PRACTICE EXAM 3

(1) Evaluate $\int \frac{t^3+t}{\sqrt{1+t^2}} dt$

(2) Evaluate $\int_3^5 x^3\sqrt{x^2-9}dx$

(3) Suppose that $\lim_{x\to a^+} g(x) = B \neq 0$ where B is finite and $\lim_{x\to a^+} h(x) = 0$, but $h(x) \neq 0$ in a neighborhood of a. Prove that

$$\lim_{x\to a^+} \left|\frac{g(x)}{h(x)}\right| = \infty.$$

(4) Let $f(x) : [0,\infty) \to \mathbb{R}^+$ be a positive continuous function such that $\lim_{x\to\infty} f(x) = 0$. Prove there exists $M \in \mathbb{R}^+$ such that $\max_{x\in[0,\infty)} f(x) = M$.

(5)
- A sequence is called *Cauchy* if for all $\epsilon > 0$ there exists $N \in \mathbb{Z}^+$ such that for all $m, n > N$, $|a_m - a_n| < \epsilon$. Prove that if $\{a_n\}$ is a convergent sequence, then it is Cauchy. (The converse is also true.)

- A function $f : \mathbb{R} \to \mathbb{R}$ is called a *contraction* if there exists $0 \leq \alpha < 1$ such that $|f(x)-f(y)| \leq \alpha|x-y|$. Let f be a contraction. For any $x \in \mathbb{R}$, prove the sequence $\{f^n(x)\}$ is Cauchy, where $f^n(x) = f\circ f\circ\cdots\circ f(x)$ (the n times composition of f with itself).