

PRACTICE EXAM 2

- (1) (10 points) Find

$$\lim_{h \rightarrow 0} \frac{\int_0^{1+h} e^{t^2} dt - \int_0^1 e^{t^2} dt}{h(3+h^2)}.$$

(If you're using a theorem, state the theorem you're using.)

- (2) (10 points) Find $(f^{-1})'(0)$ where $f(x) = \int_0^x \cos(\sin t) dt$ is defined on $[-\pi/2, \pi/2]$.
- (3) (10 points) In each case below, assume f is continuous for all x . Find $f(2)$.

$$\int_0^x f(t) dt = x^2(1+x); \quad \int_0^{f(x)} t^2 dt = x^2(1+x).$$

- (4) (15 points) Give an example of a function $f(x)$ defined on $[-1, 1]$ such that
- f is continuous and differentiable on $[-1, 1]$
 - f' is not continuous for at least one value of $x \in [-1, 1]$.
- (5) (15 points) Let $f(x)$ be continuous on $[0, 1]$ such that $f(0) = f(1)$. Show that for any $n \in \mathbb{Z}^+$ there exists at least one $x \in [0, 1]$ such that $f(x) = f(x + 1/n)$.

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