and the right theorem to reference, this will be quick!)
(6) Page 173:7

Bonus: Notes H.10:10