

## Log of a Product

**Claim:**  $L(ab) = L(a) + L(b)$ , where  $L(x) = \int_1^x \frac{dt}{t}$  is an alternately defined natural log function.

To prove this, we just plug in the formula and see what happens. On the left hand side we have:

$$L(ab) = \int_1^{ab} \frac{dt}{t} = \int_1^a \frac{dt}{t} + \int_a^{ab} \frac{dt}{t}$$

By definition,  $\int_1^a \frac{dt}{t} = L(a)$ . If we could show that  $\int_a^{ab} \frac{dt}{t} = L(b)$ , we'd be done with the proof.

It turns out that we can prove this by using a change of variables. We start with  $\int_a^{ab} \frac{dt}{t} = L(b)$ , and substitute  $t = au$  (so  $dt = a du$ ). The limits of integration are from  $u = 1$  to  $u = b$ . If we plug these into  $\int_a^{ab} \frac{dt}{t}$ , we get:

$$\int_a^{ab} \frac{dt}{t} = \int_{u=1}^{u=b} \frac{a du}{au} = \int_1^b \frac{du}{u} = L(b).$$

We can now conclude that:

$$L(ab) = L(a) + L(b)$$

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