EXAM 3 - NOVEMBER 19, 2010

(1) (10 points) Evaluate

$$\lim_{x\to 0}\left(\frac{1}{x}-\frac{1}{\log(x+1)}\right)$$

(2) (10 points) Evaluate

$$\int \frac{3x-2}{x^2-6x+10}dx$$

(3) (10 points) Let f be an infinitely differentiable function on $\mathbb{R}$. We say f is analytic on $(-1,1)$ if the sequence $\{T_n f(x)\}$ converges to $f(x)$ for all $x \in (-1,1)$, where $T_n f(x)$ is the nth Taylor polynomial of f centered at zero. Suppose there exists a constant $0 < C \leq 1$ such that

$$\left|f^{(k)}(x)\right| \leq C^k k!$$

for every positive integer k and every real number $x \in (-1,1)$. Prove that f is analytic on $(-1,1)$.

(4) (10 points) Let $f(x)$ be a function defined on $(0,\pi]$. Suppose $\lim_{n\to\infty} f(1/n) = 0$ and $\lim_{n\to\infty} f(\pi/n) = 1$. Prove that $\lim_{x\to 0^+} f(x)$ does not exist.

(5) A function f on $\mathbb{R}$ is compactly supported if there exists a constant $B > 0$ such that $f(x) = 0$ if $|x| \geq B$. If f and g are two differentiable, compactly supported functions on $\mathbb{R}$, then we define

$$(f * g)(x) = \int_{-\infty}^{\infty} f(x - y)g(y)dy.$$

Note: We define $\int_{-\infty}^{\infty} f(x)dx = \lim_{t\to\infty} \int_{-t}^{0} f(x)dx + \lim_{t\to\infty} \int_{0}^{t} f(x)dx$.

- (10 points) Prove $(f * g)(x) = (g * f)(x)$.

- (10 points) Prove $(f' * g)(x) = (g' * f)(x)$.

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