

## Types of Riemann Sums

**Question:** For Riemann sums, what's the difference between upper and lower, and right and left?

**Answer:** If you take a function like  $f(x) = \frac{1}{x}$  and break it up into pieces, the lower sum is the sum of the areas of rectangles which are always *lower* than the graph of the function; see Figure 1. The right sum is the sum of the areas

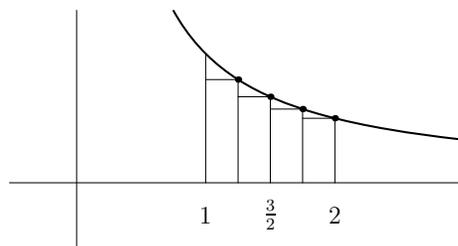


Figure 1: Lower sum.

of rectangles whose heights are  $f\left(\frac{(i+1)\Delta x}{n}\right)$ . If the values of  $f$  are positive, the upper right corner of each rectangle lies on the graph of  $f$ .

For a continuously decreasing function like  $\frac{1}{x}$ , the lower sum equals the right sum and the upper sum equals the left sum.

If the graph of the function “wiggles” up and down, the upper sum will be the sum of the areas of rectangles whose height is the maximum value the function achieves on the interval  $\left[\frac{i\Delta x}{n}, \frac{(i+1)\Delta x}{n}\right]$ . The maximum could be achieved at the left endpoint, the right endpoint or somewhere in the middle of the interval.

**Question:** If the function is increasing, then the lower sum is the left sum?

**Answer:** Correct. It's the exact opposite of the situation with an increasing function.

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